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Current fluctuation spectrum in dissipative solid-state qubits

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Abstract. We present a formalism to calculate frequency dependent electron current noise for transport through two-level systems (such as coupled quantum dots or Cooper-pair boxes) in presence of dissipation. Perturbation theories in various regimes are formulated within a matrix scheme in Laplace scheme which we evaluate in detail both for weak and strong coupling to a bosonic environment.

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1 Introduction

Electronic current noise has shot to prominence as a valuable tool for extracting information not available in conventional dc transport experiments. This is in particular true in mesoscopic systems, where energy relaxation and the loss of phase coherence is crucial for the understanding of transport properties. Transport through artificial few-level systems (coupled quantum dots, small superconducting junctions) has received a great deal of attention due to possible applications for quantum information processing.

In this contribution, we present a theoretical investigation of current noise in one of the simplest quantum systems, i.e., a quantum two-level system (TLS or qubit) coupled to a dissipative environment and to electron reservoirs. We have developed a formalism in order to calculate the full, frequency (ω) dependent charge and current noise spectrum for *arbitrary* coupling to a thermal, dissipative environment. One of our main results is the extraction of dephasing and relaxation rates of the TLS from the noise. We formulate two different perturbative regimes (weak dissipation or weak tunneling) within a single matrix formalism that allows to explore noise in regimes of different coupling strengths.

2 Model

Double quantum dots (DQD) in the strong Coulomb blockade regime [1–4] and Cooper pair boxes [5–7] can be tuned into a regime that is governed by an 'open' version of the spin–boson model (dissipative two–level system [8,9])

$$H = \frac{\varepsilon}{2}\sigma_z + T_c\sigma_x + \frac{1}{2}\sigma_z A + \sum_{\mathbf{Q}} \omega_Q a^\dagger_{\mathbf{Q}} a_{\mathbf{Q}}$$
$$A := \sum_{\mathbf{Q}} g_Q \left(a_{-\mathbf{Q}} + a^\dagger_{\mathbf{Q}}\right), \qquad (1)$$

where one additional 'transport' electron tunnels between a left (L) and a right (R) dot with energy difference ε and inter–dot coupling T_c, where $\sigma_z = |L\rangle\langle L| - |R\rangle\langle R|$ and $\sigma_x = |L\rangle\langle R| + |R\rangle\langle L|$. Our model describes a Cooper-pair box as well, the transport through the DQD being analogous to the Josephson Quasiparticle Cycle (JQP) of the superconducting single electron transistor (SSET) where the charging energy is much larger than the Josephson energy, $E_C \gg E_J$, such that only two charge states, $|2\rangle$ (one excess Cooper pair in the SSET) and $|0\rangle$ (no extra Cooper pair), are allowed, such that $\sigma_z = |0\rangle\langle 0| - |2\rangle\langle 2|$ and $\sigma_x = |0\rangle\langle 2| + |2\rangle\langle 0|$, with $2T_c \to E_J$ and $\varepsilon \to 4E_C(2n_g - 1)$ (n_g is the total polarization charge applied to the gate electrode [5–7]). The last two terms in equation (1) appear due to the coupling with the bosonic bath, where ω_Q are the frequencies of bosons, and the g_Q denote interaction constants. Although not exactly solvable, the model is quite well understood for closed systems [8] (isolated dots with one additional electron). The coupling to external leads offers the possibility to study its non–equilibrium properties.

2.1 Equations of motion

We describe the dynamics of the two level system by a reduced statistical operator $\rho(t)$, allowing for an additional

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'empty' state. In the DQD case, this describes tunneling from a left reservoir at rate Γ_L into the left dot, and from the right dot to the right reservoir at rate Γ_R. In the Cooper-pair box case, the extra 'empty' state $|1\rangle$ is needed to describe two consecutive quasiparticle events (with rates Γ_2 and Γ_1) through the cycle $|2\rangle \to |1\rangle \to |0\rangle \Leftrightarrow |2\rangle$. For simplicity, we use in the following the DQD language. The coupling to the reservoirs within Born and Markov (BM) approximation with respect to $\mathcal{H}_T$ [2,20] yields tunneling rates $\Gamma_\alpha = 2\pi \sum_{k_\alpha} |V_k^\alpha|^2 \delta(\epsilon - \epsilon_{k_\alpha})$ (we assume Fermi distributions for the reservoirs $f_L = 1$ and $f_R = 0$; large voltage regime). Then, second order perturbation theory in $\mathcal{H}_T$ becomes exact, and one obtains

$$\begin{aligned}\frac{\partial}{\partial t}\rho_{LL}(t) &= -iT_c\left[\rho_{LR}(t) - \rho_{RL}(t)\right] \\ &\quad + \Gamma_L\left[1 - \rho_{LL}(t) - \rho_{RR}(t)\right] \\ \frac{\partial}{\partial t}\rho_{RR}(t) &= -iT_c\left[\rho_{RL}(t) - \rho_{LR}(t)\right] - \Gamma_R\rho_{RR}(t).\end{aligned}$$

