Current noise of a quantum dot *p–i–n* junction in a photonic crystal

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The shot-noise spectrum of a quantum dot *p–i–n* junction embedded inside a three-dimensional photonic crystal is investigated. Radiative decay properties of quantum dot excitons can be obtained from the observation of the current noise. The characteristic of the photonic band gap is revealed in the current noise with discontinuous behavior. Applications of such a device in entanglement generation and emission of single photons are pointed out, and may be achieved with current technologies.

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Since Yablonovitch proposed the idea of photonic crystals (PCs),[1] optical properties in periodic dielectric structures have been investigated intensively.[2] Great attention has been focused on these materials not only because of their potential applications in optical devices, but also because of their ability to drastically alter the nature of the propagation of light from a fundamental perspective.[3] Among these, modification of spontaneous emission is of particular interest. Historically, the idea of controlling the spontaneous emission rate was proposed by Purcell,[4] and enhanced and inhibited spontaneous emission rates for atomic systems were intensively investigated in the 1980s (Ref. 5) by using atoms passed through a cavity. In semiconductor systems, the electron-hole pair is naturally a candidate to examine spontaneous emission, where modifications of the spontaneous emission rates of quantum dot (QD) (Ref. 6) or quantum wire (QW) (Ref. 7) excitons inside the microcavities have been observed experimentally.

Recently, the interest in measurements of shot noise in quantum transport has risen owing to the possibility of extracting valuable information not available in conventional dc transport experiments.[8] With the advances of fabrication technologies, it is now possible to embed QDs inside a *p–i–n* structure,[9] such that the electron and hole can be injected separately from opposite sides. This allows one to examine the exciton dynamics in a QD via electrical currents.[10] On the other hand, it is also possible to embed semiconductor QDs in PCs,[11] where modified spontaneous emission of QD excitons is observed over large frequency bandwidths.

In this work, we present nonequilibrium calculations for the quantum noise properties of quantum dot excitons inside photonic crystals. We obtain the current noise of QD excitons via the MacDonald formula,[12] and find that it reveals many of the characteristics of the photonic band gap (PBG). Possible applications of such a device to the generation of entangled states and the emission of single photons are also pointed out.

Model. We assume that a QD *p–i–n* junction is embedded in a three-dimensional PC. A possible structure is shown in Fig. 1. Both the hole and electron reservoirs are assumed to be in thermal equilibrium. For the physical phenomena we are interested in, the Fermi level of the $p(n)$-side hole (electron) is slightly lower (higher) than the hole (electron) subband in the dot. After a hole is injected into the hole subband in the QD, the n-side electron can tunnel into the exciton level because of the Coulomb interaction between the electron and hole. Thus, we may introduce the three dot states: $|0\rangle = |0,h\rangle$, $|\uparrow\rangle = |e,h\rangle$, and $|\downarrow\rangle = |0,0\rangle$, where $|0,h\rangle$ means there is one hole in the QD, $|e,h\rangle$ is the exciton state, and $|0,0\rangle$ represents the ground state with no hole and electron in the QD. One might argue that one cannot neglect the state $|e,0\rangle$ for real devices since the tunable variable is the applied voltage. This can be resolved by fabricating a thicker barrier on the electron side so that there is little chance for an electron to tunnel in advance.[13] Moreover, the charged exciton and biexcitons states are also neglected in our calculations, which means a low injection limit is required.[14]

Derivation of Master equation. We define the dot-operators $\widehat{n_\uparrow} \equiv |\uparrow\rangle\langle\uparrow|$, $\widehat{n_\downarrow} \equiv |\downarrow\rangle\langle\downarrow|$, $\hat{p} \equiv |\uparrow\rangle\langle\downarrow|$, $\widehat{s_\uparrow} \equiv |0\rangle\langle\uparrow|$, $\widehat{s_\downarrow}$

