Phase transitions in generalized spin-boson (Dicke) models

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(Received 7 January 2004; published 6 May 2004)

We consider a class of generalized single mode Dicke Hamiltonians with arbitrary boson coupling in the pseudo-spin *x*-*z* plane. We find exact solutions in the thermodynamic, large-spin limit as a function of the coupling angle, which allows us to continuously move between the simple dephasing and the original Dicke Hamiltonians. Only in the latter case (orthogonal static and fluctuating couplings) does the parity-symmetry induced quantum phase transition occur.

DOI: 10.1103/PhysRevA.69.053804 PACS number(s): 42.50.Fx, 73.43.Nq

I. INTRODUCTION

Spin-boson models appear in many areas of physics and are essential ingredients in theoretical quantum optics [1] (light-matter interaction), nuclear physics [2], quantum chaos [3], and quantum dissipation [4]. The spin algebra can be used to describe single $(j=N/2=1/2)$ or many $N>1$ two-level systems where, in the simplest case, the interaction is with but a single bosonic mode $(a,a^\dagger)$. Specific examples include cavity quantum electrodynamics and, more recently, "phonon cavity quantum dynamics" of electrons interacting with single phonon (oscillation) modes in nano-electromechanical systems [5–8] such as freestanding quantum dots or "molecular transistors."

A common feature of spin-boson models is that in general they are nonintegrable, with exact solutions available only for very specific cases. Examples of the latter are simplified "dephasing models," where the spin couples to both the boson and static field via only one of its components (usually chosen as J_z). Another example where exact solutions can be obtained is in the large spin limit $j\to\infty$ where bosonic representations of spin Lie algebras [2] have been known for a long time; an early example being the Holstein-Primakoff transformation [9].

In this paper, we further explore the large-spin limit by starting from the most general, single-mode, spin boson Hamiltonian with linear coupling of all (x,y,z) spin components to a static *and* a fluctuating (bosonic) term. For the specific case of the coupling of orthogonal (*x* and *z*) spin components to the static and the fluctuating term (Dicke model), we have previously found [10–12] intriguing connections between quantum chaos, entanglement, and the emergence of an instablity-induced quantum phase transition in the limit of large spin $j\to\infty$. Here, our main result will be that, surprisingly, this instability and the related parity-symmetry breaking of the ground-state wave functions only appears for "orthogonal" coupling. The Dicke Hamiltonian [13] (Rabi-Hamiltonian for spin 1/2) and its canonical equivalents therefore seem to be in a "distinguished" class of Hamiltonians with very pronounced properties. It should be mentioned from the very beginning, however, that this distinction is most visible in the strong coupling regime.

II. THE MODEL AND ITS SOLUTION

We start from a generic model Hamiltonian

$$H=\omega a^\dagger a+(\mathbf{\Omega}+a^\dagger\mathbf{\Lambda}+a\mathbf{\Lambda}^\dagger)\cdot\mathbf{J}, \quad (1)$$

describing the simplest coupling between Heisenberg-Weyl $(1,a,a^\dagger)$ and the spin algebras $J_x=\frac{1}{2}(J_++J_-)$, $J_y=\frac{1}{2i}(J_+-J_-)$, J_z, with

$$[J_z,J_\pm]=\pm J_\pm, \quad [J_+,J_-]=2J_z. \quad (2)$$

In Eq. (1), $\mathbf{\Omega}$ is a real and $\mathbf{\Lambda}$ a complex three-dimensional vector. Special cases of Eq. (1) are the Rabi or the Dicke Hamiltonian [14] $(\mathbf{\Omega}=\Omega\mathbf{e}_z,\mathbf{\Lambda}=\mathbf{\Lambda}^\dagger=\Lambda\mathbf{e}_x)$, the simple dephasing Hamiltonian [15–17] $(\mathbf{\Omega}=\Omega\mathbf{e}_i$, $\mathbf{\Lambda}=\mathbf{\Lambda}^\dagger=\Lambda\mathbf{e}_i$ with $i=x,y$, or $z)$, the Jaynes-Cummings Hamiltonian [1] $(\mathbf{\Omega}=\Omega\mathbf{e}_z$, $\mathbf{\Lambda}=\Lambda[\mathbf{e}_x-i\mathbf{e}_y])$, and the one-mode version of the dissipative spin-boson (tunneling electron) Hamiltonian [4,8,18,19] $(\mathbf{\Omega}=\omega_0\mathbf{e}_z+T\mathbf{e}_x,\mathbf{\Lambda}=\mathbf{\Lambda}^\dagger=\Lambda\mathbf{e}_z)$, where we denote the unit vectors as $\mathbf{e}_i$, $i=x,y,z$. The $j=1/2$ variant of Eq. (1) with $\mathbf{\Omega}=\omega_0\mathbf{e}_z+T\mathbf{e}_x$ and $\mathbf{\Lambda}=a\mathbf{e}_x+ib\mathbf{e}_y$ appears in quasi-one-dimensional quantum wires in the *x*-*y* plane in a constant magnetic field $B\mathbf{e}_z$ for an electron gas with spin-orbit interactions (Rashba Hamiltonian) [20].