For the remaining equation for the off–diagonal element $\rho_{LR} = \rho_{RL}^*$, one has to choose between perturbation theory in g_Q (weak coupling, PER), or in T_c in a polaron–transformed frame (strong coupling, POL). In general, the equations of motion can be written in matrix form

$$\langle \mathbf{A}(t)\rangle = \langle \mathbf{A}(0)\rangle + \int_0^t dt' \left\{M(t-t')\langle \mathbf{A}(t')\rangle + \mathbf{\Gamma}\right\} \quad (2)$$

with the matrix memory kernel M, the expectation value of the vector $\mathbf{A} \equiv (\hat{n_L}, \hat{n_R}, \hat{p}, \hat{p}^\dagger)$, and $\mathbf{\Gamma} = \Gamma_L \mathbf{e}_1$. We will take advantage of this form further below. Note that the stationary expectation values simply are

$$\langle \mathbf{A}(t \to \infty)\rangle = -M^{-1}\mathbf{\Gamma} \quad (3)$$

which follows from differentiating equation (2).

Some general remarks are in order here. No exact solution of the above model is available: this is the case even for coupling to one bosonic mode only ($g_Q \propto \delta_{Q,Q_0}$, Rabi Hamiltonian). Furthermore, for the spin–boson problem with $\Gamma_{R/L} = 0$, it is well–known that POL is equivalent to a double–path integral 'non-interacting blip approximation' (NIBA) that works well for zero bias $\varepsilon = 0$ but for $\varepsilon \neq 0$ does not coincide with PER at small couplings and low temperatures. We have compared both approaches for $\Gamma_{R/L} \neq 0$ and found [3] nearly perfect agreement for very large $\varepsilon \gg T_c$, a regime that has been tested experimentally recently [1].

2.2 Perturbation theory in the electron-boson coupling

The standard Born and Markov approximation with respect to A yields

$$\begin{aligned}\frac{d}{dt}\rho_{LR}^{\mathrm{PER}}(t) &= \left[i\varepsilon - \gamma_p - \Gamma_R/2\right]\rho_{LR}(t) + \left[iT_c - \gamma_-\right]\rho_{RR}(t) \\ &\quad - \left[iT_c - \gamma_+\right]\rho_{LL}(t).\end{aligned}$$

Here, the rates are

$$\begin{aligned}\gamma_p &:= 2\pi\frac{T_c^2}{\Delta^2}J(\Delta)\coth(\beta\Delta/2) \\ \gamma_\pm &:= -\frac{\varepsilon T_c}{\Delta^2}\frac{\pi}{2}J(\Delta)\coth(\beta\Delta/2) \mp \frac{T_c}{\Delta}\frac{\pi}{2}J(\Delta) \\ J(\omega) &:= \sum_{\mathbf{Q}} |g_Q|^2\delta(\omega - \omega_Q).\end{aligned} \quad (4)$$

where $\Delta := \sqrt{\varepsilon^2 + 4T_c^2}$ is the energy difference of the hybridized levels, and $\beta = 1/k_B T$ the inverse boson equilibrium bath temperature. Note that beside the off–diagonal decoherence rate γ_p, there appear terms $\propto \gamma_\pm$ in the diagonals which below turn out to be important for the stationary current.

The effects of the bath are encapsulated in the spectral density $J(\omega)$, where ω_Q are the frequencies of the bosons and the g_Q denote interaction constants. When showing results we will be using a generic Ohmic bath,

$$J(\omega) = 2\alpha\omega e^{-\omega/\omega_c}. \quad (5)$$

The dimensionless parameter α reflects the strength of dissipation and ω_c is a high energy cutoff [9].