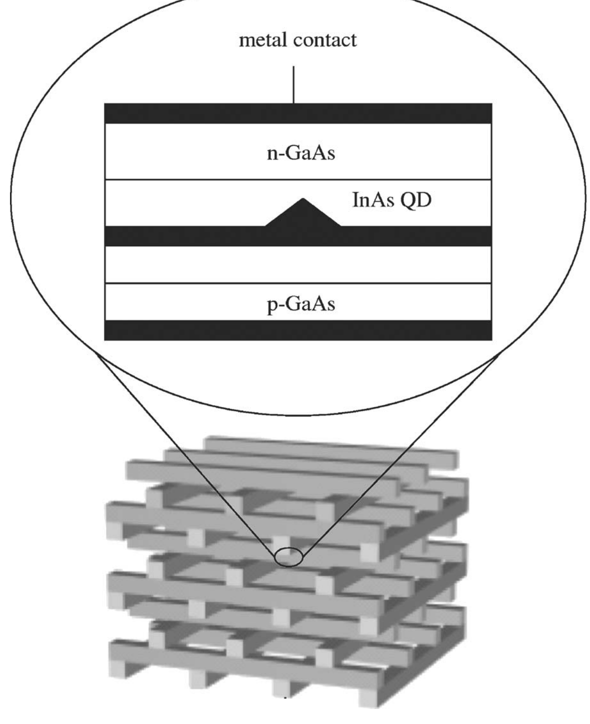

FIG. 1. Illustration of a QD inside a *p–i–n* junction surrounded by a three-dimensional PC.

$\equiv|0\rangle\langle\downarrow|$. The total Hamiltonian H of the system consists of three parts: H_0 [dot, photon bath H_p, and the electron (hole) reservoirs $H_{\rm res}$], H_T (dot-photon coupling), and the dot-reservoir coupling H_V

$$H = H_0 + H_T + H_V,$$

$$H_0 = \varepsilon_\uparrow \widehat{n_\uparrow} + \varepsilon_\downarrow \widehat{n_\downarrow} + H_p + H_{\rm res},$$

$$H_T = \sum_k D_k b_k^\dagger \hat{p} + D_k^* b_k \hat{p}^\dagger = \hat{p}X + \hat{p}^\dagger X^\dagger,$$

$$H_p = \sum_k \omega_k b_k^\dagger b_k$$

$$H_V = \sum_{\mathbf{q}} (V_{\mathbf{q}} c_{\mathbf{q}}^\dagger \widehat{s_\uparrow} + W_{\mathbf{q}} d_{\mathbf{q}}^\dagger \widehat{s_\downarrow} + \text{c.c.}),$$

$$H_{\rm res} = \sum_{\mathbf{q}} \varepsilon_{\mathbf{q}}^\uparrow c_{\mathbf{q}}^\dagger c_{\mathbf{q}} + \sum_{\mathbf{q}} \varepsilon_{\mathbf{q}}^\downarrow d_{\mathbf{q}}^\dagger d_{\mathbf{q}}. \tag{1}$$

In the above equation, $D_k = i\hbar\boldsymbol{\epsilon}\cdot\boldsymbol{\mu}\sqrt{\omega_{\mathbf{k}}/(2\epsilon_0\hbar V)}$ is the dipole coupling strength with $\boldsymbol{\epsilon}$ and $\boldsymbol{\mu}$ being the polarization vector of the photon and the dipole moment of the exciton, respectively. b_k is the photon operator, $X=\Sigma_k D_k b_k^\dagger$, and $c_{\mathbf{q}}$ and $d_{\mathbf{q}}$ denote the electron operators in the left and right reservoirs, respectively.