In the following, we restrict ourselves to $\mathbf{\Lambda}=\mathbf{\Lambda}^\dagger$ and therefore consider the Hamiltonian,

$$H=\omega a^\dagger a+\mathbf{\Omega}\cdot\mathbf{J}+(a^\dagger+a)\mathbf{\Lambda}\cdot\mathbf{J} \quad (3)$$

parametrized by *two* real three-dimensional vectors given by

$$\mathbf{\Omega}=(\Omega_x,\Omega_y,\Omega_z), \quad \mathbf{\Lambda}=\frac{2}{\sqrt{2j}}(\lambda_x,\lambda_y,\lambda_z), \quad (4)$$

where $1/\sqrt{2j}$ is inserted to ensure correct scaling in the thermodynamic limit, and the factor of 2 is for later convenience. Note that the more general case, Eq. (1), leaves three real, linearly independent three-dimensional vectors.

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The analysis would then be very similar to the following, though more cumbersome, which is why we restrict ourselves to the model Eq. (3).

We begin by rotating our coordinate axes so that we work in the x-z plane, with the coupling-vector $\mathbf{\Lambda}$ aligned along the x axis. This gives us the form with which we shall work:

$$H = \omega a^\dagger a + \Omega(J_x \cos\theta + J_z \sin\theta) + \frac{2\lambda}{\sqrt{2j}}(a^\dagger + a)J_x. \quad (5)$$

This Hamiltonian is invariant under a rotation about the z axis, under which $J_z \to J_z$ and $J_x \to -J_x$, and consequently, we shall only discuss the parameter range $0 \leqslant \theta \leqslant \pi$. In deriving exact solutions for this model in the thermodynamic limit, we shall follow the general procedure introduced for the Dicke model in Ref. [11].

First we employ the Holstein–Primakoff representation of the angular momentum operators [9], $J_+ = b^\dagger\sqrt{2j - b^\dagger b}$, $J_- = \sqrt{2j - b^\dagger b}\,b$, and $J_z = (b^\dagger b - j)$. With $J_x = \frac{1}{2}(J_+ + J_-)$, substitution gives us

$$\begin{aligned} H = {} & \omega a^\dagger a + \frac{\Omega}{2}\cos\theta(b^\dagger\sqrt{2j - b^\dagger b} + \sqrt{2j - b^\dagger b}\,b) \\ & + \Omega \sin\theta(b^\dagger b - j) \\ & + \frac{\lambda}{\sqrt{2j}}(a^\dagger + a)(b^\dagger\sqrt{2j - b^\dagger b} + \sqrt{2j - b^\dagger b}\,b). \end{aligned} \quad (6)$$

We next displace the oscillator modes $a \to a + \sqrt{\alpha}$ and $b \to b - \sqrt{\beta}$, where α and β are assumed to be of the order of j. This leads to

$$\begin{aligned} H = {} & \omega[a^\dagger a + \sqrt{\alpha}(a^\dagger + a) + \alpha] + \frac{1}{2}\Omega(\cos\theta)\sqrt{k}(b^\dagger\sqrt{\eta} + \sqrt{\eta}b \\ & - 2\sqrt{\beta}\sqrt{\eta}) + \Omega \sin\theta[b^\dagger b - \sqrt{\beta}(b^\dagger + b) + \beta - j] \\ & + \lambda\sqrt{\frac{k}{2j}}(a^\dagger + a + 2\sqrt{\alpha})(b^\dagger\sqrt{\eta} + \sqrt{\eta}b - 2\sqrt{\beta}\sqrt{\eta}), \end{aligned} \quad (7)$$

where

$$k = 2j - \beta; \quad \eta = 1 - \frac{b^\dagger b - \sqrt{\beta}(b^\dagger + b)}{k}. \quad (8)$$

