Dynamics of a Large Spin with Strong Dissipation

Till Vorrath[1] and Tobias Brandes[2]

[1]*Universität Hamburg, I. Institut für Theoretische Physik, Jungiusstrasse 9, 20355 Hamburg, Germany*

[2]*School of Physics and Astronomy, The University of Manchester, Manchester M60 1QD, United Kingdom*

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We study the generalization of the spin-boson model to spins greater than one-half in the strong-coupling regime. This model applies to dissipative large spins as well as to ensembles of identical two-state systems coupled to a common environment. Using a combination of polaron transformations and master equations, we find nonexponential spin relaxation towards one of two possible equilibrium states. For Ohmic dissipation the relaxation is approximately logarithmic in time.

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The role of macroscopic environments is central to our understanding of measurements and the question of coherence and decoherence in quantum systems. A well-studied case is quantum dissipation in the most elementary of all quantum systems, the two-level system (qubit), which due to its equivalence with a spin $1/2$ gives rise to the famous spin-boson model [1,2]. The importance of this model has never been more obvious than in the light of the recent experimental success in the generation of quantum superpositions and entanglement in noisy solid state environments. Prime examples are superconducting Cooper pair boxes or semiconductor double quantum dots [3], where quantum coherent oscillations are damped due to phonon-induced dissipation.

An additional and possibly unexpected effect of a macroscopic environment appears in an ensemble of several two-state systems—a qubit array. Not only does the environment lead to decoherence and dissipation, it also induces an indirect interaction between the two-state systems and thereby enables collective effects in ensembles of otherwise independent systems. A prominent example is the effect of superradiance, a collective spontaneous emission of two-level atoms as first pointed out by Dicke [4–6]. A large pseudospin is used for the description of such collective effects in an ensemble of identical two-state systems. The large spin is defined as the sum of all spin halves, and its state reveals the polarization of the ensemble.

Our calculations presented in this Letter combine dissipation, collective effects, and quantum tunneling within a generic "large-spin-boson model" that is motivated by various physical systems: First, there are *intrinsic spins* greater than one-half: the elements gallium and arsenic, for example, have a nuclear spin of $3/2$ and are used in many mesoscopic solid state experiments, where nuclear relaxation processes are believed to become more and more important [7].

On the other hand, large pseudospins appear in the description of *ensembles* of two-state systems. For example, collective effects in artificial qubit (double quantum dot) arrays have been predicted to become visible in the tunnel current [8]. Moreover, two-state systems can be found in crystals and amorphous solids [9], with dissipation caused by phonons [10], and experimental indications that collective effects reduce the quality of micromechanical resonators [11].

The coherence properties and the collective behavior of all these systems are strongly influenced by a dissipative environment. Although microscopic details might differ in each case, it seems desirable to investigate this interplay in a general form. For the case of strong environment coupling discussed in this Letter, we find a new relaxation dynamics with a logarithmic type of "ultraslow radiance," which is in stark contrast with the usual weak-coupling limit that has been discussed earlier [12].

Model and method.—The Hamiltonian that generalizes the spin-boson model from $J = 1/2$ to arbitrary spin J is given by

$$H = \varepsilon J_z + 2T_c J_x + J_z \sum_q \gamma_q (a_q^\dagger + a_{-q}) + \sum_q \omega_q a_q^\dagger a_q, \tag{1}$$

where the $J_{x,z}$ are components of a spin-$\boldsymbol{J}$ with bias ε and tunnel coupling T_c between the eigenstates of J_z. The macroscopic environment is modeled by a bosonic bath with creation operators $a_q^\dagger$ for a boson in mode q with frequency ω_q and coupling strength γ_q. For the special case $\varepsilon = 0$, Eq. (1) can be mapped on the Dicke Hamiltonian by a rotation around the spin z axis.

In an ensemble of identical two-state systems interacting with the same environment, the large spin is defined as the sum of the Pauli matrices describing the individual two-state systems, $\boldsymbol{J} = 1/2\Sigma_i \boldsymbol{\sigma_i}$. The expectation value of J_z is then proportional to the polarization of the ensemble. The eigenstates $|J, M\rangle$ of J_z with a fixed total spin J are used for the discussion of collective effects. These states span a subspace of the total Hilbert space of the ensemble. It is justified to consider only this subspace as the total spin is a constant of motion of the Hamiltonian (1). A fully excited ensemble of N two-state systems corresponds, for example, to the state $|J = N/2, M = N/2\rangle$ of the large pseudospin. An arbitrary initial state of the ensemble is a superposition of states with different total spins J all evolving under the Hamiltonian (1).