The couplings to the electron and hole reservoirs are given by the standard tunnel Hamiltonian H_V, where $V_{\mathbf{q}}$ and $W_{\mathbf{q}}$ couple the channels $\mathbf{q}$ of the electron and hole reservoirs. If the couplings to the electron and the hole reservoirs are weak, it is reasonable to assume that the standard Born-Markov approximation with respect to these couplings is valid. In this case, one can derive a master equation from the exact time evolution of the system. The equations of motion can be expressed as (cf. Ref. 15)

$$\frac{\partial}{\partial t}\langle\widehat{n_\uparrow}\rangle_t = -\int dt'[C(t-t') + C^*(t-t')]\langle\widehat{n_\uparrow}\rangle_{t'} + \Gamma_L[1 - \langle\widehat{n_\uparrow}\rangle_t - \langle\widehat{n_\downarrow}\rangle_t], \tag{2}$$

$$\frac{\partial}{\partial t}\langle\widehat{n_\downarrow}\rangle_t = \int dt'[C(t-t') + C^*(t-t')]\langle\widehat{n_\uparrow}\rangle_{t'} - \Gamma_R\langle\widehat{n_\downarrow}\rangle_t,$$

$$\frac{\partial}{\partial t}\langle\hat{p}\rangle_t = -\frac{1}{2}\int dt'[C(t-t') + C^*(t-t')]\langle\hat{p}\rangle_{t'} - \frac{\Gamma_R}{2}\langle\hat{p}\rangle_t,$$

where $\Gamma_L = 2\pi\Sigma_{\mathbf{q}} V_{\mathbf{q}}^2 \delta(\varepsilon_\uparrow - \varepsilon_{\mathbf{q}}^\uparrow)$, $\Gamma_R = 2\pi\Sigma_{\mathbf{q}} W_{\mathbf{q}}^2 \delta(\varepsilon_\downarrow - \varepsilon_{\mathbf{q}}^\downarrow)$, and $\varepsilon = \hbar\omega_0 = \varepsilon_\uparrow - \varepsilon_\downarrow$ is the energy gap of the QD exciton. Here, $C(t-t') \equiv \langle X_t X_{t'}^\dagger\rangle_0$ is the photon correlation function, and depends on the time interval only. We can now define the Laplace transformation for real z

$$C_\varepsilon(z) \equiv \int_0^\infty dt e^{-zt} e^{i\varepsilon t} C(t)$$

$$n_\uparrow(z) \equiv \int_0^\infty dt e^{-zt} \langle\widehat{n_\uparrow}\rangle_t \quad \text{etc.}, \quad z > 0 \tag{3}$$

and transform the whole equations of motion into z space

$$n_\uparrow(z) = -[C_\varepsilon(z) + C_\varepsilon^*(z)]n_\uparrow(z)/z + \frac{\Gamma_L}{z}[1/z - n_\uparrow(z) - n_\downarrow(z)],$$

$$n_\downarrow(z) = [C_\varepsilon(z) + C_\varepsilon^*(z)]n_\downarrow(z)/z - \frac{\Gamma_R}{z}n_\downarrow(z),$$

$$p(z) = -\frac{1}{2}[C_\varepsilon(z) + C_\varepsilon^*(z)]p(z)/z - \frac{\Gamma_R}{2z}p(z). \tag{4}$$

These equations can then be solved algebraically, and the tunnel current from the hole- or electron-side barrier

$$\widehat{I_R} = -e\Gamma_R\langle\widehat{n_\downarrow}\rangle_t, \quad \widehat{I_L} = -e\Gamma_L[1 - \langle\widehat{n_\uparrow}\rangle_t - \langle\widehat{n_\downarrow}\rangle_t] \tag{5}$$

can in principle be obtained by performing the inverse Laplace transformation on Eqs. (4). Depending on the complexity of the correlation function $C(t-t')$ in the time domain, this can be a formidable task which can however be avoided if one directly seeks the quantum noise:

Shot noise spectrum. In a quantum conductor in nonequilibrium, electronic current noise originates from the dynamical fluctuations of the current around its average. To study correlations between carriers, we relate the exciton dynamics with the hole reservoir operators by introducing the degree of freedom n as the number of holes that have tunneled through the hole-side barrier[16] and write

$$\dot{n}_0^{(n)}(t) = -\Gamma_L n_0^{(n)}(t) + \Gamma_R n_\downarrow^{(n-1)}(t),$$

$$\dot{n}_\uparrow^{(n)}(t) + \dot{n}_\downarrow^{(n)}(t) = (\Gamma_L - \Gamma_R)n_0^{(n)}(t). \tag{6}$$