We now proceed to the thermodynamic limit, by taking $j \to \infty$ and neglecting terms with powers of j in the denominator. This yields

$$\begin{aligned} H^{j\to\infty} = {} & \omega a^\dagger a + \left(\Omega \sin\theta + 2\lambda\sqrt{\frac{\alpha\beta}{2jk}} + \frac{\Omega\cos\theta}{2}\sqrt{\frac{\beta}{k}}\right)b^\dagger b \\ & + \left(\omega\sqrt{\alpha} - 2\lambda\sqrt{\frac{\beta k}{2j}}\right)(a^\dagger + a) + \left[4\lambda\sqrt{\frac{\alpha}{2jk}}(j - \beta) \right. \\ & \left. - \Omega \sin\theta\sqrt{\beta} + \Omega\cos\theta\left(\frac{j - \beta}{\sqrt{k}}\right)\right](b^\dagger + b) \\ & + \left[\frac{\lambda}{2k}\sqrt{\frac{\alpha\beta}{2jk}}(2k + \beta) + \frac{1}{4}\Omega\cos\theta\sqrt{\frac{\beta}{k}}\left(1 + \frac{\beta}{2k}\right)\right] \\ & \times(b^\dagger + b)^2 + 2\lambda\sqrt{\frac{1}{2jk}}(j - \beta)(a^\dagger + a)(b^\dagger + b) \\ & + \Omega\sin\theta(\beta - j) + \omega\alpha - \Omega\cos\theta\sqrt{k\beta} - \lambda\sqrt{\frac{\alpha\beta}{2jk}}(1 \\ & + 4k) - \frac{1}{4}\Omega\cos\theta\sqrt{\frac{\beta}{k}}. \end{aligned} \quad (9)$$

The two terms linear in bosonic operators can be eliminated by choosing the parameters α and β such that

$$\sqrt{\alpha} = \frac{2\lambda}{\omega}\sqrt{\frac{k\beta}{2j}}, \quad (10)$$

and β is given by

$$\frac{4\lambda}{k}\sqrt{\frac{\alpha k}{2j}}(j - \beta) - \Omega\sin\theta\sqrt{\beta} + \frac{1}{2}\Omega\cos\theta\sqrt{k}\left(1 - \frac{\beta}{k}\right) = 0. \quad (11)$$

Substituting the value of α into this equation and simplifying, we obtain the following equation for $\sqrt{\beta}$,

$$\frac{4\lambda^2}{\omega}\frac{j - \beta}{j}\sqrt{\beta} - \Omega\sin\theta\sqrt{\beta} + \Omega\cos\theta\frac{j - \beta}{\sqrt{2j - \beta}} = 0. \quad (12)$$

This equation is exactly soluble for $\sqrt{\beta}$, but the resulting form is extremely unwieldy, and will not be reproduced here. In a few specific cases, to be elucidated later, compact expressions can be found. With the elimination of the linear terms, our Hamiltonian assumes the form

$$H = \omega a^\dagger a + \tilde{\omega} b^\dagger b + s(b^\dagger + b)^2 + r(a^\dagger + a)(b^\dagger + b) + jE_G + k', \quad (13)$$

where the constants may be inferred by comparison with Eq. (9), with appropriate values of α and β. Hamiltonians of this form are analytically soluble via a unitary transformation, and since an example of this process was given in Ref. [11], we shall not go into the details here. Suffice to say that after a Bogoliubov transformation of the bosonic operators, the Hamiltonian becomes diagonalized,

$$H = \varepsilon_+ c_+^\dagger c_+ + \varepsilon_- c_-^\dagger c_- + jE_G + k, \quad (14)$$

where we have introduced the excitation energies of the system, $\varepsilon_\pm$, and where E_G is the scaled ground-state energy (scaled with j) and k is an unimportant constant of the order unity. In terms of the parameters introduced in Eq. (13), the excitation energies are given by

$$\varepsilon_\pm^2 = \tfrac{1}{2}(\omega^2 + \tilde{\omega}^2 + 4\tilde{\omega}s \pm \sqrt{(\tilde{\omega}^2 + 4\tilde{\omega}s - \omega^2)^2 + 16r^2\omega\tilde{\omega}}), \quad (15)$$