We now employ a combination of a unitary polaron transformation and the Born-Markov approximation to derive a master equation for the reduced density operator $\rho(t)$ of the spin. Here, the tunneling is treated perturbatively with the Hamiltonian (1) written as $H = H_0 + V$, where V is the tunneling part, $V = 2T_c J_x$. In the transformed picture, the free part of the Hamiltonian reads

$$\bar{H}_0 = e^{\sigma J_z} H_0 e^{-\sigma J_z} = \varepsilon J_z - \kappa J_z^2 + \sum_q \omega_q a_q^\dagger a_q, \quad (2)$$

with σ and κ defined as $\sigma = \Sigma_q \gamma_q/\omega_q (a_q^\dagger - a_{-q})$ and $\kappa = \Sigma_q |\gamma_q|^2/\omega_q$. We concentrate on Ohmic dissipation, where the spectral function $\rho(\omega) \equiv \Sigma_q |\gamma_q|^2 \delta(\omega - \omega_q)$ is assumed to be linear with an exponential cutoff, $\rho(\omega) = 2\alpha\omega \exp(-\omega/\omega_c)$ with α denoting the dimensionless interaction strength between the spin and the dissipative environment, and ω_c being the cutoff frequency. The parameter κ then takes a particularly simple form, $\kappa = 2\alpha\omega_c$. As a result of the transformation, spin and boson subsystems in $\bar{H}_0$ are independent and can be treated separately. However, a new nontrivial term, $-\kappa J_z^2$, appears in the spin part of the transformed Hamiltonian (2). In the spin-boson model, $J = 1/2$, this term is constant and has no physical consequences. For larger spins, however, it amounts to an energy barrier that dominates the properties of the system as we show below.

The eigenenergies E_M of the spin subsystem directly follow from $\bar{H}_0$ as $E_M = \varepsilon M - \kappa M^2$, $-J \le M \le J$, whereas in the transformed picture the tunnel term

$$\bar{V} = T_c(J_+ X + J_- X^\dagger), \qquad X = e^{\sigma}, \quad (3)$$

now contains the unitary boson displacement (“shake-up”) operators X, the temporal correlation function $C(t) = \langle \tilde{X}_t^\dagger X \rangle$ of which plays the central role in the master equation (here and in the following, the tilde refers to the time evolution in the interaction picture with respect to $\bar{H}_0$). It has been shown that the combination of the polaron transformation with a second-order Born approximation is equivalent to the noninteracting-blip approximation (NIBA) for the spin-boson model with Ohmic dissipation [13]. We apply a Markov approximation by assuming that the memory time of the environment corresponding to the width of the correlation function $C(t)$ is the shortest time scale in the problem. Note that this assumption is *not* identical to the replacement of $C(t)$ by a Delta function.

Master equation.—The master equation is derived in the basis of the eigenstates $|J, M\rangle$ of J_z. The dynamics of J_z follows from the spin density operator in the interaction picture as $\langle J_z \rangle(t) = \Sigma_M M \tilde{\rho}_{M,M}(t)$ with $\tilde{\rho}_{M,M}(t) = \langle J, M|\tilde{\rho}(t)|J, M\rangle$ (note that $\tilde{J}_z = \bar{J}_z = J_z$). The master equation for diagonal elements of the density operator becomes

$$\begin{aligned}\dot{\tilde{\rho}}_{M,M}(t) = &-2\pi T_c^2 c_{J,M}^- P(\varepsilon - \kappa(2M-1))\tilde{\rho}_{M,M}(t) \\ &- 2\pi T_c^2 c_{J,M}^+ P(-\varepsilon + \kappa(2M+1))\tilde{\rho}_{M,M}(t) \\ &+ 2\pi T_c^2 c_{J,M}^- P(-\varepsilon + \kappa(2M-1))\tilde{\rho}_{M-1,M-1}(t) \\ &+ 2\pi T_c^2 c_{J,M}^+ P(\varepsilon - \kappa(2M+1))\tilde{\rho}_{M+1,M+1}(t).\end{aligned} \quad (4)$$

