

## Shot Noise Spectrum of Open Dissipative Quantum Two-Level Systems

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We study the current noise spectrum of qubits under transport conditions in a dissipative bosonic environment. We combine (non-)Markovian master equations with correlation functions in Laplace space to derive a noise formula for both weak and strong coupling to the bath. The coherence-induced reduction of noise is diminished by weak dissipation and/or a large level separation (bias). For weak dissipation, we demonstrate that the dephasing and relaxation rates of the two-level systems can be extracted from noise. In the strong dissipation regime, the localization-delocalization transition becomes visible in the low-frequency noise.

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The way a quantum two-level system (qubit) loses coherence due to the coupling with a noisy environment has been the subject of intense research for many years [1,2]. This fundamental problem has received a great deal of attention due to recent advances in solid state devices in which quantum two-level systems (TLS) have been realized using different degrees of freedom (charge, spin, and flux) [3]. Interest in current *noise* [4], in particular, in the presence of dephasing and dissipation [5], has risen owing to the possibility of extracting valuable information not available in conventional dc transport experiments.

In this Letter, we demonstrate that current noise in coupled quantum dots or Cooper pair boxes reveal the complete dissipative, internal dynamics of qubits coupled to external electron reservoirs. We develop a formalism that allows us to make quantitative predictions for the frequency ( $\omega$ ) dependent charge and current noise for *arbitrary* dissipative environments. We find a reduction of noise by coherent oscillations, weakened by increasing the bias or weak dissipation. The latter suppresses shot noise at  $\omega = 0$  and large bias due to spontaneous boson emission. Importantly, the dephasing and relaxation rates of the TLS can be extracted from noise. Our formulation includes non-Markovian memory effects [6] and the strong coupling limit, where we observe a reestablishing of the full shot noise due to the formation of polarons as new quasiparticles.

In the following, we assume that the TLS is defined in a double quantum dot (DQD) device [7,8]. We point out, however, that our method can also be applied to charge qubits realized in a Cooper pair (CP) box [9–11]; see below. DQD's in the regime of strong Coulomb blockade can be tuned into a regime that is governed by a (pseudo) spin-boson (SB) model (dissipative two-level system [1]), coupled to reservoirs [12]  $\mathcal{H} = \mathcal{H}_{\text{SB}} + \mathcal{H}_{\text{res}} + \mathcal{H}_T$ . Here,  $\mathcal{H}_{\text{SB}}$  describes one additional “transport” electron, which tunnels between a left ( $L$ ) and a right ( $R$ ) dot with energy difference  $\varepsilon$  and interdot coupling  $T_c$ , and

is coupled to a dissipative bosonic bath ( $\mathcal{H}_B = \sum_{\mathbf{Q}} \omega_{\mathbf{Q}} a_{\mathbf{Q}}^\dagger a_{\mathbf{Q}}$ ),

$$\mathcal{H}_{\text{SB}} = \left[ \frac{\varepsilon}{2} + \sum_{\mathbf{Q}} \frac{g_{\mathbf{Q}}}{2} (a_{-\mathbf{Q}} + a_{\mathbf{Q}}^\dagger) \right] \hat{\sigma}_z + T_c \hat{\sigma}_x + \mathcal{H}_B. \quad (1)$$