and, in terms of β, the ground-state energy is given by

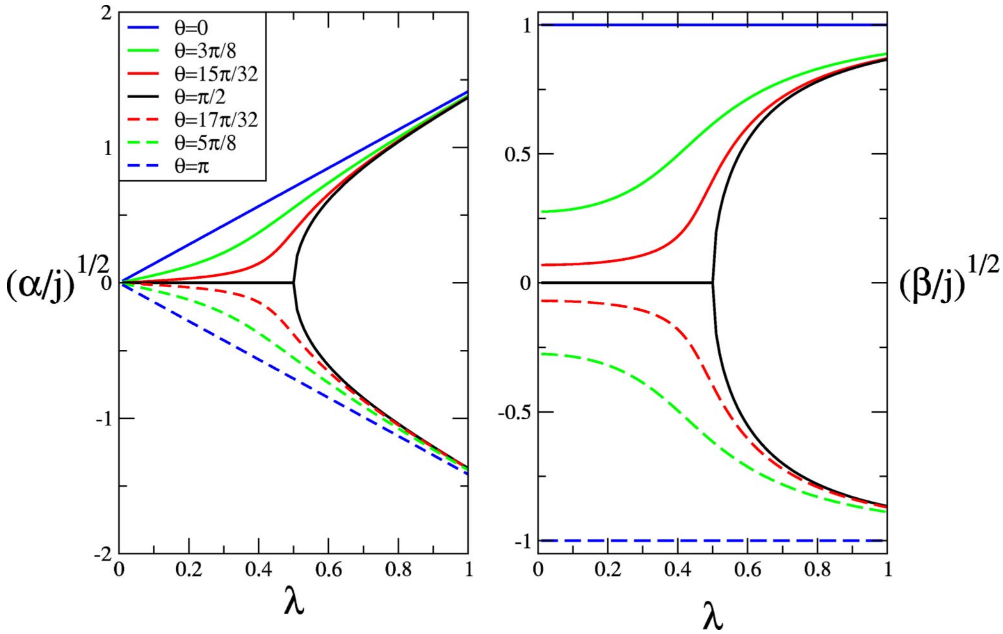

FIG. 1. The two displacement parameters $\sqrt{\alpha}$ and $\sqrt{\beta}$ as a function of the coupling λ for various different angles θ. The Hamiltonian is on scaled resonance, $\omega=\Omega=1$, $\lambda_c=0.5$.

$$jE_G = \Omega \sin\theta(\beta - j) + \frac{2\lambda^2}{j\omega}\beta(2j-\beta) - \Omega\cos\theta\sqrt{\beta(2j-\beta)}. \tag{16}$$

The general scheme in which we proceed from here is to solve Eq. (12) for β, and then use this value to compute the excitation and ground-state energies. Before considering the problem with arbitrary parameters, however, we will focus on two special cases, which will explain many of the features of the general solution. It should be pointed out that not all solutions of Eq. (12) are physically valid, and by considering the following cases we shall determine the criteria for selecting valid solutions.

III. SPECIFIC LIMITS

A. The Dicke model: $\theta=\pi/2$

In the case where the interaction and spin vectors are perpendicular we obtain the Dicke model:

$$H_{\pi/2} = \omega a^\dagger a + \Omega J_z + \frac{2\lambda}{\sqrt{2j}}(a^\dagger + a)J_x. \tag{17}$$

In this limit there exists a conserved parity Π such that $[H,\Pi]=0$, given by

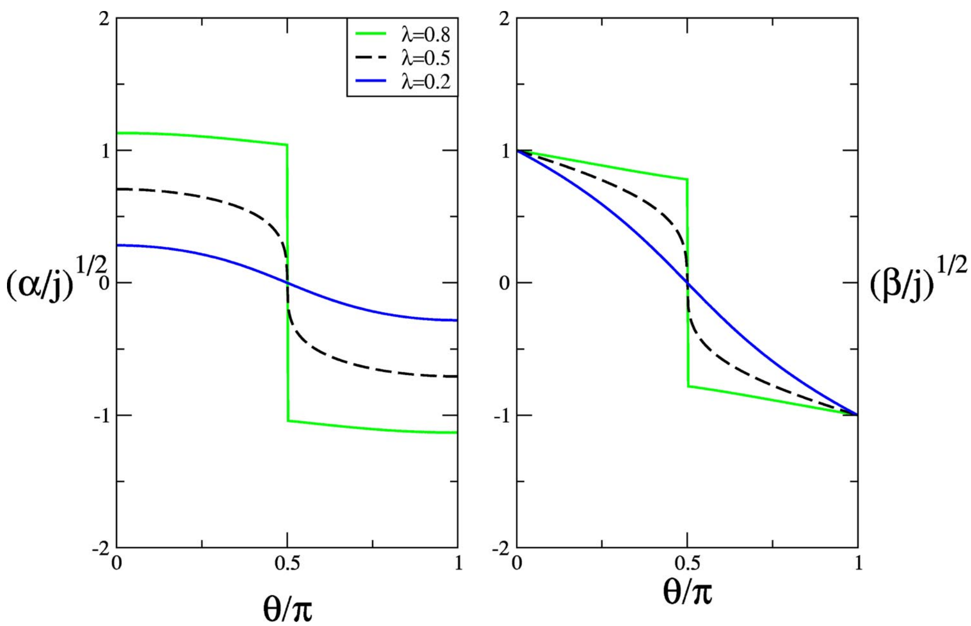

FIG. 2. The two displacement parameters $\sqrt{\alpha}$ and $\sqrt{\beta}$ as a function of the θ for representative couplings. The Hamiltonian is on scaled resonance, $\omega=\Omega=1$, $\lambda_c=0.5$.

