Shot Noise Spectrum of Open Dissipative Quantum Two-level Systems

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- Motivation: qubits, noise experiment(s)
- Noise spectrum calculation
- Some open problems

Discussions/Remarks: M. Büttiker, T. Hayashi, L. P. Kouwenhoven, T. Novotný.

Motivation

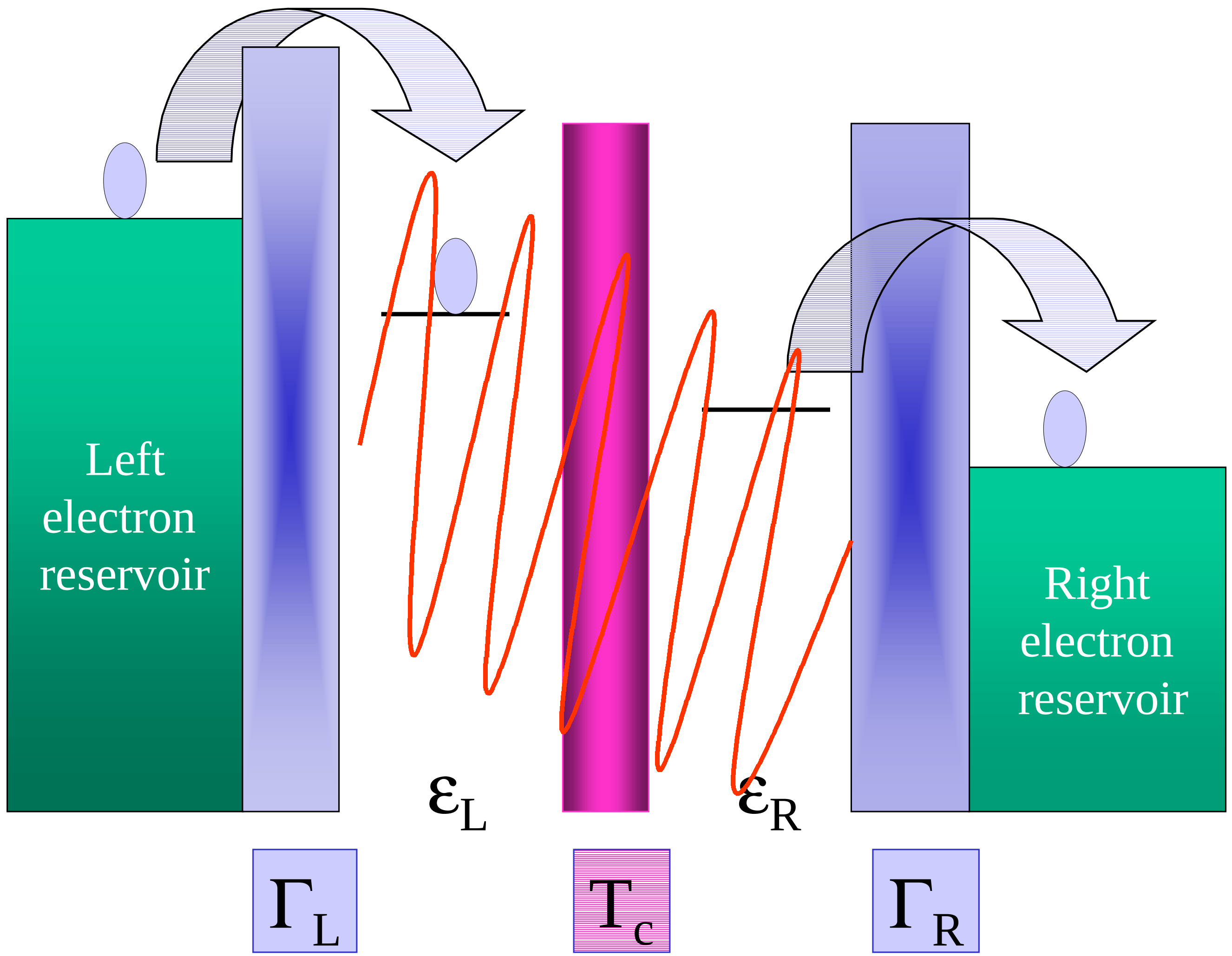
- Noise in mesoscopic conductors; M. Büttiker, Phys. Rev. B **46**, 12485 (1992); Y. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1, (2000).
- How does dissipation effect shot noise ? A. Shimizu, M. Ueda; 'Effects of Dephasing and Dissipation on Quantum Noise in Conductors', Phys. Rev. Lett. **69**, 1403 (1992); M. Ueda and A. Shimizu, J. Phys. Soc. Jpn. **62**, 2994 (1993).
- Extract quantum coherence (e.g., Δ: M.-S. Choi, F. Plastina, R. Fazio; Phys. Rev. B **67**, 045105, 2003) and dephasing time 'T_2' from ω-dependent noise spectrum. Experiment: R. Deblock, E. Onac, L. Gurevich, and L. P. Kouwenhoven, Science **301**, 203 (2003).
- Detection of particle entanglement in noise.