As in usual weak-coupling superradiance, the probability change $\dot{\tilde{\rho}}_{M,M}$ due to collective transitions to and from neighboring states $|J, M \pm 1\rangle$ is proportional to $c_{J,M}^\pm \equiv \sqrt{J(J+1) - M(M \pm 1)}$, which has a maximum at the superradiant “Dicke peak” $M = 0$, leading to nonexponential decay [14]. In addition to the superradiant factors $c_{J,M}$, the rate $P(E) = 1/(2\pi) \int dt C(t) \exp(iEt)$ for inelastic transitions due to boson emission or absorption from the dissipative environment [15] appears, with the *state-dependent* energy difference $E = \varepsilon - \kappa(2M \pm 1)$ between the states $|J, M\rangle$ and $|J, M \pm 1\rangle$. The appearance of these rates results in a spin dynamics in the strong-coupling regime that is completely different from the usual, superradiant dynamics for weak coupling. We note that in deriving Eq. (4), we used the fact that the bosonic correlation functions $\langle \tilde{X}_t \rangle$ and $\langle \tilde{X}_t X \rangle$ vanish. The validity of Eq. (4) is restricted to those parameters where the NIBA works well, which is the case for the two regimes (strong couplings, $\alpha \ge 1$ at zero temperature and intermediate coupling, $\alpha = 1/2$ at finite temperatures) studied here.

The function $P(E)$ is normalized and obeys the detailed balance symmetry, $P(-E) = \exp(-E/k_B T)P(E)$, but has to be derived for any specific realization of the dissipative environment. For Ohmic dissipation, at zero temperature absorption of energy from the environment is not possible and $P(E)$ reads

$$P_{T=0}(E) = \frac{E^{2\alpha-1}}{\omega_c^{2\alpha}\Gamma(2\alpha)} e^{-E/\omega_c}\theta(E), \qquad \alpha > \frac{1}{2}. \quad (5)$$

In the second case of $\alpha = 1/2$ studied here, $P(E)$ is calculated with the residue theorem,

$$P_{\alpha=1/2}(E > 0) = \frac{e^{-E/\omega_c}}{\omega_c \Gamma(1 + 1/\beta\omega_c)^2} \times \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \Gamma\left(n + 1 + \frac{2}{\beta\omega_c}\right) e^{-n\beta E}. \quad (6)$$

At low temperatures, $k_B T = 1/\beta \ll \omega_c$, the sum can be approximated by the geometric series,

$$P_{\alpha=1/2}(E > 0) \approx \frac{e^{-E/\omega_c}}{\omega_c \Gamma(1 + 1/\beta\omega_c)^2 (1 + e^{-\beta E})}. \quad (7)$$

We employ the exact result in the following and compute the series numerically. The rate $P(E < 0)$ follows directly from (6) with the detailed balance symmetry, whereas for zero energy one has

$$P_{\alpha=1/2}(E=0) = \frac{\Gamma(1+2/\beta\omega_c)}{2\omega_c 4^{1/\beta\omega_c}\Gamma(1+1/\beta\omega_c)^2}. \quad (8)$$

Case $J=\frac{1}{2}$.—Before turning to the dynamics of a large spin, we consider the limit $J=1/2$ where we recover the usual results of the spin–one-half boson model, with the master equation predicting an exponential relaxation of J_z to the equilibrium value $\langle J_z\rangle_\infty = (P(-\varepsilon)-P(\varepsilon))/2(P(-\varepsilon)+P(\varepsilon))$ with relaxation rate $\gamma = 2\pi T_c^2(P(-\varepsilon)+P(\varepsilon))$. Inserting the zero-temperature $P(E)$, one finds that the spin remains in its initial state at zero bias, $\varepsilon = 0$, which is the well-known localization phenomenon of the spin-boson model at $\alpha \geq 1$ [16,17]. For a finite bias, the relaxation rate becomes

$$\gamma_{T=0} = \frac{2\pi T_c^2 \varepsilon^{2\alpha-1}}{\omega_c^{2\alpha}\Gamma(2\alpha)} e^{-\varepsilon/\omega_c}, \quad (9)$$

which agrees in leading order in ε/ω_c with the relaxation rate of the spin-boson model [1]. Thus, the master Eq. (4) reproduces the correct results of the spin-boson model for zero temperature and $\alpha \geq 1$ though it does not predict the critical value of α at which the transition to the localization occurs.