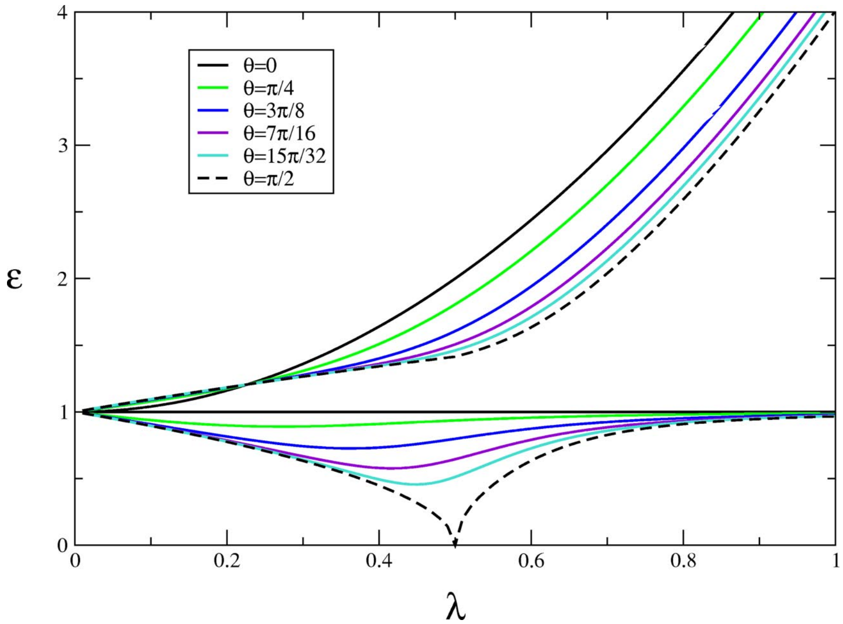

FIG. 3. The excitation energies of the system as a function of the coupling λ for various different angles θ. The Hamiltonian is on scaled resonance, $\omega=\Omega=1$, $\lambda_c=0.5$.

$$\Pi = \exp\{i\pi\hat{N}\}, \quad \hat{N} = a^\dagger a + J_z + j, \tag{18}$$

where $\hat{N}$ is the "excitation number" and counts the total number of excitation quanta in the system. Π possesses two eigenvalues, ±1, depending on whether the number of quanta is even or odd.

For the Dicke Hamiltonian, the equation for determining $\sqrt{\beta}$ becomes

$$\sqrt{\beta}[4\lambda^2(j-\beta) - j\Omega\omega] = 0. \tag{19}$$

The simplest solution sets $\sqrt{\beta}=\sqrt{\alpha}=0$, which gives rise to the effective Hamiltonian

$$H^{(1)}_{\pi/2} = \omega_0 b^\dagger b + \omega a^\dagger a + \lambda(a^\dagger + a)(b^\dagger + b) - j\omega_0, \tag{20}$$

which has the excitation energies

$$\varepsilon^{(1)2}_{\pm} = \tfrac{1}{2}\{\omega^2 + \omega_0^2 \pm \sqrt{(\omega_0^2 - \omega^2)^2 + 16\lambda^2\omega\omega_0}\}, \tag{21}$$

and ground-state energy $E^{(1)}_G = -j\omega_0$. The excitation energy $\varepsilon^{(1)}_{-}$ remains real provided that $\lambda \leqslant \lambda_c = \sqrt{\omega\omega_0}/2$, and this demarcates the range of validity of this solution. The appearance of an imaginary part of an eigenenergy is one of our criteria for distinguishing between valid and invalid solutions of Eq. (12).