The effective Hilbert space of the closed system consists of two states  $|L\rangle = |N_L + 1, N_R\rangle$  and  $|R\rangle = |N_L, N_R + 1\rangle$ , such that the system is defined by a “pseudospin” [12]  $\hat{\sigma}_z \equiv |L\rangle\langle L| - |R\rangle\langle R| \equiv \hat{n}_L - \hat{n}_R$  and  $\hat{\sigma}_x \equiv |L\rangle\langle R| + |R\rangle\langle L| \equiv \hat{p} + \hat{p}^\dagger$ . The effects of the bath can be encapsulated in the spectral density  $J(\omega) \equiv \sum_{\mathbf{Q}} |g_{\mathbf{Q}}|^2 \delta(\omega - \omega_{\mathbf{Q}})$ , where  $\omega_{\mathbf{Q}}$  are the frequencies of the bosons and the  $g_{\mathbf{Q}}$  denote interaction constants. When showing results we use  $J(\omega) = 2\alpha\omega[1 - \omega_d/\omega \sin(\omega/\omega_d)]e^{-\omega/\omega_c}$  for piezoacoustic phonons in lateral DQD's with  $\omega_d$  depending on the geometry [12], or a generic Ohmic bath ( $\omega_d \rightarrow 0$ ):  $J(\omega) = 2\alpha\omega e^{-\omega/\omega_c}$ . The dimensionless parameter  $\alpha$  reflects the strength of dissipation and  $\omega_c$  is a high energy cutoff [2]. The coupling to reservoirs  $\mathcal{H}_{\text{res}} = \sum_{k_\alpha} \epsilon_{k_\alpha} c_{k_\alpha}^\dagger c_{k_\alpha}$  is described by  $\mathcal{H}_T = \sum_{k_\alpha} (V_k^\alpha c_{k_\alpha}^\dagger s_\alpha + \text{H.c.})$ , where  $\hat{s}_\alpha = |0\rangle\langle\alpha|$  ( $\alpha = L, R$ ), and the extra state  $|0\rangle = |N_L, N_R\rangle$  describes an “empty” DQD, such that  $1 = \hat{n}_0 + \hat{n}_L + \hat{n}_R$ .

The full model described by  $\mathcal{H}$  allows one to study nonequilibrium properties, such as the inelastic stationary current or current noise, through an open dissipative TLS. We describe its dynamics by a reduced, with respect to reservoirs, statistical operator  $\rho(t)$ . Introducing the vectors  $\mathbf{A} \equiv (\hat{n}_L, \hat{n}_R, \hat{p}, \hat{p}^\dagger)^T$  and  $\mathbf{\Gamma} = (\Gamma_L, 0, 0, 0)^T \equiv \Gamma_L \mathbf{e}_1$  and a matrix memory kernel  $M$ , the equations of motion (EOM) of the expectation values [13] (with  $\langle \hat{O} \rangle \equiv \sum_{i=0,L,R} \text{Tr}_{\text{bath}} \langle i | \hat{O} \rho(t) | i \rangle$ ) read in matrix form

$$\langle \mathbf{A}(t) \rangle = \langle \mathbf{A}(0) \rangle + \int_0^t dt' \{ M(t-t') \langle \mathbf{A}(t') \rangle + \mathbf{\Gamma} \}. \quad (2)$$

Equation (2) can be solved in Laplace space as  $\langle \hat{\mathbf{A}}(z) \rangle = [z - z\hat{M}(z)]^{-1} (\langle \mathbf{A}(0) \rangle + \mathbf{\Gamma}/z)$  and serves as a starting

point for the analysis of stationary ( $1/z$  coefficient in Laurent series for  $z \rightarrow 0$ ) and nonstationary quantities. The memory kernel has a block structure

$$z\hat{M}(z) = \begin{bmatrix} -\hat{G} & \hat{T}_c \\ \hat{D}_z & \hat{\Sigma}_z \end{bmatrix}, \quad \hat{G} \equiv \begin{pmatrix} \Gamma_L & \Gamma_L \\ 0 & \Gamma_R \end{pmatrix}, \quad (3)$$

where  $\hat{T}_c \equiv -iT_c(1 - \sigma_x)$ , and the coupling to the reservoirs within Born and Markov (BM) approximation with respect to  $\mathcal{H}_T$  [12,14] is given by  $\Gamma_\alpha = 2\pi \sum_{k_\alpha} |V_k^\alpha|^2 \delta(\epsilon - \epsilon_{k_\alpha})$  (we assume Fermi distributions for the reservoirs  $f_L = 1$  and  $f_R = 0$ ; large voltage regime). The blocks  $\hat{D}_z$  and  $\hat{\Sigma}_z$  are determined by the EOM for the coherences (off-diagonal elements)  $\langle \hat{p} \rangle = \langle \hat{p}^\dagger \rangle^*$  and contain the complete information on dephasing of the system. In general, no exact solution is available but we present approximate results now: for weak coupling to the bosons, one can use perturbation theory (PER) in  $\alpha$  in the correct basis of the hybridized states of the TLS. In BM approximation, the resulting expressions are

$$\hat{D}^{\text{PER}} = \hat{T}_c + \begin{pmatrix} \gamma_+ & -\gamma_- \\ \gamma_+ & -\gamma_- \end{pmatrix}, \quad \hat{\Sigma}^{\text{PER}} = \begin{pmatrix} E & 0 \\ 0 & E^* \end{pmatrix}, \quad (4)$$

