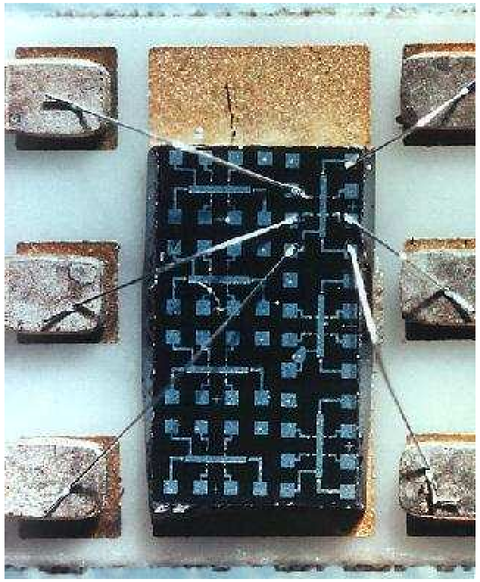
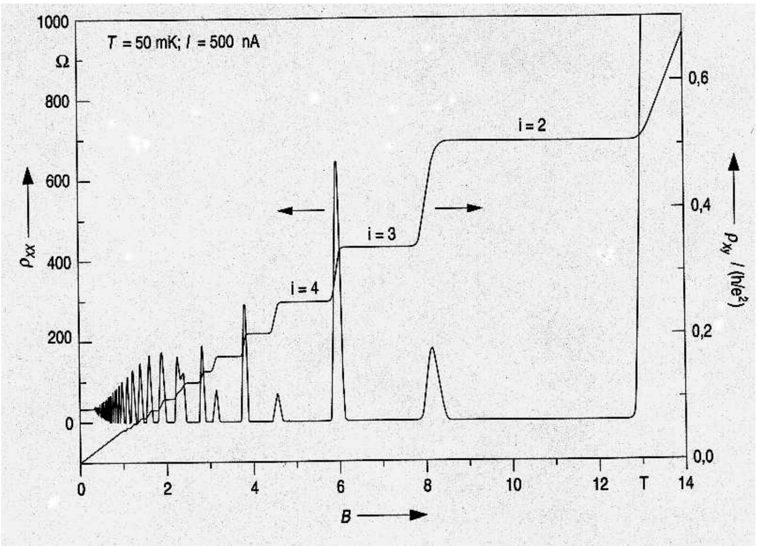
Electronic transport and noise in dissipative few-level systems

T. Brandes

- Introduction: Electronic Transport
- Quantum Mechanical Transport
- Quantum Noise and Dissipation