2.3 Polaron transformation

The polaron transformation [2] leads to an integral equation

$$\begin{aligned}\rho_{LR}^{POL}(t) = -\int_0^t dt' e^{i\varepsilon(t-t')}\Bigg[&\frac{\Gamma_R}{2}C(t-t')\rho_{LR}(t') \\ &+ iT_c\left\{C(t-t')\rho_{LL}(t') - C^*(t-t')\rho_{RR}(t')\right\}\Bigg],\end{aligned}$$

where $C(t)$ is an equilibrium correlation function with respect to the bosonic bath (inverse temperature $\beta = 1/k_B T$, spectral density $\rho(\omega)$, Eq. (4)),

$$\begin{aligned}C(t) &:= \langle X(t)X^\dagger\rangle_B = e^{-\Phi(t)} \\ \Phi(t) &= \int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\left[(1-\cos\omega t)\coth\left(\frac{\beta\omega}{2}\right) + i\sin\omega t\right].\end{aligned} \quad (6)$$

In this case, the matrix $M(\tau)$ is time-dependent. We define the Laplace transform

$$\hat{C}(z) = \int_0^\infty dt e^{-zt}C(t). \quad (7)$$

Laplace-transforming equation (6), we find

$$\begin{aligned}\hat{\rho}_{LR}(z) &= -\frac{\Gamma_R}{2}\hat{C}_\varepsilon(z)\hat{\rho}_{LR}(z) \quad (8) \\ &\quad - iT_c\hat{C}_\varepsilon(z)\hat{\rho}_{LL}(z) + iT_c\hat{C}^*_{-\varepsilon}(z)\hat{\rho}_{RR}(z) \\ \hat{\rho}_{RL}(z) &= -\frac{\Gamma_R}{2}\hat{C}^*_\varepsilon(z)\hat{\rho}_{RL}(z) \quad (9) \\ &\quad + iT_c\hat{C}^*_\varepsilon(z)\hat{\rho}_{LL}(z) - iT_c\hat{C}_{-\varepsilon}(z)\hat{\rho}_{RR}(z), \quad (10)\end{aligned}$$

where we abbreviated $\hat{C}_\varepsilon(z) = \hat{C}(z - i\varepsilon)$, $\hat{C}_{-\varepsilon}(z) = \hat{C}(z + i\varepsilon)$, $\hat{C}^*_{-\varepsilon}(z) = [\hat{C}(z^* + i\varepsilon)]^*$, and $\hat{C}^*_\varepsilon(z) = [\hat{C}(z^* - i\varepsilon)]^*$. We re-arrange these equations into

$$z\hat{\rho}_{LR}(z) = \left[z - \frac{1}{C_\varepsilon(z)} - \frac{\Gamma_R}{2}\right]\hat{\rho}_{LR}(z) - iT_c\hat{\rho}_{LL}(z) + iT_c\frac{C^*_{-\varepsilon}(z)}{C_\varepsilon(z)}\hat{\rho}_{RR}(z) \quad (11)$$

$$z\hat{\rho}_{RL}(z) = \left[z - \frac{1}{C^*_\varepsilon(z)} - \frac{\Gamma_R}{2}\right]\hat{\rho}_{LR}(z) \quad (12)$$

$$+ iT_c\hat{\rho}_{LL}(z) - iT_c\frac{C_{-\varepsilon}(z)}{C^*_\varepsilon(z)}\hat{\rho}_{RR}(z). \quad (13)$$

Note that in the limit of vanishing electron-boson coupling, $C_\varepsilon(z) = C^*_{-\varepsilon}(z) = (z - i\varepsilon)^{-1}$. Furthermore, we have

$$\mathrm{Re}[C_\varepsilon(z)]|_{z=\pm i\omega} = \pi P(\varepsilon \mp \omega), \quad (14)$$

where $P(\varepsilon)$ is the probability for inelastic tunneling with energy transfer ε [9]. For Ohmic dissipation at $T = 0$, one has

$$C_\varepsilon(0) = -i/\omega_c(-\varepsilon/\omega_c)^{2\alpha-1}e^{-\varepsilon/\omega_c}\Gamma(1-2\alpha, -\varepsilon/\omega_c), \quad (15)$$

such that

$$P(\varepsilon) = (\varepsilon/\omega_c)^{2\alpha-1}e^{-\varepsilon/\omega_c}\theta(\varepsilon)/(\omega_c\Gamma(2\alpha)). \quad (16)$$

Here, $\Gamma(x, y)$ is the incomplete Gamma function.