In the second regime, $\alpha = 1/2$, the spin-boson model corresponds to the Toulouse limit of the anisotropic Kondo model. This case is of special significance as it marks the coherent-incoherent transition at zero temperature in the limit $T_c/\omega_c \to 0$ [2]. At zero bias, the relaxation rate follows from (8) as

$$\gamma_{\varepsilon=0} = 4^{-1/\beta\omega_c} \frac{2\pi T_c^2 \Gamma(1+2/\beta\omega_c)}{\omega_c |\Gamma(1+1/\beta\omega_c)|^2}, \quad (10)$$

which again correctly converges to the zero-temperature result of the spin-boson model, $\gamma = 2\pi T_c^2/\omega_c$; cf. [18]. Furthermore, for finite bias we checked numerically that the exponential relaxation in our model is very close to the solution of the spin-boson model in the scaling limit [2].

The equilibrium value of $\langle J_z\rangle_\infty$ as predicted by Eq. (4) is just the NIBA result that becomes unphysical at low temperatures, from which we conclude that the combination of the polaron transformation with the Born-Markov approximation reproduces the results of the spin-boson model apart from those parameters at which the NIBA fails. We therefore expect that this approach is likewise reliable for greater spins, $J > 1/2$.

Case $J > \frac{1}{2}$.—We now turn to our central results for the dynamics of a large spin, which differs entirely from that of a spin one-half. The zero-temperature regime is considered first. While a spin one-half is localized for $\alpha \geq 1$ and $\varepsilon = 0$, greater spins relax towards one of the two minima, i.e., the polarized states $|J, \pm J\rangle$, on the inverted parabola; cf. Fig. 1. The most striking feature here is the dependence of the long-time behavior of the spin on its *initial value,* which determines to which of the two minima the spin relaxes. This is the direct consequence of the effective potential term $-\kappa J_z^2$ in the transformed Hamiltonian (2).

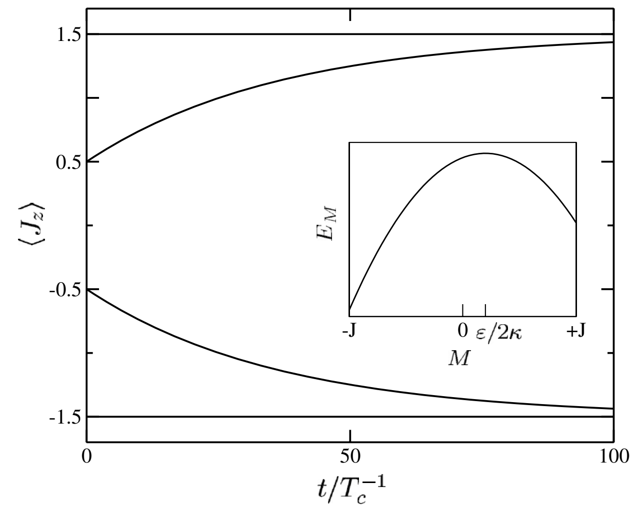

FIG. 1. Dynamics of J_z for a spin 3/2 with different initial values ($\varepsilon = 0$, $\alpha = 1$, $\omega_c = 50T_c$, and $k_BT = 0$). The inset shows the eigenenergies of the unperturbed system.

The two branches $M > 0$ and $M < 0$ are perfectly symmetric at $\varepsilon = 0$. For an initial value $|J, M_0\rangle$ on the ascending branch, $M_0 > 0$, the master equation describes transitions $M \to M+1$ with rate

$$\Gamma_{M\to M+1} = 2\pi T_c^2 c_{J,M}^{+} P(-\varepsilon + \kappa(2M+1)). \quad (11)$$

For Ohmic dissipation, Eq. (5), this rate obeys $\Gamma_{M\to M+1} \ll \Gamma_{M-1\to M}$ such that each transition happens much slower than the previous one. Hence, the transition $M-1 \to M$ is basically finished before the next one, $M \to M+1$, becomes effective. The total relaxation of the large spin to the equilibrium value $+J$ appears as a cascade of exponential relaxations. The time $t(M)$ the spin needs to relax to a state $|J, M\rangle$ is then approximately independent of the initial state and governed only by the last transition. Taking the inverse of the rate (11) as a measure for that time yields

$$t(M) \approx \frac{1}{2\pi T_c^2 c_{J,M}^{-} P(-\varepsilon + \kappa(2M-1))}. \quad (12)$$