Electronic Transport

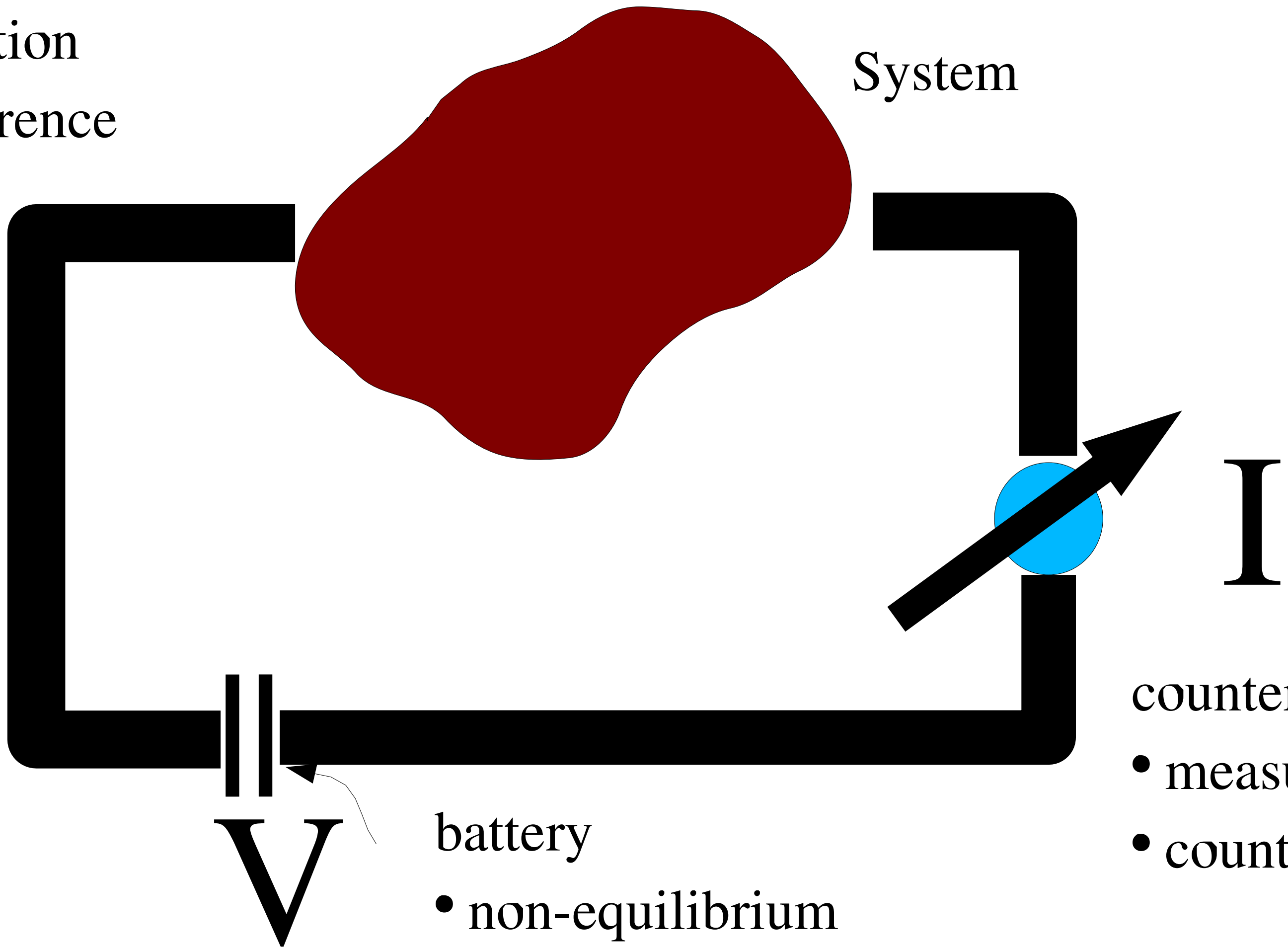




$$R_K = \frac{h}{e^2} = 25812,807\Omega.$$

leads,environment

- dissipation
- decoherence



counter

- measurement
- counting statistics

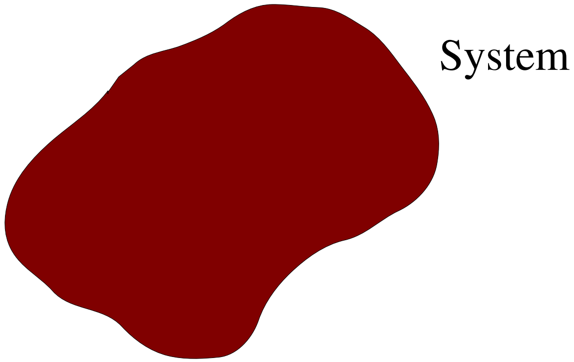
battery

- non-equilibrium

TRANSPORT = system + non-equilibrium + external world

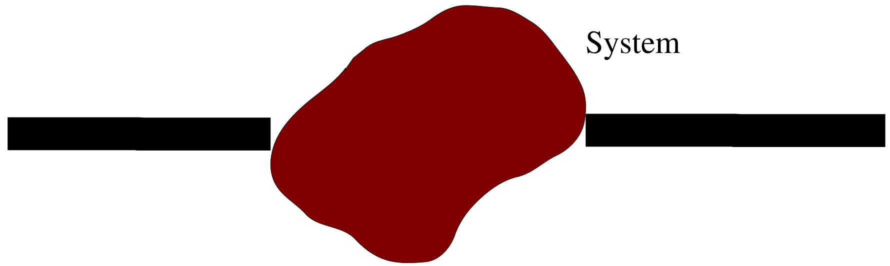
Of course, there are some well-known, quite general methods:

- linear-response theory.



Kubo: $$\chi_{AB}(t) = i\theta(t)\langle[\tilde{A}(t), B]\rangle_0$$

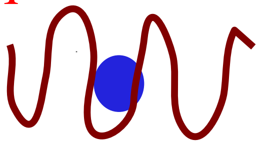
- Landauer-Büttiker scattering approach.



$$G = 2\frac{e^2}{h}\mathrm{Tr}T^\dagger T.$$

PHYSICAL REVIEW VOLUME 97, NUMBER 3 FEBRUARY 1, 1955

Equilibrium...



Slow Electrons in a Polar Crystal

R. P. FEYNMAN
California Institute of Technology, Pasadena, California
(Received October 19, 1954)

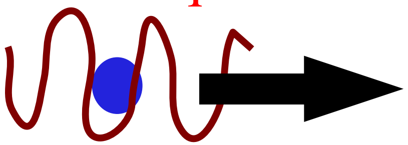
A variational principle is developed for the lowest energy of a system described by a path integral. It is

PHYSICAL REVIEW B VOLUME 1, NUMBER 10 15 MAY 1970

Velocity Acquired by an Electron in a Finite Electric Field in a Polar Crystal

K. K. THORNBER*† AND RICHARD P. FEYNMAN
California Institute of Technology, Pasadena, California 91109
(Received 24 November 1969)

...and non-equilibrium



(one electron)

procedure was quite successful in treating the ground-state energy of the polaron.[4] However, neither in FHIP nor in the present paper has a variational principle emerged for the impedance, in the former study, or for the field-velocity dependence developed here.

A physically very reason[illegible] for the

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Electronic Transport

Things are difficult. Start from something simple?

SMALL STUFF:

- Dimension 2 (2DEG), 1 (wires), 0 (few-level quantum systems).
- Single Electron Transistor.
- charge/flux/spin qubits (controllable two-level systems)

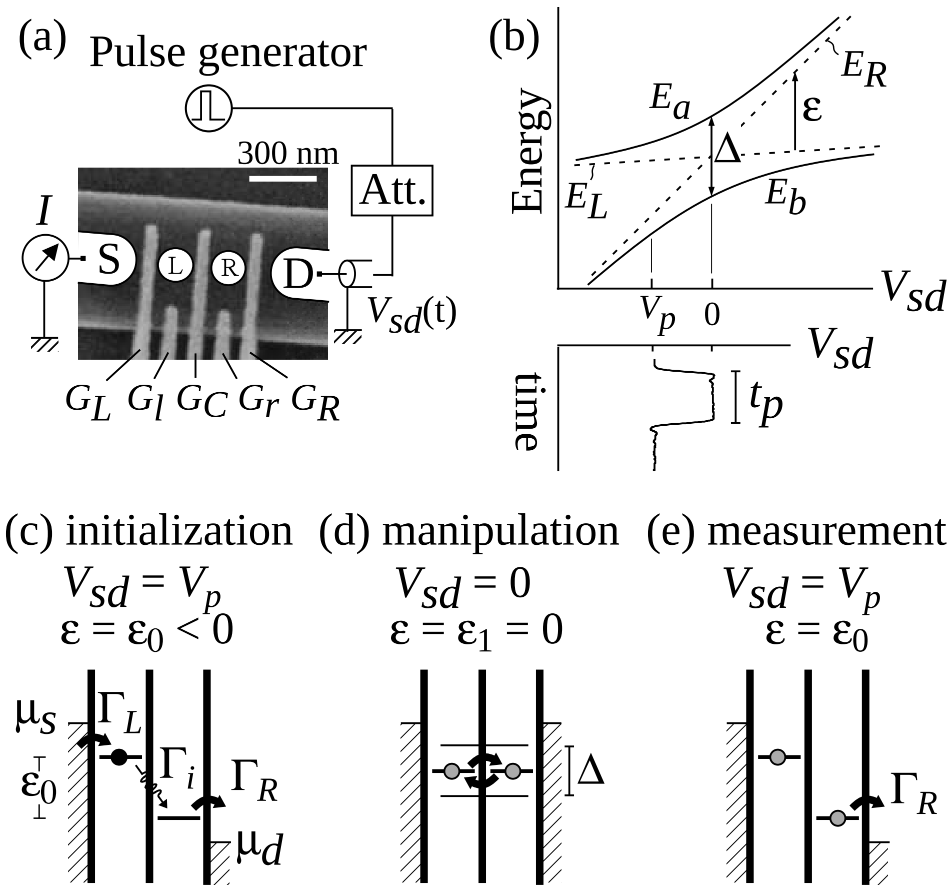
- tunneling $\rightsquigarrow$ quantum superpositions
- interactions $\rightsquigarrow$ entanglement
- environment $\rightsquigarrow$ decoherence