2.4 Matrix notation

We can introduce a convenient matrix notation that comprises both the PER and the POL case. Equation (2) can be solved in Laplace space as

$$\langle\hat{\mathbf{A}}(z)\rangle = [z - z\hat{M}(z)]^{-1}(\langle\mathbf{A}(0)\rangle + \mathbf{\Gamma}/z) \quad (17)$$

and serves as a starting point for the analysis of stationary ($1/z$ coefficient in Laurent series for $z \to 0$) and non-stationary quantities. The memory kernel has a block structure

$$z\hat{M}(z) = \begin{bmatrix}-\hat{G} & \hat{T}_c\\ \hat{D}_z & \hat{\Sigma}_z\end{bmatrix}, \quad \hat{G} \equiv \begin{pmatrix}\Gamma_L & \Gamma_L\\ 0 & \Gamma_R\end{pmatrix}, \quad (18)$$

where $\hat{T}_c \equiv -iT_c(1 - \sigma_x)$.

In the Born-Markov (PER) approximation, the resulting expressions are:

$$\hat{D}^{\mathrm{PER}} = \hat{T}_c + \begin{pmatrix}\gamma_+ & -\gamma_-\\ \gamma_+ & -\gamma_-\end{pmatrix}, \quad \hat{\Sigma}^{\mathrm{PER}} = \begin{pmatrix}E & 0\\ 0 & E^*\end{pmatrix}, \quad (19)$$

where $E = i\varepsilon - \gamma_p - \frac{\Gamma_R}{2}$ and the rates equation (4) completely determine dephasing and relaxation in the system.

In the POL (strong coupling) approximation, the resulting matrices in z-space are

$$\hat{D}_z^{\mathrm{POL}} = iT_c\begin{pmatrix}-1 & \hat{C}^*_{-\varepsilon}/\hat{C}_\varepsilon\\ 1 & -\hat{C}_{-\varepsilon}/\hat{C}^*_\varepsilon\end{pmatrix}, \quad \hat{\Sigma}_z^{\mathrm{POL}} = \begin{pmatrix}\tilde{E} & 0\\ 0 & \tilde{E}^*\end{pmatrix}, \quad (20)$$

with $\tilde{E}^{[*]} \equiv z - 1/C^{[*]}_\varepsilon(z) - \Gamma_R/2$. In contrast to the PER solution, where $M(\tau) = M = z\hat{M}(z)$ is time-independent, $M^{\mathrm{POL}}(\tau)$ is time-dependent and $z\hat{M}(z)$ depends on z in the POL approach.

3 Spectral density of the current fluctuations

Usually, current noise is described by the power spectral density

$$\mathcal{S}_I(\omega) \equiv 2\int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\mathcal{S}_I(\tau) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle\{\Delta\hat{I}(\tau), \Delta\hat{I}(0)\}\rangle$$
$$\Delta\hat{I}(t) \equiv \hat{I}(t) - \langle\hat{I}(t)\rangle. \quad (21)$$

However, quantum noise can be asymmetric in the frequency ω due to the non-commutativity of current operators, such that the noise spectrum may be defined as:

$$\mathcal{S}_I^{asym}(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle\hat{I}(\tau), \hat{I}(0)\rangle - \langle\hat{I}\rangle\langle\hat{I}\rangle. \quad (22)$$

It has been recently shown that such an asymmetric quantum shot noise spectrum can be detected in situations where a quantum of energy $\hbar\omega$ is transferred from the system to the measurement apparatus [21–23] which was demonstrated experimentally [16].

In order to keep our discussion as general as possible we consider here the *symmetrized* version of the quantum noise, namely equation (21), and leave the study of *non-symmetrized* noise for future research.

We define the Fano factor

$$\gamma \equiv \frac{\mathcal{S}_I(0)}{2qI}, \quad (23)$$

which quantifies deviations from the Poissonian noise, $\mathcal{S}_I(0) = 2qI$ (uncorrelated carriers with charge q).