where  $E = i\varepsilon - \gamma_p - \frac{\Gamma_R}{2}$ ,  $\gamma_p \equiv 2\pi \frac{T_c^2}{\Delta^2} J(\Delta) \coth(\beta\Delta/2)$ , and  $\gamma_\pm \equiv -\frac{\varepsilon T_c}{\Delta^2} \frac{\pi}{2} J(\Delta) \coth(\beta\Delta/2) \mp \frac{T_c}{\Delta} \frac{\pi}{2} J(\Delta)$  completely determine dephasing and relaxation in the system. Here,  $\Delta \equiv \sqrt{\varepsilon^2 + 4T_c^2}$  is the hybridization splitting and  $\beta = 1/k_B T$ .

On the other hand, for strong electron-boson coupling, one has to start from a polaron-transformed frame (strong coupling, POL), leading to an integral equation [12] which involves the boson correlation function  $C(t) \equiv \exp\{-\int_0^\infty d\omega \frac{J(\omega)}{\omega^2} [(1 - \cos\omega t) \coth(\frac{\beta\omega}{2}) + i \sin\omega t]\}$ . Introducing  $C_\varepsilon^{[*]}(z) \equiv \int_0^\infty dt e^{-zt} e^{[-i\varepsilon t]} C^{[*]}(t)$ , the resulting matrices in  $z$  space are

$$\hat{D}_z^{\text{POL}} = iT_c \begin{pmatrix} -1 & \hat{C}_{-\varepsilon}^*/\hat{C}_\varepsilon \\ 1 & -\hat{C}_{-\varepsilon}/\hat{C}_\varepsilon^* \end{pmatrix}, \quad \hat{\Sigma}_z^{\text{POL}} = \begin{pmatrix} \tilde{E} & 0 \\ 0 & \tilde{E}^* \end{pmatrix}, \quad (5)$$

with  $\tilde{E}^{[*]} \equiv z - 1/C_\varepsilon^{[*]}(z) - \Gamma_R/2$ . In contrast to the PER solution, where  $M(\tau) = M = z\hat{M}(z)$  is time independent,  $M^{\text{POL}}(\tau)$  is time dependent and  $z\hat{M}(z)$  depends on  $z$  in the POL approach [15]. We note that  $\text{Re}[C_\varepsilon(z)]|_{z=\pm i\omega} = \pi P(\varepsilon \mp \omega)$ , where  $P(\varepsilon)$  is the probability for inelastic tunneling with energy transfer  $\varepsilon$  [2].

As mentioned above, our model describes a CP box as well, the transport through the DQD being analogous to the Josephson quasiparticle cycle of the superconducting single electron transistor (SSET) with  $E_C \gg E_J$ , such that only two charge states,  $|2\rangle$  (one excess CP in the SSET) and  $|0\rangle$  (no extra CP), are allowed. Two consecutive quasiparticle events (with rates  $\Gamma_2$  and  $\Gamma_1$ ) couple  $|2\rangle$  and  $|0\rangle$  with another state  $|1\rangle$  through the cycle  $|2\rangle \rightarrow |1\rangle \rightarrow |0\rangle \Leftrightarrow |2\rangle$ . Interdot tunneling is analogous to coherent tunneling of a CP through one of the junctions, and

tunneling to and from the DQD is analogous to the two quasiparticle events through the probe junction in the SSET [11].

Current noise, which is described by the power spectral density  $S_I(\omega) \equiv 2 \int_{-\infty}^\infty d\tau e^{i\omega\tau} S_I(\tau) = \int_{-\infty}^\infty d\tau e^{i\omega\tau} \langle \{\Delta\hat{I}(\tau), \Delta\hat{I}(0)\} \rangle$ , with  $\Delta\hat{I}(t) \equiv \hat{I}(t) - \langle \hat{I}(t) \rangle$ , is a sensitive tool to study correlations between carriers [4]. The Fano factor ( $\gamma \equiv \frac{S_I(0)}{2qI}$ ) quantifies deviations from the Poissonian noise,  $S_I(0) = 2qI$ , which characterizes uncorrelated carriers with charge  $q$ . Importantly,  $S_I(\omega)$  has to be calculated from the autocorrelations of the *total* current  $I(t)$ , i.e., particle plus displacement current [4]. Using current conservation together with the Ramo-Shockley theorem,  $I(t) = aI_L(t) + bI_R(t)$  ( $a$  and  $b$ , with  $a + b = 1$ , depend on each junction capacitance [4]), one can express  $S_I(\omega)$  in terms of the spectra of particle currents and the charge noise spectrum  $S_Q(\omega)$  [16],