Model: 'Transport' Spin-Boson Hamiltonian

- Transport through **double quantum dot** / Cooper pair box / single molecule: different systems, but similar formalism.
- States $|L\rangle$, $|R\rangle$, $|0\rangle$.
- Hamiltonian

$$
\begin{aligned}
\mathcal{H} &= \mathcal{H}_{SB} + \mathcal{H}_{res} + \mathcal{H}_T \\
\mathcal{H}_{SB} &= \left[\frac{\varepsilon(t)}{2} + \sum_{\mathbf{Q}} \frac{g_Q}{2}\left(a_{-\mathbf{Q}} + a^\dagger_{\mathbf{Q}}\right)\right]\hat{\sigma}_z + T_c(t)\hat{\sigma}_x + \mathcal{H}_B \\
\mathcal{H}_T &= \sum_{k_\alpha}(V_k^\alpha c^\dagger_{k_\alpha} s_\alpha + H.c.), \quad \hat{s}_\alpha = |0\rangle\langle\alpha|, \alpha = L, R \\
\hat{\sigma}_z &\equiv |L\rangle\langle L| - |R\rangle\langle R| \equiv \hat{n}_L - \hat{n}_R, \quad \hat{\sigma}_x \equiv |L\rangle\langle R| + |R\rangle\langle L| \equiv \hat{p} + \hat{p}^\dagger.
\end{aligned}
$$

- $\mathcal{H}_{res}$ electron reservoirs, $\mathcal{H}_B$ boson Hamiltonian.


Left electron reservoir
εL
εR
Right electron reservoir
ΓL
Tc
ΓR

Dissipation vs. Nano-Mechanics

- **'Dissipative Qubit':** infinitely many bosonic modes.

$$\mathcal{H}_{SB} = \left[\frac{\varepsilon}{2} + \sum_{\mathbf{Q}} \frac{g_Q}{2}\left(a_{-\mathbf{Q}} + a^{\dagger}_{\mathbf{Q}}\right)\right]\hat{\sigma}_z + T_c\hat{\sigma}_x + \mathcal{H}_B$$

$$J(\omega) \equiv \sum_{\mathbf{Q}} |g_Q|^2 \delta(\omega - \omega_Q) = \begin{cases} 2\alpha\omega_{\mathbf{ph}}^{1-s}\omega^{s}e^{-\omega/\omega_c} \\ \text{other spectral density} \end{cases}$$

- **'Nano-mechanics model':** single bosonic mode. 'Transport' Rabi Hamiltonian, TB, N. Lambert, Phys. Rev. B **67**, 125323 (2003).

$$\mathcal{H}_{SB} = \left[\frac{\varepsilon}{2} + \frac{g}{2}\left(a + a^{\dagger}\right)\right]\hat{\sigma}_z + T_c\hat{\sigma}_x + \mathcal{H}_B$$

  – Noise in shuttles; D. Mozyrsky, I. Martin, PRL **89**, 018301 (2002); A. D. Armour, cond-mat/0401387 (2004); T. Novotný *et al.*, PRL **92**, 248302 (2004); C. Flindt, T. Novotný, A.-P. Jauho, cond-mat/0405512.

Memory Kernel

- Equations of Motion for expectation values $\mathbf{A} \equiv (\hat{n}_L, \hat{n}_R, \hat{p}, \hat{p}^\dagger)$ with $\hat{p} \equiv |L\rangle\langle R|$ is

$$\langle \mathbf{A}(t)\rangle = \langle \mathbf{A}(0)\rangle + \int_0^t dt' \left\{ M(t-t')\langle \mathbf{A}(t')\rangle + \Gamma_L \mathbf{e}_1 \right\}.$$

- Memory kernel $M(t-t')$ contains complete information on dissipative two-state dynamics. In Laplace space

$$z\hat{M}(z) = \begin{bmatrix} -\hat{G} & \hat{T}_c \\ \hat{D}_z & \hat{\Sigma}_z \end{bmatrix}, \quad \hat{G} \equiv \begin{pmatrix} \Gamma_L & \Gamma_L \\ 0 & \Gamma_R \end{pmatrix}, \quad \hat{T}_c \equiv iT_c(\sigma_x - 1)$$

- Reservoirs coupling in Born and Markov approximation, $\Gamma_\alpha = 2\pi \sum_{k_\alpha} |V_k^\alpha|^2 \delta(\epsilon - \epsilon_{k_\alpha})$, $\alpha = L/R$. Exact for source-drain-bias $\mu_L - \mu_R \to \infty$ (Gurvitz, Prager 1996, Stoof, Nazarov 1996, Gurvitz 1998.)
- Blocks $\hat{D}_z$, $\hat{\Sigma}_z$ are determined by the EOM for the coherences $\langle \hat{p}\rangle = \langle \hat{p}^\dagger\rangle^*$. Contain the complete information on dephasing and relaxation.