3.1 Current conservation, quantum regression theorem

To calculate $\mathcal{S}_I(\omega)$, we need to relate the reduced dynamics of the qubit to reservoir operators like the current operator. Note that $S_I(\omega)$ has to be calculated from the autocorrelations of the *total* current I, i.e. particle plus displacement current [24]. Using current conservation together with the Ramo-Shockley theorem,

$$I = aI_L + bI_R \quad (24)$$

(a and b, with $a + b = 1$, depend on each junction capacitance [24]), one can express $\mathcal{S}_I(\omega)$ in terms of the spectra of particle currents and the charge noise spectrum $S_Q(\omega)$,

$$S_I(\omega) = aS_{I_L}(\omega) + bS_{I_R}(\omega) - ab\omega^2 S_Q(\omega). \quad (25)$$

Here, the charge-charge correlation function $S_Q(\omega)$ is defined as

$$S_Q(\omega) \equiv \lim_{t\to\infty} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle \{\hat{Q}(t), \hat{Q}(t+\tau)\}\rangle = 2\mathrm{Re}\left\{\hat{f}(z=i\omega) + \hat{f}(z=-i\omega)\right\}, \quad (26)$$

where $\hat{Q} = \hat{n}_L + \hat{n}_R$ and $\hat{f}(z)$ is the Laplace transform of

$$f(\tau) = \sum_{i,j=L,R} \langle \hat{n}_i(t)\hat{n}_j(t+\tau)\rangle. \quad (27)$$

This is evaluated with the help of the charge correlation functions,

$$\mathbf{C}_\alpha(\tau) \equiv \langle \hat{n}_\alpha(t)\mathbf{A}(t+\tau)\rangle$$
$$f(\tau) = (\mathbf{e}_1 + \mathbf{e}_2)[\mathbf{C}_L(\tau) + \mathbf{C}_R(\tau)]. \quad (28)$$

The EOM for $\mathbf{C}_\alpha(\tau)$ can be obtained from the quantum regression theorem [17] which in our case here reads

$$\mathbf{C}_i(\tau) = \mathbf{C}_i(0) + \int_0^\tau d\tau' \left\{M(\tau-\tau')\mathbf{C}_i(\tau') + \langle n_i(t)\rangle \mathbf{\Gamma}\right\}. \quad (29)$$

Its solution is expressed with the help of the resolvent $[z - z\hat{M}(z)]^{-1}$.

3.2 'Quasiparticle counting' and MacDonald formula

We relate the qubit dynamics with reservoir operators by introducing a counting variable n which represents the number of electrons that have tunneled through the right barrier [13,25]. We define generalized expectation values as

$$O^{(n)} \equiv \sum_{i=0,L,R} \mathrm{Tr}_{\mathrm{bath}} \langle n,i|\hat{O}\rho(t)|n,i\rangle. \quad (30)$$

The usual expectation values are recovered as

$$\langle \hat{O}\rangle = \sum_n O^{(n)}. \quad (31)$$

One now writes

$$\dot{n}_0^{(n)} = -\Gamma_L n_0^{(n)} + \Gamma_R n_R^{(n-1)}$$
$$\dot{n}_{L/R}^{(n)} = \pm\Gamma_{L/R} n_0^{(n)} \pm iT_c\left(p^{(n)} - [p^{(n)}]^\dagger\right) \quad (32)$$

and corresponding equations for $p^{(n)}$ and $[p^{(n)}]^\dagger$. A similar derivation holds for the left barrier. Equations (32) allow to calculate the particle current and the noise spectrum from

$$P_n(t) = n_0^{(n)}(t) + n_L^{(n)}(t) + n_R^{(n)}(t), \quad (33)$$

which gives the total probability of finding n electrons in the collector by time t. In particular, $I_R(t) = e\sum_n n\dot{P}_n(t)$ and S_{I_R} can be calculated from [12]

$$S_{I_R}(\omega) = 2\omega e^2 \int_0^\infty dt \sin(\omega t)\frac{d}{dt}\left[\langle n^2(t)\rangle - (t\langle I\rangle)^2\right]. \quad (34)$$

Here,

$$\frac{d}{dt}\langle n^2(t)\rangle = \sum_n n^2 \dot{P}_n(t) = \Gamma_R \sum_{n=0}^\infty n n_R^{(n)}(t) + \Gamma_R \sum_{n=0}^\infty n_R^{(n)}(t). \quad (35)$$

Solving equations (32) we obtain

$$S_{I_R}(\omega) = 2eI\left\{1 + \Gamma_R\left[\hat{n}_R(-i\omega) + \hat{n}_R(i\omega)\right]\right\}$$
$$z\hat{n}_R(z) = \Gamma_L g_+(z)/N(z)$$
$$N(z) \equiv [z + \Gamma_R + g_-(z)](z+\Gamma_L) + (z + \Gamma_R + \Gamma_L)g_+(z). \quad (36)$$

with

$$g_{+[-]}(z) = \pm iT_c(\mathbf{e}_1 - \mathbf{e}_2)\left[z - \hat{\Sigma}_z\right]^{-1}\hat{D}_z \mathbf{e}_{1[2]}. \quad (37)$$