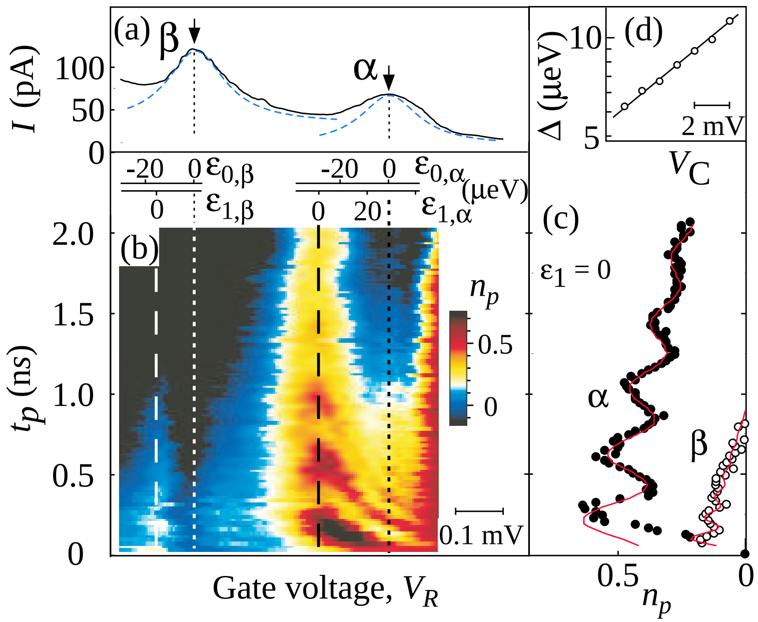
$\rightsquigarrow$ arena of *Mesoscopic Physics*.

VOLUME 91, NUMBER 22 PHYSICAL REVIEW LETTERS week ending 28 NOVEMBER 2003

Coherent Manipulation of Electronic States in a Double Quantum Dot

T. Hayashi,[1] T. Fujisawa,[1] H. D. Cheong,[2] Y. H. Jeong,[3] and Y. Hirayama[1,4]
[1]NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, 243-0198, Japan





cf. TB, T. Vorrath, PRB **66**, 075341 (2003); U. Hartmann, F. K. Wilhelm, PRB **67**, 161307 (2003), M. Thorwart, J. Eckel, E.R. Mucciolo cond-mat/0505621 (2005).

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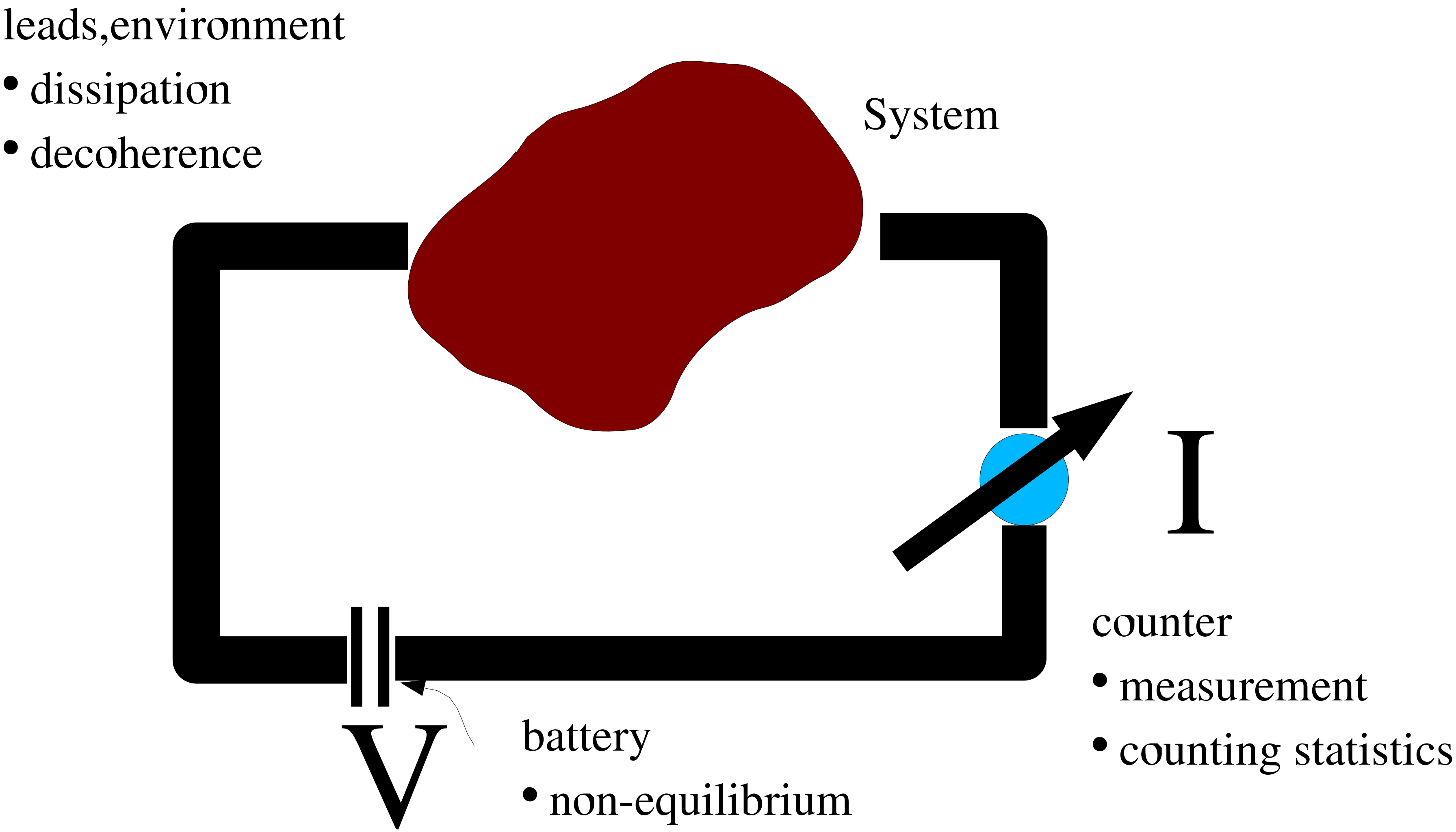
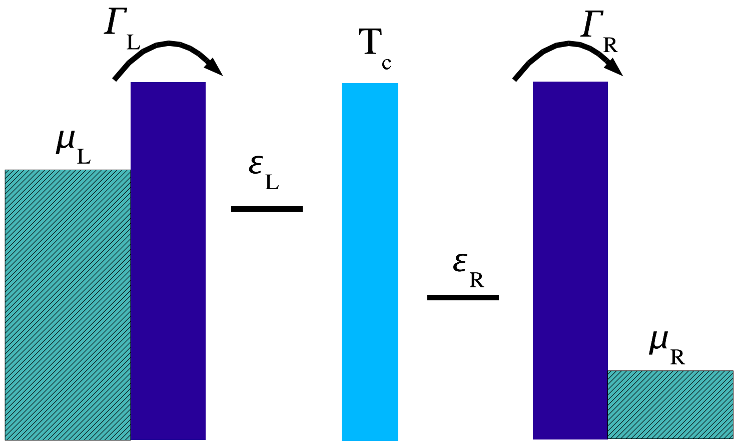
These are also useful in order to understand transport ‘from scratch’.