$$S_I(\omega) = aS_{I_L}(\omega) + bS_{I_R}(\omega) - ab\omega^2 S_Q(\omega). \quad (6)$$

Note that in symmetric configurations,  $a \approx b$ , the charge noise reduces the contribution from particle currents to the noise spectrum. For  $a \approx 1$  or  $b \approx 1$  the main contribution to noise comes from particle currents. At zero frequency  $S_I(0) = S_{I_L}(0) = S_{I_R}(0)$ .  $S_Q(\omega)$  is defined as

$$S_Q(\omega) \equiv \lim_{t \rightarrow \infty} \int_{-\infty}^\infty d\tau e^{i\omega\tau} \langle \{\hat{Q}(t), \hat{Q}(t + \tau)\} \rangle \\ = 2\text{Re}\{\hat{f}(z = i\omega) + \hat{f}(z = -i\omega)\}, \quad (7)$$

where  $\hat{Q} = \hat{n}_L + \hat{n}_R$  and  $\hat{f}(z)$  is the Laplace transform of

$$f(\tau) = \sum_{i,j=L,R} \langle \hat{n}_i(t) \hat{n}_j(t + \tau) \rangle \quad (8)$$

and can be evaluated with the help of the charge correlation functions  $\mathbf{C}_\alpha(\tau) \equiv \langle \hat{n}_\alpha(t) \mathbf{A}(t + \tau) \rangle$ , as  $f(\tau) = (\mathbf{e}_1 + \mathbf{e}_2)[\mathbf{C}_L(\tau) + \mathbf{C}_R(\tau)]$ . The EOM for  $\mathbf{C}_\alpha(\tau)$  can be obtained from the quantum regression theorem [17] whose solution is again expressed with the help of the resolvent  $[z - z\hat{M}(z)]^{-1}$ ; cf. Eq. (3).

To calculate the contribution of particle currents to noise, we need to relate the reduced dynamics of the qubit described by Eqs. (2) and (3) to reservoir operators. For  $S_{I_R}(\omega)$ , we introduce the number  $n$  of electrons that have tunneled through the right barrier [9,18,19] which defines generalized expectation values as  $O^{(n)} \equiv \sum_{i=0,L,R} \text{Tr}_{\text{bath}} \langle n, i | \hat{O} \rho(t) | n, i \rangle$  (such that  $\langle \hat{O} \rangle = \sum_n O^{(n)}$ ) and write

$$\dot{n}_0^{(n)} = -\Gamma_L n_0^{(n)} + \Gamma_R n_R^{(n-1)}, \quad (9) \\ \dot{n}_{L/R}^{(n)} = \pm \Gamma_{L/R} n_0^{(n)} \pm iT_c (p^{(n)} - [p^{(n)}]^\dagger)$$

and correspondingly for  $p^{(n)}$  and  $[p^{(n)}]^\dagger$  and the left barrier. Equations (9) allow one to calculate the particle current and the noise spectrum from  $P_n(t) = n_0^{(n)}(t) + n_L^{(n)}(t) + n_R^{(n)}(t)$ , which gives the total probability of finding  $n$  electrons in the collector by time  $t$ . In particular,