Equations (36, 37) demonstrate the dependence of the current noise on the dephasing via the two-by-two blocks $\hat{D}_z$ and $\hat{\Sigma}_z$, cf. equations (18, 19, 20). Explicitly,

$$g_\pm^{\mathrm{PER}}(z) \equiv 2T_c \frac{T_c(\gamma_p + \Gamma_R/2 + z) - \varepsilon\gamma_\pm}{(\gamma_p + \Gamma_R/2 + z)^2 + \varepsilon^2}$$
$$g_{+[-]}^{\mathrm{POL}}(z) \equiv T_c^2\left[\frac{C_{[-]\varepsilon}^{[*]}(z)}{1 + \frac{\Gamma_R}{2}C_\varepsilon(z)} + (C \leftrightarrow C^*)\right]. \quad (38)$$

A similar derivation yields $S_{I_L}(\omega) = S_{I_R}(\omega)$.

4 Results

4.1 Zero frequency shot noise

In the zero frequency limit $z \to 0$, one obtains

$$S_I(0) = 2eI\left(1 + 2\Gamma_R \frac{d}{dz}\left[z\hat{n}_R(z)\right]_{z=0}\right). \quad (39)$$

Without bath ($\alpha = 0$), we recover previous results of reference [13] (shot noise of DQD's) and reference [11] (shot noise of the CP box). In particular, we obtain

$$[z\hat{n}_R(z)]'_{z=0} = -\frac{4T_c^2\Gamma_L}{\Gamma_R} \times \frac{4\varepsilon^2(\Gamma_R - \Gamma_L) + 3\Gamma_L\Gamma_R^2 + \Gamma_R^3 + 8\Gamma_R T_c^2}{\left[\Gamma_L\Gamma_R^2 + 4\Gamma_L\varepsilon^2 + 4T_c^2(\Gamma_R + 2\Gamma_L)\right]^2}, \quad (40)$$

For $\alpha = 0$ and $\Gamma \equiv \Gamma_L = \Gamma_R$ (Fig. 1a, thick solid line), the smallest Fano factor is reached for $\varepsilon = 0$ where quantum coherence strongly suppresses noise. The maximum suppression ($\gamma = 1/5$) is reached for $\Gamma = 2\sqrt{2}T_c$. For large $\varepsilon > 0$ ($\varepsilon < 0$) the charge becomes localized in the right (left) level, $S_I(0)$ is dominated by only one Poisson process, namely the noise of the right (left) barrier, and $\gamma \to 1$.

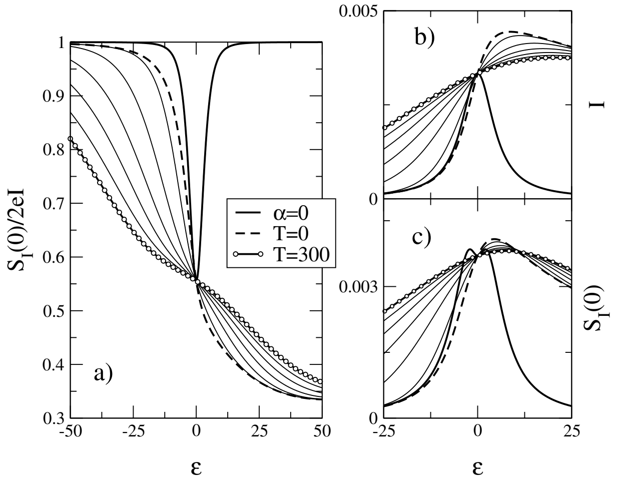

Fig. 1. a) Fano factor vs. bias ε for dissipative coupling $\alpha = 0.005$ and different temperatures from $T = 0$ (dashed line) to $T = 300$ (line with circles) in intervals of 50. For comparison, the case $\alpha = 0$ is also shown (thick solid line). Parameters $T_c = 3$, $\Gamma_L = \Gamma_R = 0.01$, $\omega_c = 500$, (in μeV) correspond to typical experimental values [1] in double quantum dots. b) Current vs. bias ε. c) Shot noise vs. bias ε.