The remaining two solutions of Eq. (19) are given by the displacements

$$\sqrt{\alpha} = \pm\frac{2\lambda}{\omega}\sqrt{\frac{j}{2}(1-\mu^2)}, \quad \sqrt{\beta} = \pm\sqrt{j(1-\mu)}, \tag{22}$$

where we have defined $\mu \equiv \omega\omega_0/4\lambda^2 = \lambda_c^2/\lambda^2$. The Hamiltonians obtained with these solutions (one for each sign) are

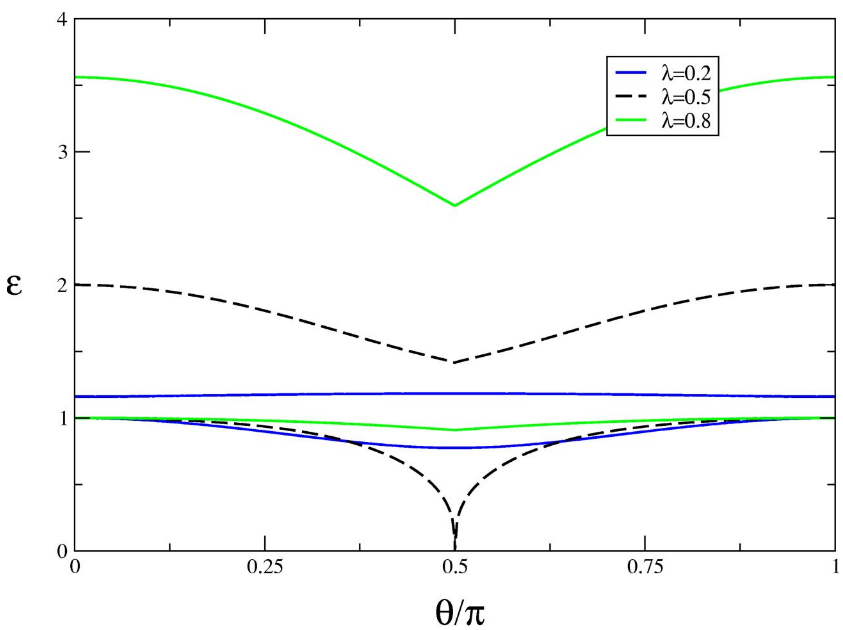

FIG. 4. The excitation energies of the system as a function of angle θ for representative values of coupling λ. The Hamiltonian is on scaled resonance, $\omega=\Omega=1$, $\lambda_c=0.5$.

identical and have the same excitation energies

$$\varepsilon_{\pm}^{(2)2} = \frac{1}{2}\left\{\frac{\omega_0^2}{\mu^2} + \omega^2 \pm \sqrt{\left[\frac{\omega_0^2}{\mu^2} - \omega^2\right]^2 + 4\omega^2\omega_0^2}\right\}, \quad (23)$$

and ground-state energy,

$$E_G^{(2)} = -\left\{\frac{2\lambda^2}{\omega} + \frac{\omega_0^2\omega}{8\lambda^2}\right\}, \quad (24)$$

and we thus see that these two solutions are completely degenerate. By considering the reality of $\varepsilon_-^{(2)}$, we conclude that these second two solutions are only valid providing $\lambda \geqslant \lambda_c$.

As described in Ref. [10] and to be discussed later, the existence of these different solutions, one with zero displacement, and two with finite and opposite displacements, describes a quantum phase transition in the Dicke model, which occurs at the critical coupling λ_c. The nature of this quantum phase transition is such that the parity symmetry becomes broken above λ_c, which explains the appearance of the two degenerate, broken symmetry solutions.

B. One dimension: $\theta=0$

With interaction and spin aligned, the full Hamiltonian of Eq. (5) becomes

$$H_0(j) = \omega a^\dagger a + \Omega J_x + \frac{2\lambda}{\sqrt{2j}}(a^\dagger + a)J_x. \quad (25)$$

This Hamiltonian is integrable for arbitrary j since its eigenstates are clearly also eigenstates of J_x, which allows us to replace the operator with its eigenvalue $m_x=-j,-j+1,\ldots,j-1,j$, such that

$$H_0(j) = \omega a^\dagger a + \Omega m_x + \frac{2\lambda}{\sqrt{2j}}(a^\dagger + a)m_x. \quad (26)$$

This leaves us with a single-mode bosonic Hamiltonian which may be diagonalized via a simple displacement $a \rightarrow a-(2\lambda m_x)/(\sqrt{2j}\omega)$. This results in the diagonal form

$$H_0(j) = \omega a^\dagger a + \Omega m_x - 2\frac{\lambda^2 m_x^2}{j\omega}, \quad (27)$$

which has the energy

$$E_{n,m_x} = \omega n + \Omega m_x - 2\frac{\lambda^2 m_x^2}{j\omega}. \quad (28)$$

We proceed to the thermodynamic limit by writing $m_x=k_x-j$, and neglecting terms with j in the denominator. Whence,

$$E_{n,k_x}^{j\rightarrow\infty} = \omega n + \left(\Omega + 4\frac{\lambda^2}{\omega}\right)k_x - j\left(\Omega + 2\frac{\lambda^2}{\omega}\right), \quad (29)$$

from which we immediately see that the excitation energies are $\varepsilon_-=\omega$ and $\varepsilon_+=\Omega+4\lambda^2/\omega$, and the scaled ground-state energy is $E_G=-(\Omega+2\lambda^2/\omega)$.