$I_R(t) = e \sum_n n \dot{P}_n(t)$  and  $S_{I_R}$  can be calculated from [20]

$$S_{I_R}(\omega) = 2\omega e^2 \int_0^\infty dt \sin(\omega t) \frac{d}{dt} [\langle n^2(t) \rangle - \langle n(t) \rangle^2], \quad (10)$$

where  $\frac{d}{dt} \langle n^2(t) \rangle = \sum_n n^2 \dot{P}_n(t) = \Gamma_R \sum_{n=0}^\infty n n_R^{(n)}(t) + \Gamma_R \sum_{n=0}^\infty n_R^{(n)}(t)$ . Solving Eqs. (9) with the initial condition  $n_R^{(n)}(0) = \delta_{n,0} n_R(0)$ , where  $n_R(0)$  is the stationary solution of Eqs. (2) [19], we get

$$S_{I_R}(\omega) = 2eI \{1 + \Gamma_R [\hat{n}_R(-i\omega) + \hat{n}_R(i\omega)]\}, \quad (11)$$

with  $z\hat{n}_R(z) = \Gamma_L g_+(z)/N(z)$ , where  $N(z) \equiv [z + \Gamma_R + g_-(z)][z + \Gamma_L] + (z + \Gamma_R + \Gamma_L)g_+(z)$  and

$$g_{+[-]}(z) = \pm iT_c(\mathbf{e}_1 - \mathbf{e}_2)[z - \hat{\Sigma}_z]^{-1} \hat{D}_z \mathbf{e}_1 [2]. \quad (12)$$

Equations (11) and (12) demonstrate the dependence of the current noise on the dephasing via the two-by-two blocks  $\hat{D}_z$  and  $\hat{\Sigma}_z$ ; cf. Eqs. (3)–(5). Explicitly,

$$g_{\pm}^{\text{PER}}(z) \equiv 2T_c \frac{T_c(\gamma_p + \Gamma_R/2 + z) - \varepsilon\gamma_{\pm}}{(\gamma_p + \Gamma_R/2 + z)^2 + \varepsilon^2}, \quad (13)$$

$$g_{+[-]}^{\text{POL}}(z) \equiv T_c^2 \left[ \frac{C_{[-]}^{[*]}(z)}{1 + \frac{\Gamma_R}{2} C_{\varepsilon}(z)} + (C \leftrightarrow C^*) \right].$$

A similar derivation yields  $S_{I_L}(\omega) = S_{I_R}(\omega)$ . The explicit expressions Eqs. (11)–(13), together with the inverse of a 4 by 4 matrix for the charge noise Eqs. (7), yield our key quantity  $S_I(\omega)$ , Eq. (6).

*Zero frequency (shot noise).*—In the zero-frequency limit  $z \rightarrow 0$ , one obtains

$$S_I(0) = 2eI \left( 1 + 2\Gamma_R \frac{d}{dz} [z\hat{n}_R(z)]_{z=0} \right). \quad (14)$$

Equation (14) allows one to investigate the shot noise of open dissipative TLS's for *arbitrary environments*. In contrast to noninteracting mesoscopic conductors, the noise cannot be written in the Khlus-Lesovik form  $S_I(0) = 2e^2 \int \frac{dE}{2\pi} t(E)[1 - t(E)]$  with an effective transmission coefficient  $t(E)$ . Without bath, we recover the results of Ref. [18] (shot noise of DQD's) and Ref. [11] (shot noise of the CP box). For  $\alpha = 0$  and  $\Gamma \equiv \Gamma_L = \Gamma_R$  [Fig. 1(a), solid line], the smallest Fano factor is reached for  $\varepsilon = 0$  where quantum coherence strongly suppresses noise. The maximum suppression ( $\gamma = 1/5$ ) is reached for  $\Gamma = 2\sqrt{2}T_c$ . For large  $\varepsilon > 0$  ( $\varepsilon < 0$ ) the charge becomes localized in the right (left) level,  $S_I(0)$  is dominated by only one Poisson process, namely, the noise of the right (left) barrier, and  $\gamma \rightarrow 1$ . For  $\alpha \neq 0$ , this mechanism is strongly affected by the possibility of exchanging energy quanta with the bath. The effect of a bosonic bath on noise in a mesoscopic scatterer was first discussed in [21]. For the TLS discussed here, spontaneous emission (for  $\varepsilon > 0$ ) occurs even at very low temperatures [8,12] and the noise is reduced [5] well below the Poisson limit [Fig. 1(a)]. The maximum suppression is now reached when the elastic and inelastic rates coincide, i.e.,  $\gamma_p =$