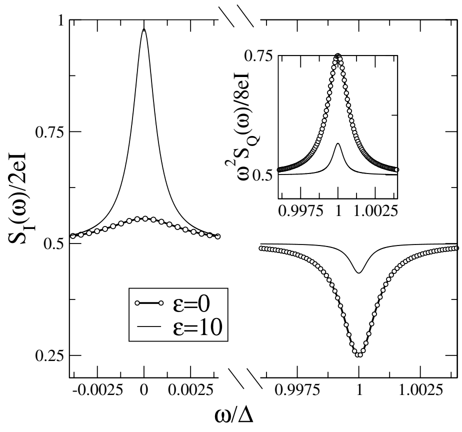

Fig. 2. Frequency dependent current noise (no dissipation, $T = 0$, $\Gamma_L = \Gamma_R = 0.01$). Inset: charge noise contribution.

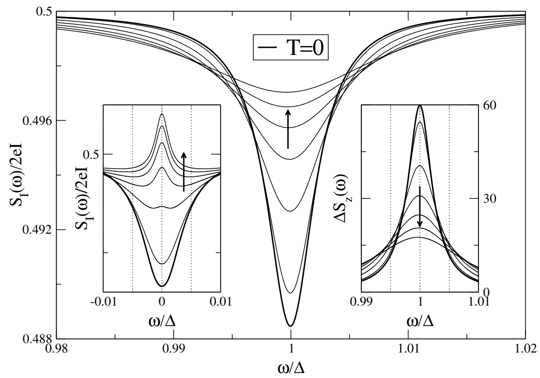

Fig. 3. Effect of decoherence on current noise near resonance (ohmic dissipation, bias $\varepsilon = 10$, $\Gamma_R = \Gamma_L = \Gamma = 0.01$, $\alpha = 0.005$ and different temperatures from $T = 0$ (thick solid line) to $T = 300$ (in intervals of 50), corresponding to $T \approx 0.4\Delta - 2.2\Delta$. Right inset: the pseudospin correlation function $S_z(\omega)$. Left inset: low frequencies region near shot noise limit $\omega = 0$. Arrows indicate the direction of increasing temperatures.

For $\alpha \neq 0$, the system has a finite possibility to exchange energy quanta with the bath. The effect of a bosonic bath on noise in a mesoscopic scatterer was first discussed in [14]. For the TLS discussed here, spontaneousemission (for $\varepsilon > 0$) occurs even at very low temperatures [1,2], and the noise is reduced [10] well below the Poisson limit (Fig. 1a, dashed line).

The maximum suppression is now reached when the elastic and inelastic rates coincide, i.e., $\gamma_p = \Gamma_R$, as we have checked numerically. For large couplings, spontaneous emission leads to a very asymmetric Fano factor that goes from $\gamma \approx 1$ to $\gamma \approx 0.5$ as ε changes sign (not shown here).

At finite temperatures, absorption of energy quanta from the the bath is possible and the Fano factor for $\varepsilon < 0$ is also reduced below the Poisson limit. Note that for $\varepsilon > 0$ the opposite behavior is found and the Fano factor increases as the temperature increases. Interestingly, at $\varepsilon = 0$ both the current and shot noise become temperature-independent.

4.2 Finite frequency noise

For finite ω, we show the numerical results for $\alpha = 0$ in Figure 2. The background noise is half the Poisson value as one expects for a symmetric structure. γ deviates from this value around $\omega = 0$ where the noise has a peak and $\omega = \Delta$ where the noise is suppressed. This noise supression (dip in the Fano factor) directly reflects the resonant shape of $S_Q(\omega)$ around Δ (inset), cf. equation (25). The physical reason for this is the fact that an increase of ε localizes the qubit and, thus, the zero-frequency noise reaches $\gamma \to 1$. Moreover, the dip in the high frequency noise at $\omega = \Delta$ is progressively destroyed (reduction of quantum coherence) as ε increases.

A similar reduction of the dip at $\omega = \Delta$ occurs at fixed ε and Γ with increasing dissipation (Fig. 3) in the weak coupling (PER) regime. This behavior demonstrates that $S_I(\omega)$ reveals the complete internal dissipative dynamics of the TLS.

In order to substantiate the above argument, we plot the symmetrized pseudospin correlation function

$$S_z(\omega) = 1/2 \int_{-\infty}^{\infty} d\omega e^{i\omega\tau} \langle \{\hat{\sigma}_z(\tau), \hat{\sigma}_z\} \rangle \qquad (41)$$

(Fig. 3, right inset). This function is often used to investigate the dynamics of the SB problem [9]. Both functions reflect (in the same fashion) how the coherent dynamics of the system progressively gets damped by the bosonic bath.