TRANSPORT = system + non-equilibrium + external world

Three-State Transport Model

- Transport model for the smallest quantum system: $SU(2)$ plus one empty state.
- $|L\rangle = |N_L+1, N_R\rangle$ 'left', $|R\rangle = |N_L, N_R+1\rangle$ 'right', $|0\rangle = |N_L, N_R\rangle$ 'empty'.



- internal bias $\varepsilon = \varepsilon_L - \varepsilon_R$, tunnel coupling T_c.

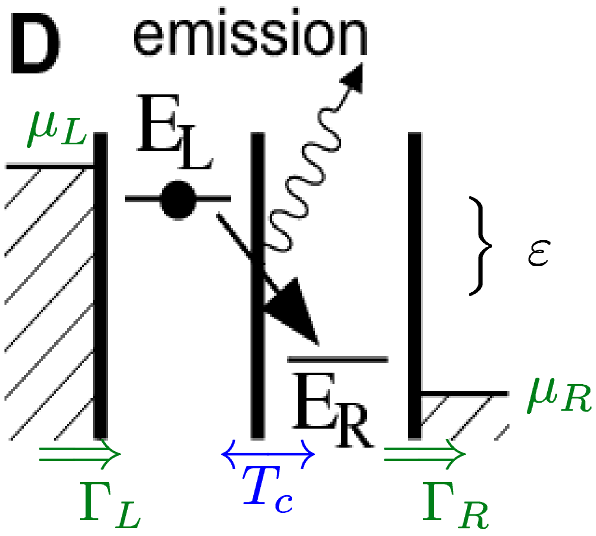
$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_{res} + \mathcal{H}_T, \quad \mathcal{H}_S = \frac{\varepsilon}{2}\hat{\sigma}_z + T_c\hat{\sigma}_x$$

$$\mathcal{H}_T = \sum_{k_i}(V_k^i c_{k_i}^\dagger |0\rangle\langle i| + H.c.), i = L, R.$$

One goal: calculate density operator ρ for $t \to \infty$. ρ has 4 (not 3) real parameters,

$$\rho = \begin{pmatrix} \rho_{00} & 0 & 0 \\ 0 & \rho_{LL} & \rho_{LR} \\ 0 & \rho_{RL} & \rho_{RR} \end{pmatrix}, \quad \rho_{00} = 1 - \rho_{LL} - \rho_{RR}.$$

Double Quantum Dots



3 states $|L\rangle$, $|R\rangle$, $|0\rangle$

$\hat{\sigma}_z \equiv |L\rangle\langle L| - |R\rangle\langle R|, \quad \hat{\sigma}_x \equiv |L\rangle\langle R| + |R\rangle\langle L|.$

$$\mathcal{H} = \mathcal{H}_{SB} + \mathcal{H}_{res} + \mathcal{H}_T$$

$$\mathcal{H}_T = \sum_{k_\alpha} (V_k^\alpha c_{k_\alpha}^\dagger |0\rangle\langle\alpha| + H.c.), \alpha = L, R$$

$$\mathcal{H}_{SB} = \left[\frac{\varepsilon}{2} + \sum_{\mathbf{Q}} \frac{g_Q}{2}\left(a_{-\mathbf{Q}} + a_{\mathbf{Q}}^\dagger\right)\right]\hat{\sigma}_z + T_c\hat{\sigma}_x + \mathcal{H}_B.$$

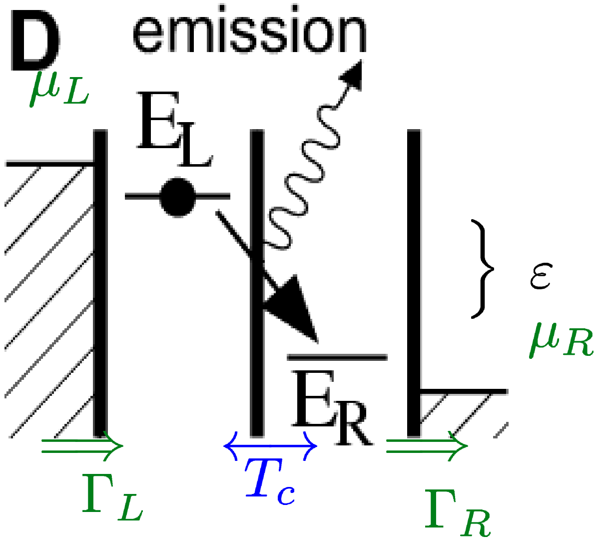
Loc-Deloc Transition at $\alpha = 1$, *Leggett et al 87*

- 'Internal' Parameter ε, T_c;

$$J(\omega) \equiv \sum_{\mathbf{Q}} |g_Q|^2 \delta(\omega - \omega_Q) = \begin{cases} 2\alpha\omega_{\text{ph}}^{1-s}\omega^s e^{-\frac{\omega}{\omega_c}} \\ \text{microscopic model: } \textit{Phonons...} \end{cases}$$

- 'External' parameters $\mu_L, \mu_R, \Gamma_\alpha(\varepsilon) = 2\pi \sum_{k_\alpha} |V_k^\alpha|^2 \delta(\varepsilon - \varepsilon_{k_\alpha}), \alpha = L/R$.