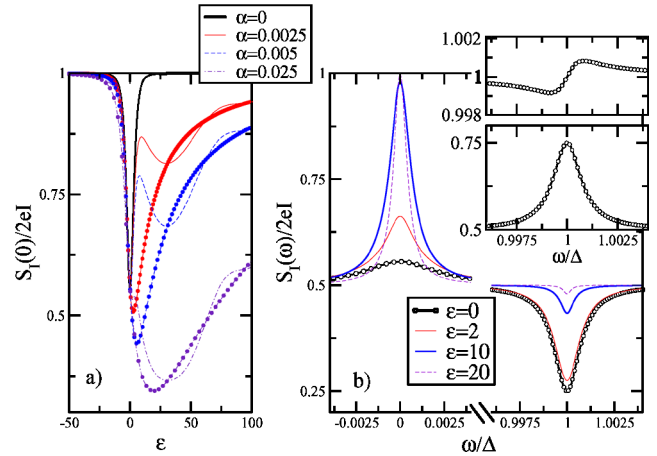


FIG. 1 (color online). (a) Fano factor vs bias  $\varepsilon$  for different dissipative couplings  $\alpha$ . Parameters  $T_c = 3$ ,  $\Gamma = 0.15$ ,  $\omega_c = 500$ ,  $\omega_d = 10$ , and  $T = 2$  (in  $\mu\text{eV}$ ) correspond to typical experimental values [8] in double quantum dots. Lines: acoustic phonons; circles: generic Ohmic environment  $\omega_d = 0$  (see the text). (b) Frequency dependent current noise ( $\alpha = 0$ ,  $T = 0$ , and  $\Gamma = 0.01$ ). Inset: (top) contribution to noise from particle currents  $S_{I_R}(\omega)/2eI$ ; (bottom) charge noise contribution  $\omega^2 S_Q(\omega)/8eI$ .  $a = b = 1/2$ .

$\Gamma_R$ , as we have checked numerically. For large couplings, spontaneous emission leads to a very asymmetric Fano factor that goes from  $\gamma \approx 1$  to  $\gamma \approx 0.5$  as  $\varepsilon$  changes sign (not shown here).

*Finite frequencies.*—For finite  $\omega$ , the numerical results for  $\alpha = 0$  are shown in Fig. 1(b) where we plot  $S_I(\omega)$  for different values of  $\varepsilon$ . The background noise is half the Poisson value as one expects for a symmetric structure.  $\gamma$  deviates from this value around  $\omega = 0$  where the noise has a peak and  $\omega = \Delta$  where the noise is suppressed. The dip in the Fano factor directly reflects the resonance of the subtracted charge noise  $S_Q(\omega)$  around  $\Delta$  [inset Fig. 1(b)]; cf. Eq. (6). An increase of  $\varepsilon$  localizes the qubit and, thus, the zero-frequency noise reaches  $\gamma \rightarrow 1$ . Moreover, the dip in the high frequency noise at  $\omega = \Delta$  [Fig. 1(b)] is progressively destroyed (reduction of quantum coherence) as  $\varepsilon$  increases which is consistent with the previous argument. A similar reduction of the dip at  $\omega = \Delta$  occurs at fixed  $\varepsilon$  and  $\Gamma$  with increasing dissipation (Fig. 2) in the weak coupling (PER) regime. This behavior demonstrates that  $S_I(\omega)$  reveals the complete internal dissipative dynamics of the TLS. The above argument can be further substantiated by plotting also the symmetrized pseudospin correlation function  $S_z(\omega) = 1/2 \int_{-\infty}^\infty d\omega e^{i\omega\tau} \langle \{\hat{\sigma}_z(\tau), \hat{\sigma}_z\} \rangle$  (Fig. 2, right inset) which is commonly used to investigate the dynamics of the SB problem [2]. Both functions reflect in the same fashion how the coherent dynamics of the system progressively gets damped by the bosonic bath.

In particular, the *dephasing rate* can be extracted from the half-width of  $S_I(\omega)$  around  $\omega = \Delta$ . For an Ohmic environment,  $\gamma_d^b = \gamma_p/2 + 2\pi\alpha(\frac{\varepsilon}{\Delta})^2 k_B T$ , such