We now seek to obtain these results using the general procedure outlined in Sec. II. The equation for the determination of β becomes

$$(j-\beta)[j\Omega\omega + 4\lambda^2\sqrt{\beta(2j-\beta)}] = 0. \quad (30)$$

Setting the second factor in this expression to zero leads to values of $\sqrt{\beta}$ and $\sqrt{\alpha}$ which give rise to complex excitation energies for all parameter values. These solutions are unphysical and we discard them as we did for the Dicke model. Considering the other solution, we have $\beta=j$, which gives $\sqrt{\beta}=\pm\sqrt{j}$ and $\sqrt{\alpha}=\pm(\lambda/\omega)\sqrt{2j}$. With these choices, the Hamiltonian of Eq. (13) becomes

$$H_0^{j\rightarrow\infty} = \omega a^\dagger a + \left(\frac{2\lambda^2}{\omega} \pm \frac{\Omega}{2}\right)\left(b^\dagger b + \frac{3}{4}(b^\dagger + b)^2 - \frac{1}{2}\right) - j\left(\frac{2\lambda^2}{\omega} \pm \Omega\right). \quad (31)$$

Note that the two modes are now decoupled. The b mode may be diagonalized via the squeezing transformation,

$$b \rightarrow \frac{1}{\sqrt{1-\sigma^2}}(b^\dagger + \sigma b), \quad b^\dagger \rightarrow \frac{1}{\sqrt{1-\sigma^2}}(b + \sigma b^\dagger), \quad (32)$$

with the squeezing parameter $\sigma=-1/3$. In this way we arrive at the final form of the Hamiltonian

$$H_0^{j\rightarrow\infty} = \omega a^\dagger a + \left(\frac{4\lambda^2}{\omega} \pm \Omega\right)b^\dagger b - j\left(\frac{2\lambda^2}{\omega} \pm \Omega\right). \quad (33)$$

The excitation energies of this Hamiltonian are clearly always real. However, only the Hamiltonian with the upper sign (corresponding to $\sqrt{\beta}=+\sqrt{j}$) has the same excitation and ground-state energies as our previous calculation. The solution with $\sqrt{\beta}=-\sqrt{j}$ leads to a Hamiltonian with the incorrect energies, and is thus seen to be spurious. This solution is obviously unphysical for $\lambda<\sqrt{\omega\Omega}/2$, as here the coefficient of the second oscillator becomes negative. The origin of this spurious solution can be easily understood by considering the $\theta=\pi$ limit of the Hamiltonian. In this case, the Hamiltonian is the same as that of Eq. (25), except that Ω is replaced by $-\Omega$. Exchanging J_x for its eigenvalue as above and diagonalizing the atomic mode, we obtain the energies

$$E_{n,m_x} = \omega n + -\Omega m_x - 2\frac{\lambda^2 m_x^2}{j\omega}. \quad (34)$$

The problem with this Hamiltonian arises when we take the thermodynamic limit under the assumption that $m_x=-j$ is the spin-quantum number of the ground state. This leads to the energy

$$E_{n,k_x}^{j\rightarrow\infty} = \omega n + (-\Omega + 4\lambda^2/\omega)k_x - j(-\Omega + 2\lambda^2/\omega), \quad (35)$$

which is the same as the spurious solutions obtained above. Clearly, the correct ground state of the $\theta=0$ Hamiltonian actually has the quantum number $m_x=+j$. So we see that the origin of this type of spurious solution is due to the incorrect counting of the states labeled with m_x as we go to the thermodynamic limit. The solutions with the incorrect sign always have a ground-state energy that is higher than the correct solution, and thus we are easily able to discard the solutions which arise from misidentifying the ground state.