Formulation



- EOM for reduced density operator

$$\langle \mathbf{A}(t)\rangle = \langle \mathbf{A}(0)\rangle + \int_0^t dt' \left\{ M(t,t')\langle \mathbf{A}(t')\rangle + \Gamma_L \mathbf{e}_1 \right\}.$$

- $\mu_L - \mu_R \to \infty$ (Gurvitz, Prager 1996, Stoof, Nazarov 1996, Gurvitz 1998.)

- 'Memory Kernel'

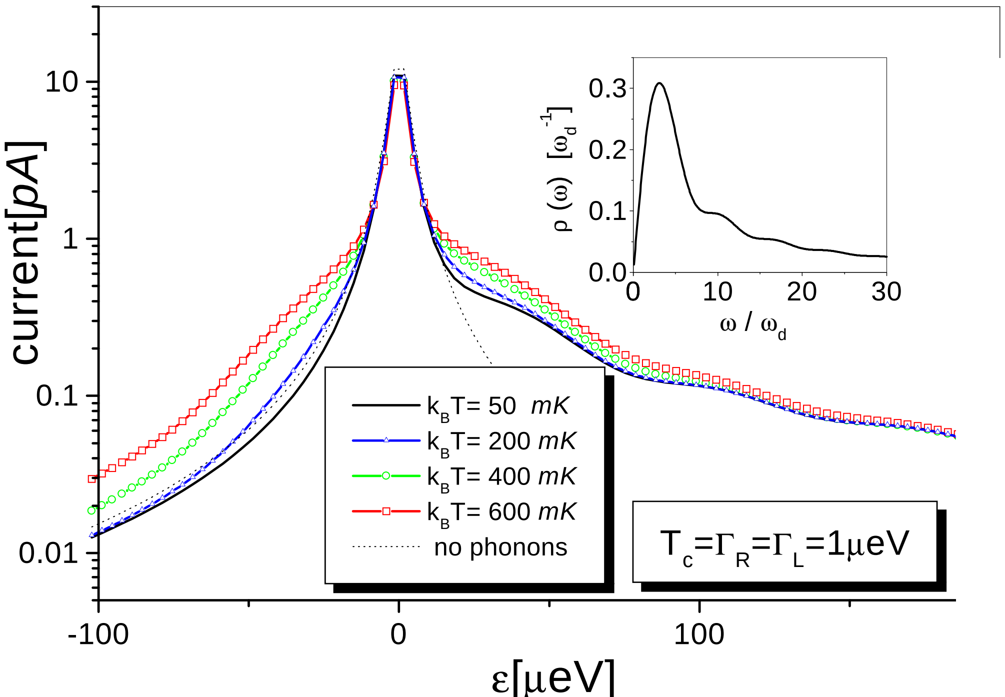
$$z\hat{M}(z) = \begin{bmatrix} -\hat{G} & \hat{T}_c \\ \hat{D}_z & \hat{\Sigma}_z \end{bmatrix}, \quad \hat{G} \equiv \begin{pmatrix} \Gamma_L & \Gamma_L \\ 0 & \Gamma_R \end{pmatrix}, \quad \hat{T}_c \equiv iT_c(\sigma_x - 1)$$

- Blocks $\hat{D}_z$, $\hat{\Sigma}_z$: Dephasing, Relaxation $\begin{cases} \text{PER} \\ \text{POL} \end{cases}$

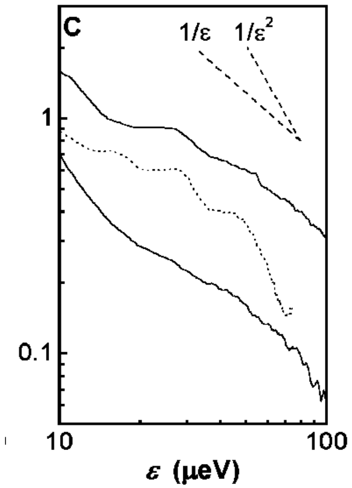
- Polaron-Transformation (POL) $\equiv$ NIBA (non-interacting blib approximation): calculate $\hat{D}_z$ and $\hat{\Sigma}_z$ using bosonic correlation function

$$C_\varepsilon^{[*]}(z) \equiv \int_0^\infty dt e^{-zt} e^{[-]i\varepsilon t} \exp(-\int_0^\infty d\omega \frac{J(\omega)}{\omega^2} \left[(1-\cos\omega t)\coth\left(\frac{\beta\omega}{2}\right) \pm i \sin\omega t\right]).$$

- Polaron tunneling $\rightsquigarrow$ 'boson shake-up' effect
- Re$[C_\varepsilon(z)]|_{z=\pm i\omega} = \pi P(\varepsilon \mp \omega)$: P(E)-Theory.



TB *et al* 98-00; Guinea et al 00; Keil, Schöller 02



T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tarucha, and L. P. Kouwenhoven, Science **282**, 932 (1998)

$\propto \varepsilon^{1+2\alpha}, \alpha \approx 0.1$

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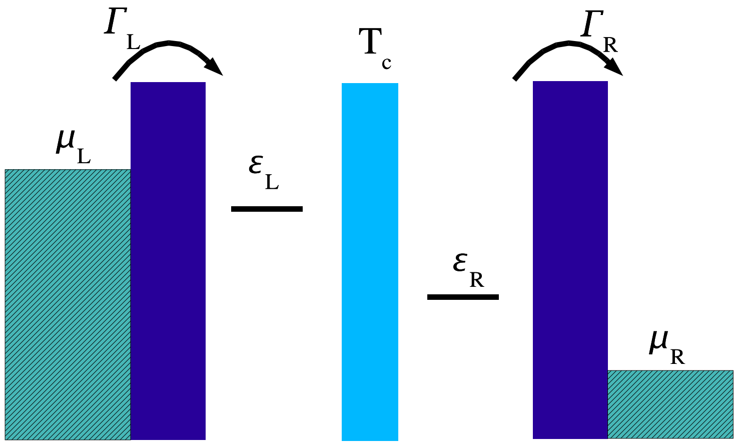
- Double quantum dots, strong Coulomb blockade $U \to \infty$.



