7.1 Math Practise

7.1.1 Finite Series

1. Calculate $S_1 = 2 + 4 + 6 + ... + 100$.
2. Calculate $S_2 = 1 + 2 + 4 + 8 + 16 + ... + 1024$.
3. Calculate the binomials (n is an integer, use $0! = 1$)

$$\left(\begin{array}{c} 4 \\ 2 \end{array} \right), \quad \left(\begin{array}{c} n \\ n \end{array} \right), \quad \left(\begin{array}{c} n \\ 0 \end{array} \right), \tag{7.1}$$

4. Prove by induction:

$$\sum_{k=1}^{n} (2k-1) = n^2. \tag{7.2}$$

7.1.2 Infinite Series

1. Calculate

$$S_1 = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + ... \tag{7.3}$$

2. Calculate

$$S_2 = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} \pm ... \tag{7.4}$$

3. Calculate (a and b arbitrary)

$$S_3 = 1 + ab + (ab)^2 + (ab)^3 + (ab)^4 + ... \tag{7.5}$$

Which condition must be fulfilled for S_3 to converge?

7.1.3 Sum (Σ) Symbol

1. Simplify the expressions (a_k arbitrary)

$$S_1 = \sum_{k=0}^{n} a_k - \sum_{m=1}^{n+2} a_m, \quad S_2 = \sum_{k=m}^{n} a_k - \sum_{k=m+1}^{n+2} a_{k-1}, \quad n > m \tag{7.6}$$

7.2 Math Problems

7.2.1 Taylor Series

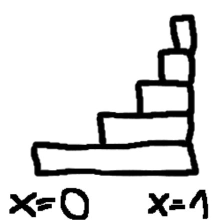
1. Calculate the Taylor series of $f(x) = e^x$ around $x = 0$.
2. Calculate the n–th derivative $(n = 0, 1, 2, 3, ...)$ of $f(x) = \sin(x)$ at $x = 0$. Use the result to calculate the Taylor series of $\sin(x)$ around $x = 0$.
3. Calculate the n–th derivative $(n = 0, 1, 2, 3, ...)$ of $f(x) = \cos(x)$ at $x = 0$. Use the result to calculate the Taylor series of $\cos(x)$ around $x = 0$.
4. (Knowledge question) What is the Taylor series of $f(x) = -7 + x^2 + 1000x^6$ around $x = 0$?

7.2.2 * Induction

Prove Bernoulli's inequality $(1 + x)^n \geq 1 + nx, \quad x \geq -1$ for all integers n by induction $n \to n + 1$.

7.3 Physics

7.3.1 ** Staircase



Calculate the center of mass coordinate $\mathbf{r}_N = (X_N, Y_N)$ of a staircase of $N > 1$ rectangular blocks, each one on top of one with double size but the same height (figure shows the case $N = 5$, all blocks have equal mass density).
Calculate $\lim_{N\to\infty} X_N$.

7.3.2 * Photon Torpedo

A photon 'torpedo' (Star Trek) is fired along the x–axis from $x = 0$ onto a looking–glass planet at $x = 1$ (right side). A part $0 < R < 1$ of the photon intensity is reflected to the left, a part $T = 1 - R$ of the intensity is transmitted to the right $x > 1$. The reflected part hits another looking–glass planet on the opposite side at $x = -1$, and again the part R of the torpedo gets reflected to the right (R^2 of the original intensity), and the part T is transmitted to the left side $x < -1$ (which is TR of the original intensity). The torpedo is thus bouncing back and forth between the two planets and gets weaker and weaker.
a) Make a sketch of this 'experiment'.
b) Calculate the total transmitted photon 'torpedo' intensity T_r on the right $(x > 1)$ and T_l on the left $(x < -1)$ side after infinitely many reflections. Show that there is a 'sum rule' for the sum $T_r + T_l$ of both total transmitted photon 'torpedo' intensities.