Equations (6) allow us to calculate the particle current and the noise spectrum from $P_n(t) = n_0^{(n)}(t) + n_\uparrow^{(n)}(t) + n_\downarrow^{(n)}(t)$ which gives the total probability of finding n electrons in the collector by time t. In particular, the noise spectrum S_{I_R} can be calculated via the MacDonald formula[12,17]

$$S_{I_R}(\omega) = 2\omega e^2 \int_0^\infty dt \sin(\omega t)\frac{d}{dt}[\langle n^2(t)\rangle - (t\langle I\rangle)^2], \tag{7}$$

where $(d/dt)\langle n^2(t)\rangle = \Sigma_n n^2 \dot{P}_n(t)$. Solving Eqs. (4) and (6), we obtain

$$S_{I_R}(\omega) = 2eI\{1 + \Gamma_R[\hat{n}_\downarrow(z = -i\omega) + \hat{n}_\downarrow(z = i\omega)]\}. \tag{8}$$

In the zero-frequency limit, Eq. (6) reduces to

$$S_{I_R}(\omega = 0) = 2eI\left\{1 + 2\Gamma_R\frac{d}{dz}[z\hat{n}_\downarrow(z)]_{z=0}\right\}. \tag{9}$$

As can be seen, there is no need to evaluate the correlation function $C(t-t')$ in the time domain such that all one has to do is to solve Eq. (4) in z space.

Results and discussions. The above derivation shows that the noise spectrum of the QD excitons depends strongly on

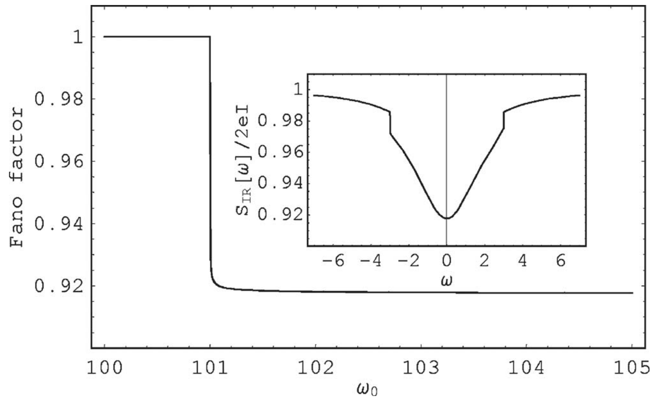

FIG. 2. Current noise (Fano factor) of QD excitons in a one-band PC as a function of the exciton band gap ω_0. The PBG frequency ω_c is set equal to 101β. The inset shows frequency-dependent noise, in which ω_0 is fixed to 104β.

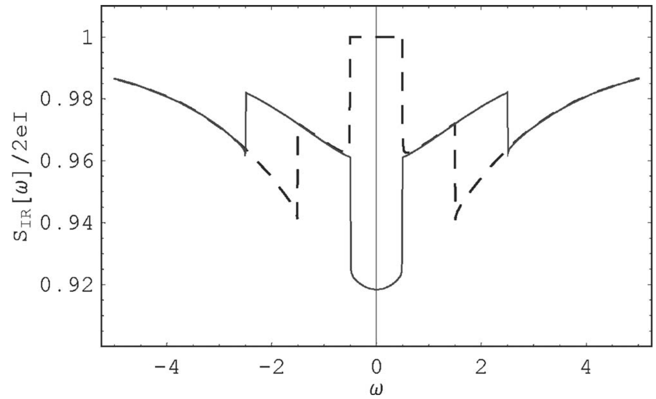

FIG. 3. Shot-noise spectrum of QD excitons in a two-band PC with ω_{c_1} and ω_{c_2} set equal to 101β and 99β, respectively. To demonstrate the ability of extracting information from the PC, the exciton band gap ω_0 in gray and dashed curves is chosen as above ω_{c_2} (ω_0=101.5β) and between the two band edge frequencies (ω_0 =100.5β), respectively.

