3rd-Year Projects 2004

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1. GENERAL

1.1 Requirements, Skills needed

- Readyness to work hard.
- A keen interest in Theoretical Physics.
- Good knowledge of Quantum Mechanics 1 (see my lecture notes).
- Good mathematical skills: longer algebraic manipulations, matrices, complex numbers, differential equations.
- Good computational skills: ability to write and test codes (Fortran, C, C++ or similar, this is up to you and your responsibility) for standard, smaller numerical problems like the numerical inversion of matrices with complex elements, reading in/out of data, producing 2d plots from numerically generated data, iterations etc.
- Theoretical skills: physical understanding of the theoretical models (Hamiltonians), being able to explain them in simple physical terms, being able to discuss possible improvements of these models or to develop new models. Being able to explain and assess the numerical results.

1.2 General Remarks

The level of my 3rd year projects is in general relatively high, but weaker students will always find a 'security net' built into the project that will allow them to successfully complete their project. Those who are more ambitious will soon realise (and like) the 'open-ended' research-style character of their projects.

2. PROJECT DESCRIPTIONS

2.1 Preparation (all projects)

2.1.1 Revision of quantum mechanical scattering in one dimension

(QM lecture notes) Transmission and reflection coefficient. Read the corresponding sections in the lecture notes. Back up with QM textbooks: Baym, Merzbacher.

2.1.2 TASK: Transmission Coefficient of a Delta Barrier

(QM lecture notes) Revise the properties of the Dirac Delta function $\delta(x)$. Consider the Hamiltonian

$$\mathcal{H} = \frac{p^2}{2m} + g\delta(x). \tag{2.1}$$

Write down the corresponding stationary Schrödinger for scattering states $\Psi(x)$ of energy E, derive the jump of the first derivative $\Psi'(x)$ at $x = 0$, and derive explicit expressions for the reflection and transmission coefficient as a function of E. You may use the reference [1] and John Robinson's project report [2].

2.2 Project 1: Spin-Dependent Scattering

2.2.1 Revision of spin $\frac{1}{2}$, two-by-two Pauli matrices

QM textbooks.

2.2.2 Model 1

Consider the Hamiltonian

$$\mathcal{H}_1 = \frac{p^2}{2m} + \frac{\omega_0}{2}\sigma_z + g\sigma_x\delta(x) \tag{2.2}$$

describing the interaction of a mass-m particle in one dimension (position x, momentum p) with a spin $\frac{1}{2}$. Write down the corresponding stationary Schrödinger equation for scattering states, i.e. two-component vectors (spinors) $\Psi_\uparrow(x)$ and $\Psi_\downarrow(x)$ of energy E. Generalise the approach from the usual Delta-barrier case to derive explicit expressions for the reflection and transmission coefficient as a function of E for various scattering conditions (incoming particle from the left).

2.2.3 Model 2

Carry out the same programme with the spin-dependent Fabry-Perot Hamiltonian

$$\mathcal{H}_2 = \frac{p^2}{2m} + \frac{\omega_0}{2}\sigma_z + g_1\sigma_x\delta(x - x_1) + g_2\sigma_x\delta(x - x_2). \tag{2.3}$$

2.2.4 Model 3

Will be discussed depending on progress made that far.

2.3 Project 2: Oscillating Fabry-Perot Resonator

2.3.1 Revision of harmonic oscillator

(QM lecture notes). Creation, annihilation operators etc.

2.3.2 Model

Consider the Hamiltonian

$$\mathcal{H} = \frac{p^2}{2m} + \omega a^\dagger a + g_1 \delta(x)[a^\dagger + a] + g_2 \delta(x - x_2), \tag{2.4}$$

describing the interaction of a mass-m particle in one dimension (position x, momentum p) with a single bosonic (harmonic oscillator) degree of freedom. Study in detail the calculations in [1] which correspond to the case $g_2 = 0$. Generalise these calculations to the case $g_2 \neq 0$ and calculate, in analogy to [1], the energy-dependent transmission coefficient.

2.4 Project 3: Oscillator and Delta-Barrier

2.4.1 Revision of harmonic oscillator

(QM lecture notes). Solution of stationary Schrödinger equation.

2.4.2 Model

Consider the Hamiltonian

$$\mathcal{H} = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + g\delta(x - x_0) \tag{2.5}$$

describing a mass-m particle in one dimension (position x, momentum p) within a harmonic potential with an additional delta barrier. Try to work out the stationary states and the eigenvalues of the energy.

2.4.3 Discussion

Time-evolution. Coherent states. More details to follow depending on progress that far.

BIBLIOGRAPHY

[1] T. Brandes and J. Robinson, phys. stat. sol. (b) **234**, 378 (2002).

[2] J. Robinson, 3rd year project report, cf. http://brandes.phy.umist.ac.uk/teaching, 2002.