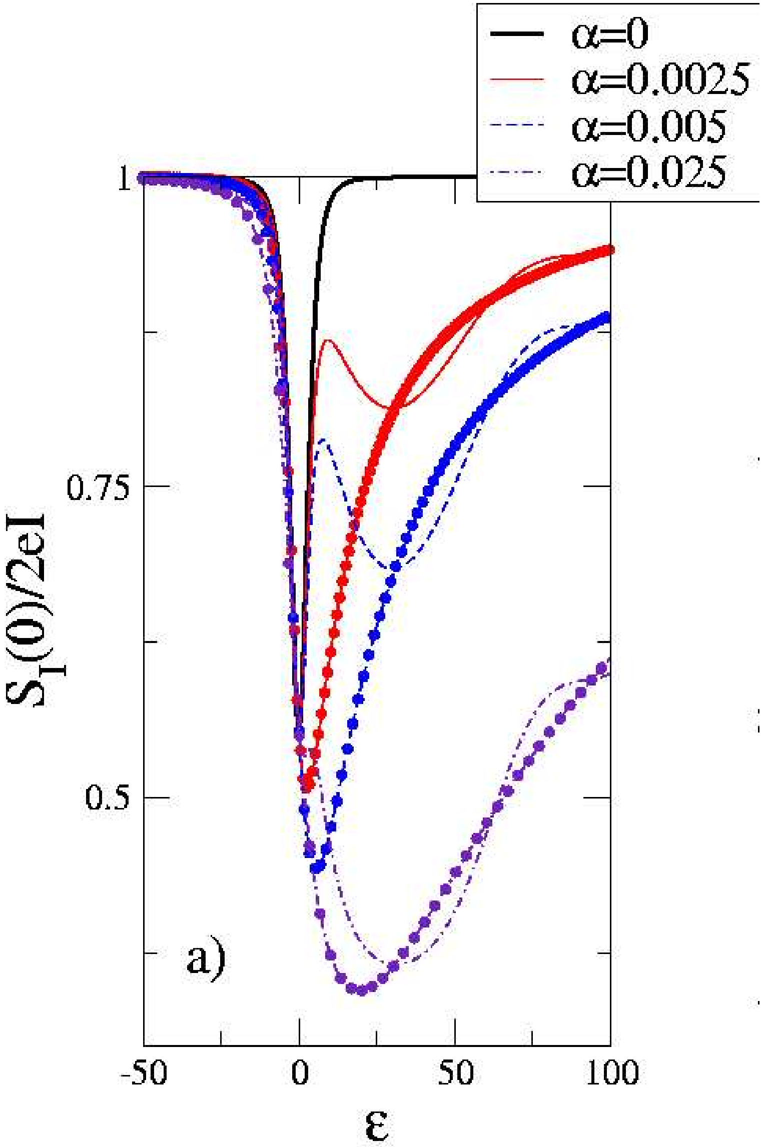
- Complicated problem for any bias $|\mu_L - \mu_R| < \infty$.
- Only $\mu_L - \mu_R \to \infty$ relatively easy. Then, exact (?) solution in Markovian limit (flat tunneling DOS, no memory).
- External tunnel rates; $\Gamma_i(\varepsilon) = 2\pi \sum_{k_i} |V_k^i|^2 \delta(\varepsilon - \varepsilon_{k_i})$.

- Solve Liouville-von-Neumann eq. $\rightsquigarrow$ stationary current (Stoof-Nazarov 1996, Gurvitz 1996)

$$\langle \hat{I} \rangle_{t\to\infty}^{\mathrm{SN}} = -e \frac{T_c^2 \Gamma_R}{\Gamma_R^2/4 + \varepsilon^2 + T_c^2(2 + \Gamma_R/\Gamma_L)},$$

- Just Breit-Wigner. However, zero current for $\Gamma_R \to \infty$... quantum Zeno effect (continuous measurement version, Koshino, Shimizu PRL 2004): right lead as detector with ∞ bandwidth.