$C_\varepsilon(z)$. Let us now turn our attention to the spontaneous emission of a QD exciton in a three-dimensional PC, where the vacuum dispersion relation is strongly modified: An anisotropic band-gap structure is formed on the surface of the first Brillouin zone in the reciprocal lattice space. In general, the band edge is associated with a finite collection of symmetrically placed points $\mathbf{k}_0^i$ leading to a three-dimensional band structure.[3] In our study, the transition energy of the QD exciton is assumed to be near the band edge ω_c. The dispersion relation for those wave vectors $\mathbf{k}$ whose directions are near one of the $\mathbf{k}_0^i$ can be expressed approximately by $\omega_{\mathbf{k}}=\omega_c+A|\mathbf{k}-\mathbf{k}_0^i|^2$, where A is a model dependent constant.[18] Thus, the correlation function $C_\varepsilon(z)=\Sigma_{\mathbf{k}}|gD_k|^2/[z+i(\omega_{\mathbf{k}}-\omega_0)]$ can be calculated around the directions of each $\mathbf{k}_0^i$ separately, and is given by

$$C_\varepsilon(z)=\frac{-i\omega_0^2\beta^{3/2}}{\sqrt{\omega_c}+\sqrt{-iz-(\omega_0-\omega_c)}}, \tag{10}$$

with $\beta^{3/2}=d^2\Sigma_i\sin^2\theta_i/8\pi\epsilon_0\hbar A^{3/2}$.[19] Here, $\hbar\omega_0$ is the transition energy of the QD exciton, d is the magnitude of the dipole moment, and θ_i is the angle between the dipole vector of the exciton and the ith $\mathbf{k}_0^i$.

The shot-noise spectrum of QD excitons inside a PC is displayed in Fig. 2, where the tunneling rates Γ_L and Γ_R are assumed to be equal to 0.1β and β, respectively. We see that the Fano factor $[F\equiv S_{I_R}(\omega=0)/2e\langle I\rangle]$ displays a discontinuity as the exciton transition frequency is tuned across the PBG frequency ($\omega_c=101\beta$). It also reflects the fact that below the band edge frequency ω_c, spontaneous emission of the QD exciton is inhibited. To observe this experimentally, a dc electric field (or magnetic field) could be applied in order to vary the band-gap energy of the QD exciton. Another way to examine the PBG frequency is to measure the frequency-dependent noise as shown in the inset of Fig. 2, where the exciton band gap is set equal to 104β. As can be seen, discontinuities also appear as ω is equal to the *detuned* frequency between PBG and QD exciton.

When the atomic resonant transition frequency is very close to the edge of the band and the band gap is relatively large, the above one-band model is a good approximation. If the band gap is narrow, one must consider both upper and lower bands. For a three-dimensional anisotropic PC with point-group symmetry, the dispersion relation near two band edges can be approximated as

$$\omega_k=\begin{cases}\omega_{c_1}+C_1|\mathbf{k}-\mathbf{k}_{10}^i| & (\omega_k>\omega_{c_1}),\\ \omega_{c_2}-C_2|\mathbf{k}-\mathbf{k}_{20}^j| & (\omega_k>\omega_{c_1}).\end{cases} \tag{11}$$

Here, $\mathbf{k}_{10}^i$ and $\mathbf{k}_{20}^j$ are two finite collections of symmetry related points, which are associated with the upper and lower band edges,[20] and C_1 and C_2 are model-dependent constants. Following the derivation for the one-band PC, the correlation function can now be written as

$$C_\varepsilon(z)=\sum_{n=1}^{2}\frac{(-1)^n i\omega_0^2\beta_n^{3/2}}{\sqrt{\omega_{c_n}}+\sqrt{(-1)^n[iz+(\omega_0-\omega_{c_n})]}}, \tag{12}$$

where $\beta_n^{3/2}=d^2\Sigma_i\sin^2\theta_i^{(n)}/8\pi\epsilon_0\hbar C_n^{3/2}$ with the corresponding collections of angles $\theta_i^{(n)}$, $n=1$, 2.