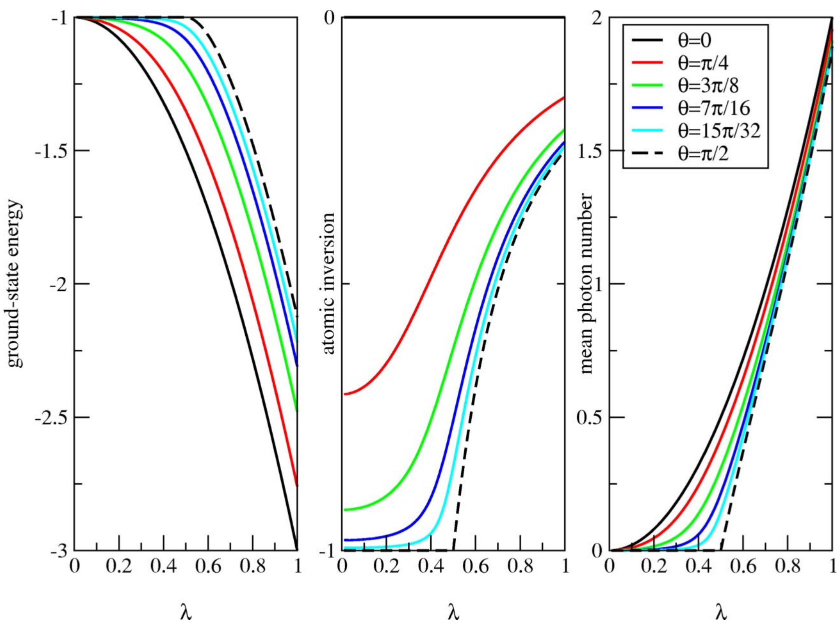

FIG. 5. The ground-state energy, atomic inversion and mean photon number of the ground state as a function of the coupling λ for various different angles θ. The Hamiltonian is on scaled resonance, $\omega = \Omega = 1$, $\lambda_c = 0.5$.

IV. RESULTS AND DISCUSSION

To determine the behavior of the system away from these two specific limits, we first solve for $\sqrt{\alpha}$ and $\sqrt{\beta}$. Figures 1 and 2 show the values of these two displacement parameters as functions of both λ and θ. Our first observation is that for all $\theta \neq \pi/2$, there is only one solution for a given λ. Furthermore, the sign of $\sqrt{\alpha}$ and $\sqrt{\beta}$ is given by that of cos θ. The divide between the regions of positive and negative displacements is spanned by the special case of $\theta = \pi/2$, which is the previously discussed Dicke model. In this case we have $\sqrt{\alpha} = \sqrt{\beta} = 0$ below λ_c, and *two* solutions of opposite sign above λ_c. The displacement parameters α and β determine the center(s) of the collective ground-state wave function of the coupled systems in a position-momentum representation of the two bosonic modes a and b [10]. The appearance of two solutions for $\theta = \pi/2$ then corresponds to a breaking up of the wave function into two macroscopically separated parts for $j \to \infty$. This parity breaking phase transition therefore occurs only at $\theta = \pi/2$ which demonstrates that the Dicke model $H_{\pi/2}$ with its "orthogonal" coupling is unique within the whole class of Hamiltonians H_θ. It is only in this special case that the super-radiant phase will exhibit macroscopically coherent (Schrödinger's cat) behavior when j remains finite.

This conclusion is corroborated by considering the excited states of our models. The nature of the system is characterized by the behavior of its two excitation energies, which are plotted in Figs. 3 and 4. In Fig. 3 the limiting cases of $\theta = 0$ and $\theta = \pi/2$ are clearly identifiable, and serve to provide bounds for the other solutions away from these values. The most crucial consequence of this is that again, only for $\theta = \pi/2$ and $\lambda = \lambda_c$ does ε_- identically vanish, and so it is only for these parameter values that a quantum phase transition occurs.

A further check is made in Fig. 5, where we plot the values of important observables of the system. The expression for the ground-state energy has been given in Eq. (24). The atomic inversion and mean-field occupation are given by

$$\langle J_z \rangle / j = \beta / j - 1, \quad \langle a^\dagger a \rangle = \alpha / j. \tag{36}$$

Again, singular behavior in the form of nonanalyticities of the curves at $\lambda = \lambda_c = 1/2$ is observed only at $\theta = \pi/2$ in agreement with the above result.

To summarize, the existence of the quantum phase transition for spin-boson models H_θ is dependent on the two vectors Λ and Ω being exactly perpendicular, which one might not have expected at the outset. In conclusion, we briefly discuss the implications these findings have for spin-boson systems. One obvious consequence is that "non orthogonal" coupling terms always would smear out phase transitions or their precursors when tuning from a weak to a strong coupling regime in, e.g., photon or phonon cavities. At first sight, this looks like bad news for the possible realization of critical behavior in realistic systems where one would always expect perturbative terms leading to a general, not necessarily orthogonal coupling, unless some symmetry prevents this from occurring. On the other hand, it would be desirable to explore tunable systems where one can vary the parameter θ (for example by using external electric or magnetic fields), in order to test some of our predictions.

ACKNOWLEDGMENTS

This work was supported by Project Nos. EPSRC GR44690/01 and DFG Br1528/4-1, the WE Heraeus Foundation, and the Dutch Science Foundation NWO/FOM.

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