For Ohmic dissipation, Eq. (5) gives an approximation for the spin dynamics,

$$\langle J_z\rangle \approx \frac{1}{4\alpha}\ln(t) + C, \quad (13)$$

where all other parameters are absorbed in the constant C. The logarithmic relaxation is clearly visible in Fig. 2, which shows the dynamics of a spin of size $J = 9/2$, the oscillations reflecting the step-by-step relaxation. Because of the symmetry of the unbiased spin, the previous statements equally apply to the negative branch, $M < 0$. A finite bias lifts this symmetry and shifts the vertex of the parabola from zero to $\varepsilon/2\kappa$. This ratio, however, is less than one-

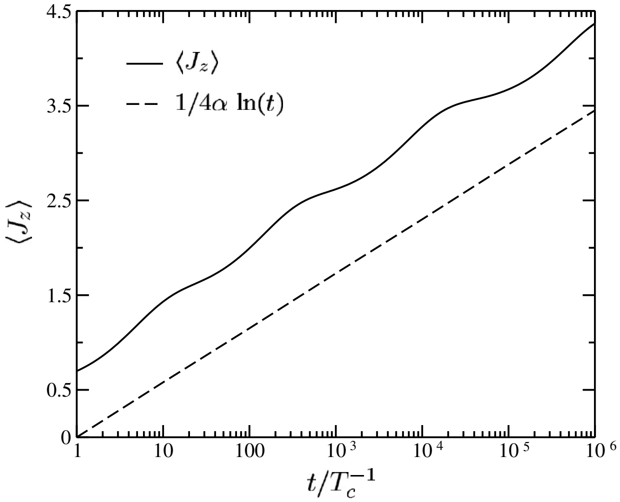

FIG. 2. Logarithmic plot of the relaxation of a spin of size $J = 4.5$ with initial value $\langle J_z \rangle_0 = 1/2$ ($\varepsilon = 0$, $\alpha = 1$, $\omega_c = 50T_c$, and $k_B T = 0$) and the approximation (13) with $C = 0$.

half for Ohmic dissipation as long as $\varepsilon < \omega_c$ and does not result in a qualitatively different dynamics.

At finite temperatures, the spin can also absorb energy from the environment. Thus, transitions in both directions become possible, $M \leftrightarrow M + 1$. Because of the detailed balance relation, however, the absorption rate is much smaller than the emission rate and does not alter the behavior as described above. This is confirmed by the numerical solution of the master equation with $P(E)$ as given in (6). In particular, the logarithmic relaxation to the equilibrium, Eq. (13), is similarly found at finite temperatures and finite bias. The equilibrium values, though, are only approximately given by the polarized spin $\pm J$. As the large spin shares a similar dynamics in both regimes of parameters studied in this work, we may speculate that this behavior is generally found for strong couplings, $\alpha > 1/2$, at finite temperatures, although at this point we are not able to prove this conjecture.

In conclusion, the dynamics of a large spin with strong dissipation differs qualitatively from that of a spin one-half as studied in the usual spin-boson model. This difference is a result of the angular momentum algebra for $J > \frac{1}{2}$ and can alternatively be understood as a collective effect since the large spin equally describes an ensemble of identical two-state systems. For strong dissipation, we found that the spin relaxes to one of two polarized states. This strongly resembles the broken parity symmetry of the one-mode Dicke superradiance model in the strong-coupling limit [19]. For Ohmic dissipation with an exponential cutoff of the spectral function the relaxation is logarithmic. In particular, the relaxation becomes slower with increasing spin size, which is in contrast to the dynamics of a large spin with weak interaction to the environment [12] where the equilibrium value is unique and the spin exhibits superradiance, i.e., the relaxation becomes *faster* with increasing spin size. Apparently, the transition between these different behaviors occurs at intermediate coupling strengths, $0 < \alpha < 1/2$. This regime, however, is not accessible by the method chosen in this work since the tunneling and the dissipation become equally important and none can be treated perturbatively.

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