Figure 3 illustrates the frequency-dependent noise of QD excitons embedded inside a two-band PC. The two-band edge frequencies ω_{c_1} and ω_{c_2} are set equal to 101β and 99β, respectively. There are three regimes for the choices of the exciton band gap: $\omega_0>\omega_{c_1}$, $\omega_0<\omega_{c_2}$, and $\omega_{c_1}>\omega_0>\omega_{c_2}$. When ω_0 is tuned above the upper band-edge ω_{c_1} (or below the lower band-edge ω_{c_2}), the QD exciton is allowed to decay, such that the shot noise spectrum (gray curve) is suppressed in the range of $|\omega|<|\omega_0-\omega_{c_1}|$. On the other hand, however, if ω_0 is between the two band edges, spontaneous emission is inhibited. As shown by the dashed curve, the current noise in the central region is increased with its value equal to unity. Similar to the one-band PC, the curves of the shot noise spectrum reveal two discontinuities at $|\omega|=|\omega_0-\omega_{c_1}|$ or $|\omega_0-\omega_{c_2}|$, demonstrating the possibility to extract information from a PC by the current noise.

A few remarks about the application of the QDs inside a PC should be mentioned here. As is known, controlling the propagation of light (waveguide) is one of the optoelectronic applications of PCs.[21] By controlling the exciton band-gap ω_0 across the PBG frequency with appropriate tunneling rates of the electron and hole, one may achieve the emission of a single photon at predetermined times and directions (waveguides),[22] which are important for the field of quantum information technology. Furthermore, it has been demonstrated recently that a precise spatial and spectral overlap between a single self-assembled quantum dot and a photonic crystal membrane nanocavity can be implemented by a deterministic approach.[23] One of the immediate applications is the coupling of two QDs to a single common cavity mode.[24] Therefore, if two QD *p–i–n* junctions can also be incorporated inside a PC (and on the way of light propagation), the cavitylike effect may be used to create an entangled state between two QD excitons with remote separation.[13] The obvious advantages then would be a suppression of decoherence of the entangled state by the PBG, and the observation of the enhanced shot noise could serve in order to identify the entangled state.[10]

In summary, we have derived the nonequilibrium current noise of QD excitons incorporated in a *p–i–n* junction surrounded by a one-band or two-band PC. We found that characteristic features of the PBG can be obtained from the shot noise spectrum. Generalizations to other types of PCs are expected to be relatively straightforward, which makes QD *p–i–n* junctions good detectors of quantum noise.[25]