Fano Factor



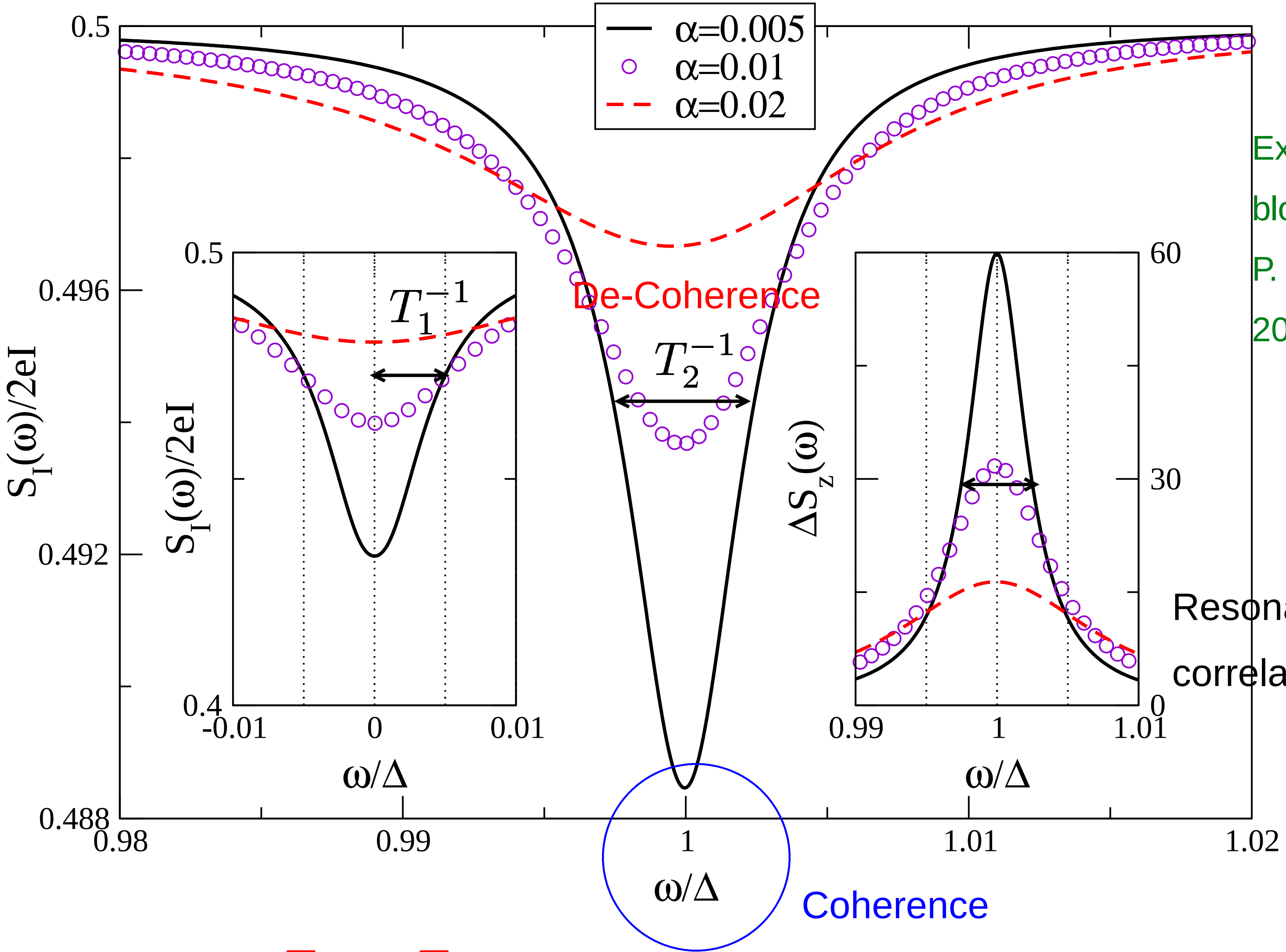
- Non-equilibrium noise spectrum

$$\mathcal{S}_{\mathcal{II}}(\omega) \equiv \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle \{\Delta\hat{I}(\tau), \Delta\hat{I}(0)\}\rangle.$$

- (dissipation $\alpha = 0$) coherence suppresses noise: minimum at internal bias $\varepsilon = 0$.
- ($\alpha = 0$) large $|\varepsilon|$ 'localises' charge (L or R).
- ($\alpha \neq 0$) for $\varepsilon > 0$: dissipation suppresses noise (like in disordered metallic systems, A. Shimizu, M. Ueda (1992)). Maximal for $\gamma_p = \Gamma_R$.

Frequency dependent noise spectrum

R. Aguado, TB, Phys. Rev. Lett. **92**, 206601 (2004), Eur. Phys. J. B **40**, 357 (2004).

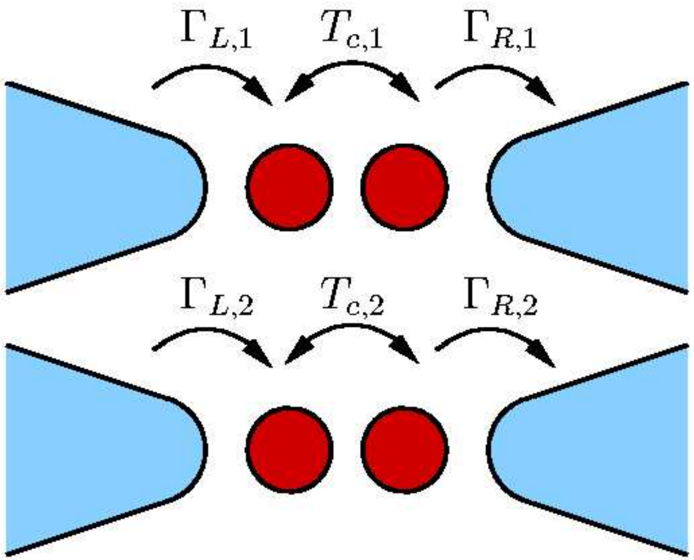


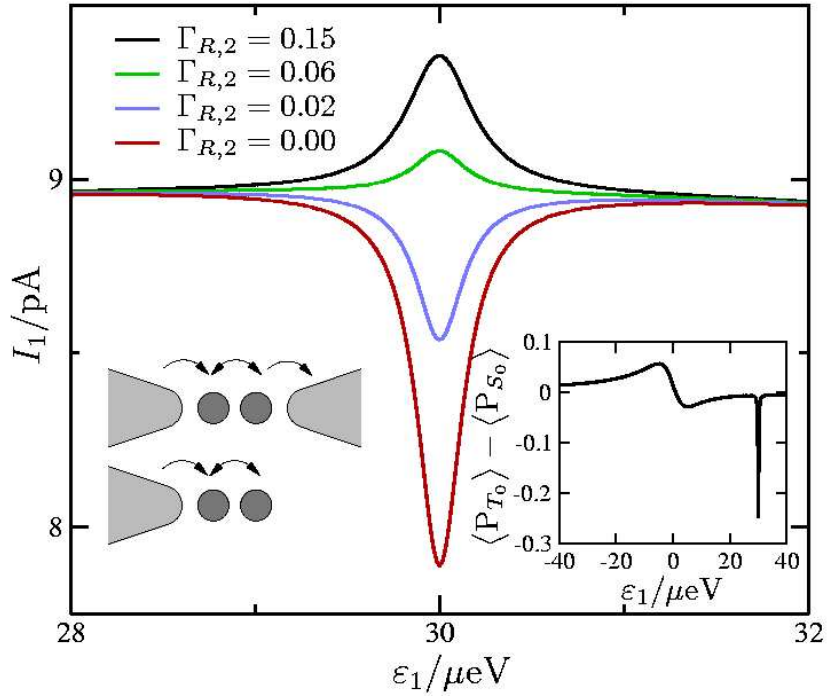
Exp. Cooper-Pair Box: R. Deblock, E. Onac, L. Gurevich, L. P. Kouwenhoven, Science **301**, 203 (2003)

Resonance as Pseudo-Spin-correlation function

contains T_1 und T_2 (PER) !