In the past, there have been various schemes for the extraction of inelastic rates (dephasing and relaxation rates) from transport quantities in double quantum dots, e.g. by adiabatic transfer or pumping [18,19].

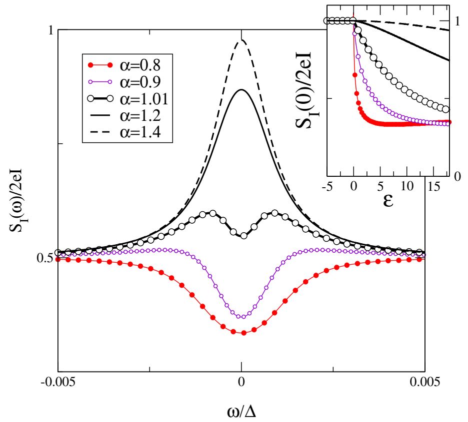

Fig. 4. Low frequency current noise in qubit with strong ohmic dissipation ($T = 0$, $\varepsilon = 10$, $T_c = 3$, $\Gamma_L = \Gamma_R = 0.01$). Inset: Shot noise for $\omega = 0$.

One central prediction of our theory here is that for open two-level systems, these rates can be extracted from a *current noise measurement* at low temperatures by determining the half-width of the correlation function $S_I(\omega)$ around $\omega = \Delta$ or $\omega = 0$ [26]. For an Ohmic environment, $\gamma_d^b = \gamma_p/2 + 2\pi\alpha(\frac{\varepsilon}{\Delta})^2 k_B T$, such that the total *dephasing rate* is $\gamma_d(T = 0) = \gamma_d^b + \Gamma/2 = (\gamma_p + \Gamma)/2$ Close to $\omega = 0$, the peak in $S_I(\omega)$ for $\alpha = 0$ changes into a dip around $\omega = 0$ reflecting incoherent relaxation dynamics for $\alpha \neq 0$. The half-width is now given by the *relaxation rate* such that the full-width of $S_I(\omega)$ around $\omega = 0$, at $T = 0$, is *twice* that of the high frequency noise (Fig. 3, thick solid line of the left inset).

As temperature increases, the Lorentzian dip around $\omega = \Delta$ broadens. This can be understood in terms of an increase of the dephasing rate with temperature. The noise at low frequencies does not follow this simple behavior (Fig. 3, left inset). Instead, its shape changes as one increases the temperature. Although we do not have a rigurous explanation for this, one can expect a strong dependence of the shot noise around $\omega = 0$ with temperature because in the range studied $T \approx 0.4\Delta - 2.2\Delta$ the frequencies are always smaller than temperature. Moreover, as we explained above, the Fano factor decreases with temperature for $\varepsilon < 0$ whereas the opposite behavior is found for $\varepsilon > 0$.

4.3 Strong dissipation noise

The results for the strong coupling (POL) regime are presented in Figure 4. Near $\omega = 0$, POL and PER yield nearly identical results for the noise $S_I(\omega)$ at very small α (not shown here). The cross-over to Poissonian noise near $\omega = 0$ with increasing α indicates the formation of localized polarons. The delocalisation-localisation transition [8,9] of the spin-boson model at $\alpha = 1$ is reflected in a change of the analytic behaviour of C_ε and the shot noise near zero bias (Fig. 4, inset). Similar physics has been found recently in the suppression of the persistent current $I(|\varepsilon|) \propto \mathrm{Im}C_{-|\varepsilon|}$ through a strongly dissipative quantum ring containing a quantum dot with bias ε [15]. Although POL becomes less reliable for $\alpha < 1$ and smaller bias, the non-symmetry in ε of the shot noise and the inelastic current $\propto \mathrm{Re}C_\varepsilon$ reflects the 'open' topology of our TLS in the non-linear transport regime.

5 Conclusion

We have shown that the current noise spectrum contains detailed information about the internal, dissipative dynamics of open quantum two-level system such as double quantum dots or Cooper pair boxes. On the theoretical side, the challenging question of how to go beyond the two perturbative methods presented here remains open. One way could be the use of path-integral techniques beyond the NIBA, another could be exact solutions for certain values of the coupling α, although at present it is unclear how feasable these approaches are in the transport situation discussed here.

On the experimental side, promising progress has been made for noise measurements [16] and experiments in tunable double quantum dots [4] and Cooper-pair boxes [7], where we hope our predictions to be tested in the near future.

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