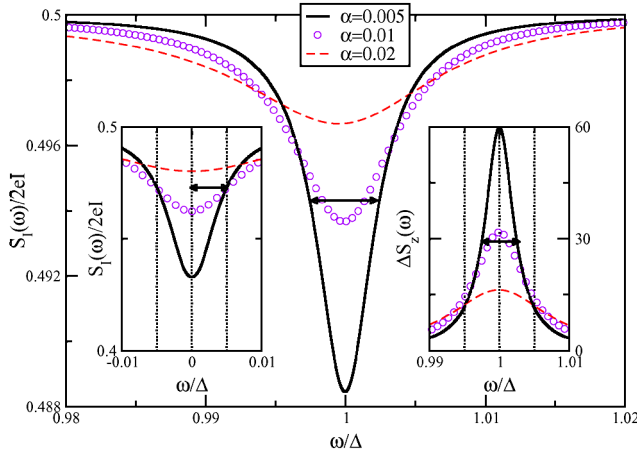


FIG. 2 (color online). Effect of Ohmic dissipation on current noise near resonance ( $\varepsilon = 10$ ,  $\Gamma = 0.01$ , and  $\alpha = 0.005, 0.01, 0.02$  corresponding to  $\gamma_p \approx 4.74\Gamma, 9.47\Gamma, 18.95\Gamma$ ). Right inset: pseudospin correlation function  $S_z(\omega)$ . Arrows indicate relaxation rate  $(\Gamma + \gamma_p)/\Delta \approx 0.005$  for  $\alpha = 0.005$ . Left inset: low frequency region near shot noise limit  $\omega = 0$ .

that the total dephasing rate is  $\gamma_d(T=0) = \gamma_d^b + \Gamma/2 = (\gamma_p + \Gamma)/2$  (Fig. 2, arrows denote full-width, i.e.,  $2\gamma_d \approx \gamma_p$  as  $\alpha$  increases). Close to  $\omega = 0$ , the peak in  $S_I(\omega)$  for  $\alpha = 0$  changes into a dip around  $\omega = 0$  reflecting incoherent relaxation dynamics for  $\alpha \neq 0$ . The half-width is now given by the relaxation rate such that the full-width of  $S_I(\omega)$  around  $\omega = 0$  is twice that of the high frequency noise (Fig. 2, left inset).

The results for the strong coupling (POL) regime are presented in Fig. 3. Near  $\omega = 0$ , POL and PER yield nearly identical results for the noise  $S_I(\omega)$  at very small  $\alpha$  (not shown here). The crossover to Poissonian noise near  $\omega = 0$  with increasing  $\alpha$  indicates the formation of localized polarons. The delocalization-localization transition [1,2] of the spin-boson model at  $\alpha = 1$  is reflected in a change of the analyticity of  $C_\varepsilon$  and the shot noise near zero bias (Fig. 3, inset). Similar physics has been found

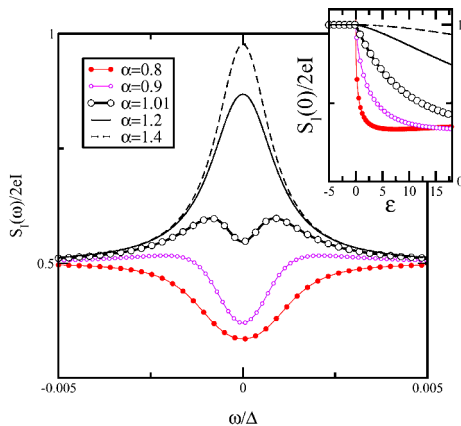


FIG. 3 (color online). Low frequency current noise in qubit with strong Ohmic dissipation ( $T = 0$ ,  $\varepsilon = 10$ ,  $T_c = 3$ , and  $\Gamma_L = \Gamma_R = 0.01$ ). Inset: shot noise for  $\omega = 0$ .

recently in the suppression of the persistent current  $I(|\varepsilon|) \propto \text{Im}C_{-|\varepsilon|}$  through a strongly dissipative quantum ring containing a quantum dot with bias  $\varepsilon$  [22]. Although POL becomes less reliable for  $\alpha < 1$  and smaller bias, the nonsymmetry in  $\varepsilon$  of the shot noise and the inelastic current  $\propto \text{Re}C_\varepsilon$  reflects the “open” topology of our TLS in the nonlinear transport regime.

To conclude, our results demonstrate that frequency-dependent current noise provides detailed information about the internal, dissipative dynamics of an open quantum two-level system such as double quantum dots or Cooper pair boxes. The weak coupling regime should be close to current experiments [23] in these systems, where we expect our predictions to be tested in the near future.

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