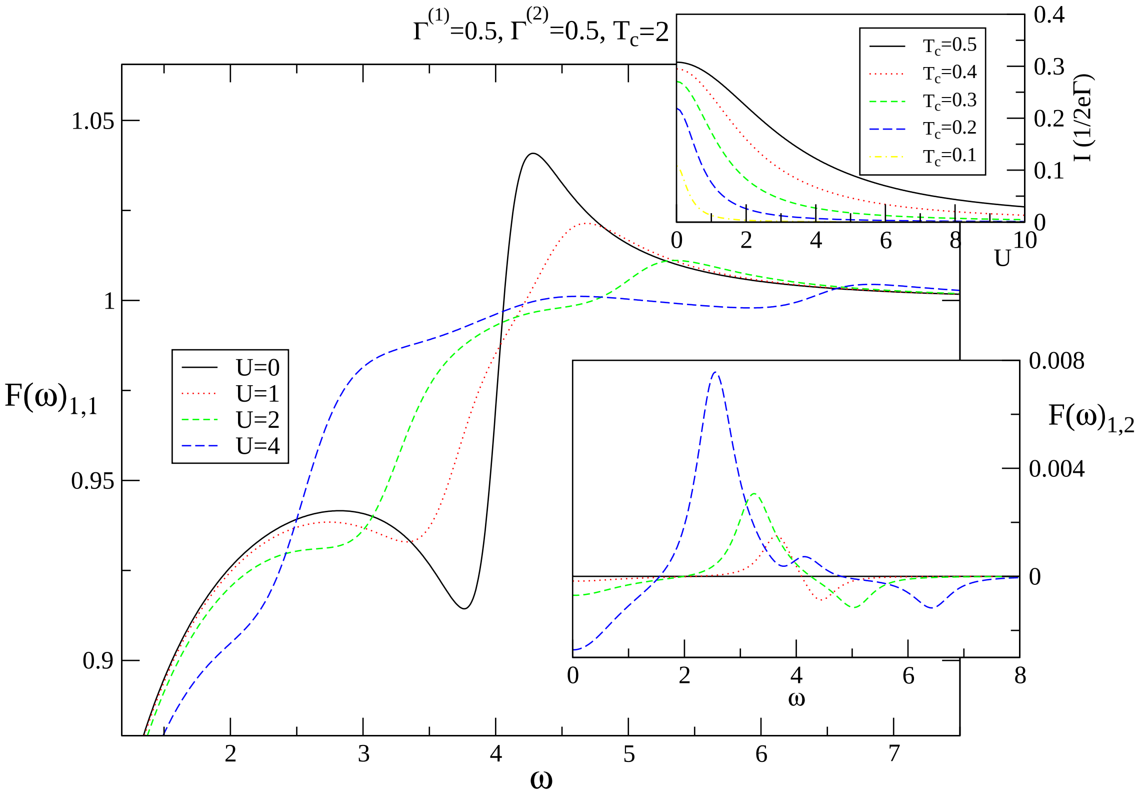
Transport through coupled 2-Qubits





- *Phonon* coupling: effective interaction, Dicke effect T. Vorrath, TB, PRB 2003.
- *Coulomb* coupling: many-body ‘toy model’ (two site Hubbard with spin) N. Lambert, TB 2005.

Two Double Quantum Dots: Coulomb-Coupling U



- Current suppressed with increasing U.
- Noise resonances at

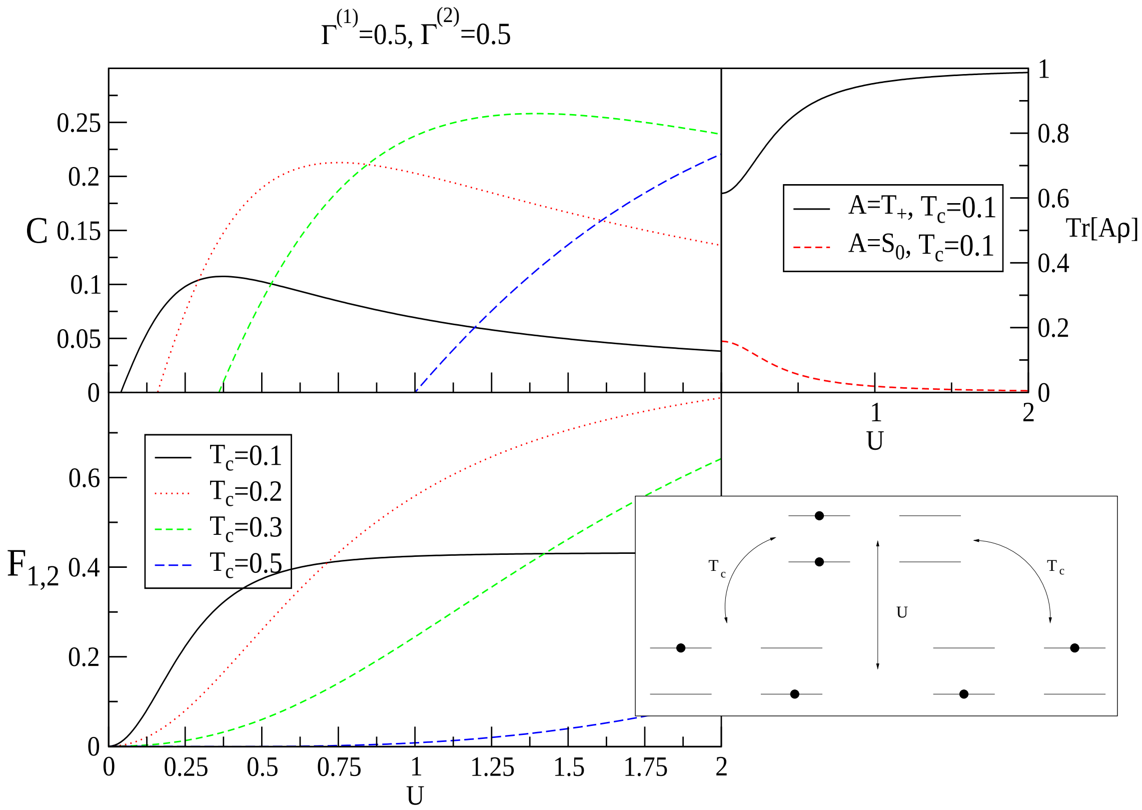
$$\omega = \frac{1}{2}(U \pm \sqrt{16T_c^2 + U^2}).$$

- Cross noise spectrum.

(cf. quantum dot stack; Sprekeler, Kießlich, Wacker Schöll 2004).

N. Lambert (2005).

Two Double Quantum Dots: Entanglement



- *Concurrence* and cross-Fano-factor.
- Zero concurrence below certain U: state too mixed in order to be entangled.
- Better analytical grip needed - but how.

N. Lambert (2005).

Three-State Transport Model

Looks simple but has some very intriguing properties (some are yet to be discovered...)

- Infinite bias, but additional, internal dissipation etc:

- ‘transport pseudo-spin-boson’ model: quantum noise, Full Counting Statistics.
- QIP tasks, Q-Optics effects, NEMS stuff (single phonon).
- ...

- Finite bias: not much is known...

- Higher order lead tunneling: orbital Kondo effect + precursors.
- Various techniques: Schrieffer-Wolff trafo. Real-time RG; numerical RG; ...

Here, some future progress is expected.

TB, Phys. Rep. **408**, 315 (2005).