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[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[2] S. Sakoda, *Optical Properties of Photonic Crystals* (Springer, New York, 2001).
[3] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[4] E. M. Purcell, Phys. Rev. **69**, 681 (1946).
[5] P. Goy, J. M. Raimond, M. Gross, and S. Haroche, Phys. Rev. Lett. **50**, 1903 (1983); G. Gabrielse and H. Dehmelt, *ibid.* **55**, 67 (1985); R. G. Hulet, E. S. Hilfer, and D. Kleppner, *ibid.* **55**, 2137 (1985); D. J. Heinzen, J. J. Childs, J. E. Thomas, and M. S. Feld, *ibid.* **58**, 1320 (1987).
[6] M. Bayer, F. Weidner, A. Larionov, A. McDonald, A. Forchel, and T. L. Reinecke, Phys. Rev. Lett. **86**, 3168 (2001); G. S. Solomon, M. Pelton, and Y. Yamamoto, *ibid.* **86**, 3903 (2001).
[7] C. Constantin, E. Martinet, D. Y. Oberli, E. Kapon, B. Gayral, and J. M. Gerard, Phys. Rev. B **66**, 165306 (2002).
[8] C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997); Y. M. Blanter and M. Buttiker, Phys. Rep. **336**, 1 (2000).
[9] G. Kießlich, A. Wacker, E. Schöll, S. A. Vitusevich, A. E. Belyaev, S. V. Danylyuk, A. Förster, N. Klein, and M. Henini, Phys. Rev. B **68**, 125331 (2003).
[10] Y. N. Chen, T. Brandes, C. M. Li, and D. S. Chuu, Phys. Rev. B **69**, 245323 (2004).
[11] P. Lodahl, A. Floris van Driel, Ivan S. Nikolaev, Arie Irman, Karin Overgaag, Daniël Vanmaekelbergh, and Willem L. Vos, Nature (London) **430**, 654 (2004); T. Yoshie, A. Scherer, J. Hendrickson, G. Khitrova, H. M. Gibbs, G. Rupper, C. Ell, O. B. Shchekin, and D. G. Deppe, *ibid.* **432**, 200 (2004).
[12] D. K. C. MacDonald, Rep. Prog. Phys. **12**, 56 (1948).
[13] Y. N. Chen, D. S. Chuu, and T. Brandes, Phys. Rev. Lett. **90**, 166802 (2003).
[14] Z. Yuan, B. E. Kardynal, R. M. Stevenson, A. J. Shields, C. J. Lobo, K. Cooper, N. S. Beattie, D. A. Ritchie, and M. Pepper, Science **295**, 102 (2002).
[15] T. Brandes and B. Kramer, Phys. Rev. Lett. **83**, 3021 (1999); Y. N. Chen and D. S. Chuu, Phys. Rev. B **66**, 165316 (2002).
[16] Actually, the total current noise should be expressed in terms of the spectra of particle currents and the charge noise spectrum: $S_I(\omega)=aS_{I_L}(\omega)+bS_{I_R}(\omega)-ab\omega^2 S_Q(\omega)$, where a and b are capacitance coefficient $(a+b=1)$ of the junctions. Since we have assumed a highly asymmetric setup $(a \ll b)$, it is plausible to consider the hole-side spectra $S_{I_R}(\omega)$ only.
[17] R. Aguado and T. Brandes, Phys. Rev. Lett. **92**, 206601 (2004).
[18] In fact, A is dependent on the structures of the photonic crystals. For a simple three-dimensional periodic dielectric, A can be approximated as $A \simeq \omega_c/(\mathbf{k}_0^i)^2$, where $\mathbf{k}_0^i=\pi/L$ with L being the lattice constant of the photonic crystal (Ref. 18). In order not to lose the generality, we let A being a constant, such that β becomes the unit for the numerical calculations.
[19] S. John and T. Quang, Phys. Rev. A **50**, 1764 (1994); S. Y. Zhu, Y. Yang, H. Chen, H. Zheng, and M. S. Zubairy, Phys. Rev. Lett. **84**, 2136 (2000); N. Vats, S. John, and K. Busch, Phys. Rev. A **65**, 043808 (2002).
[20] Y. Yang, M. Fleischhauer, and S. Y. Zhu, Phys. Rev. A **68**, 043805 (2003).
[21] E. Chow, S. Y. Lin, S. G. Johnson, P. R. Villeneuve, J. D. Joannopoulos, J. R. Wendt, G. A. Vawter, W. Zubrzycki, H. Hou, and A. Alleman, Nature (London) **407**, 983 (2000).
[22] E. Lidorikis, M. L. Povinelli, S. G. Johnson, and J. D. Joannopoulos, Phys. Rev. Lett. **91**, 023902 (2003).
[23] A. Badolato, K. Hennessy, M. Atatüre, J. Dreiser, E. Hu, P. M. Petroff, and A. Imamoglu, Science **308**, 1158 (2005).
[24] A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. **83**, 4204 (1999).
[25] R. Aguado and L. P. Kouwenhoven, Phys. Rev. Lett. **84**, 1986 (2000).