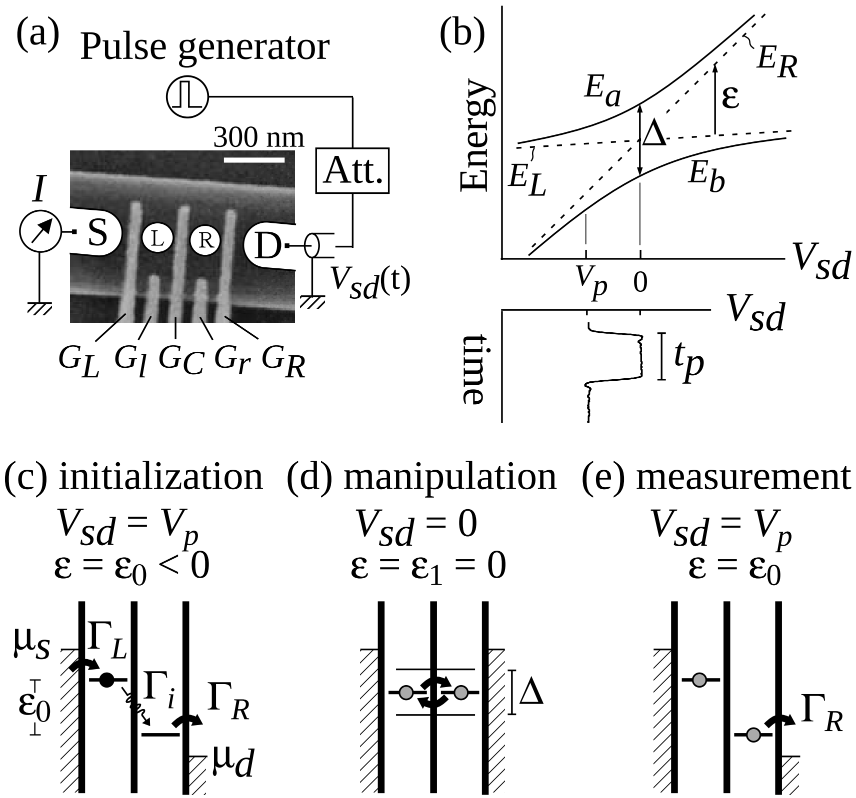
PER and POL Approach

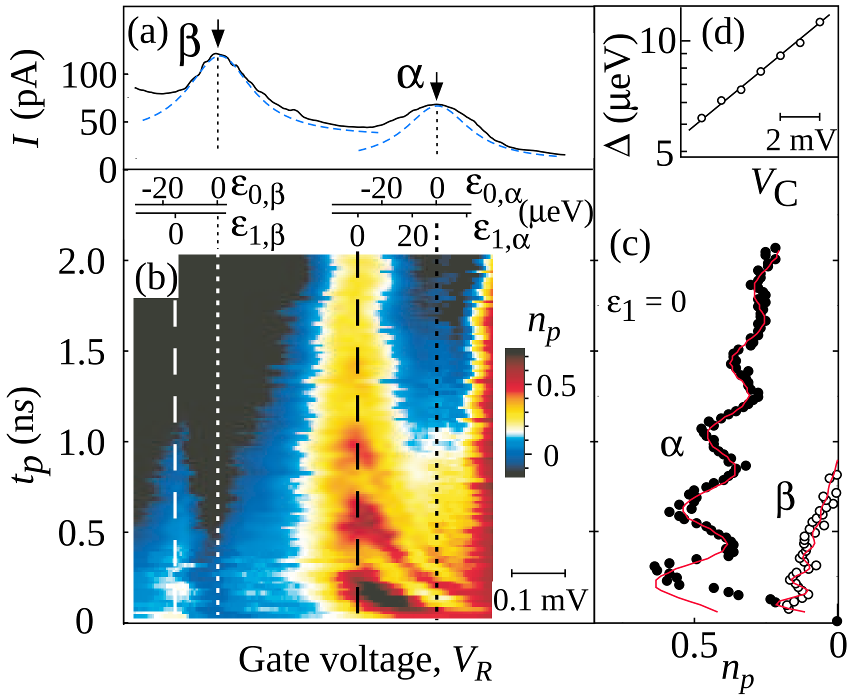
- Choice between
  - perturbation theory (PER) for weak dissipation g_Q in correct basis of the hybridised states of TLS
  - polaron transformation (POL) for strong dissipation (perturbative in coherent tunneling T_c)
- PER expression uses scattering rates

$$\hat{D}^{\mathrm{PER}} = \hat{T}_c + \begin{pmatrix} \gamma_+ & -\gamma_- \\ \gamma_+ & -\gamma_- \end{pmatrix}, \quad \hat{\Sigma}^{\mathrm{PER}} = \begin{pmatrix} E & 0 \\ 0 & E^* \end{pmatrix}, \text{ where}$$

$E = i\varepsilon - \gamma_p - \frac{\Gamma_R}{2}$, $\gamma_p := 2\pi \frac{T_c^2}{\Delta^2} J(\Delta) \coth(\beta\Delta/2)$ and
$\gamma_\pm \equiv -\frac{\varepsilon T_c}{\Delta^2} \frac{\pi}{2} J(\Delta) \coth(\beta\Delta/2) \mp \frac{T_c}{\Delta} \frac{\pi}{2} J(\Delta)$ with $\Delta = \sqrt{\varepsilon^2 + 4T_c^2}$.

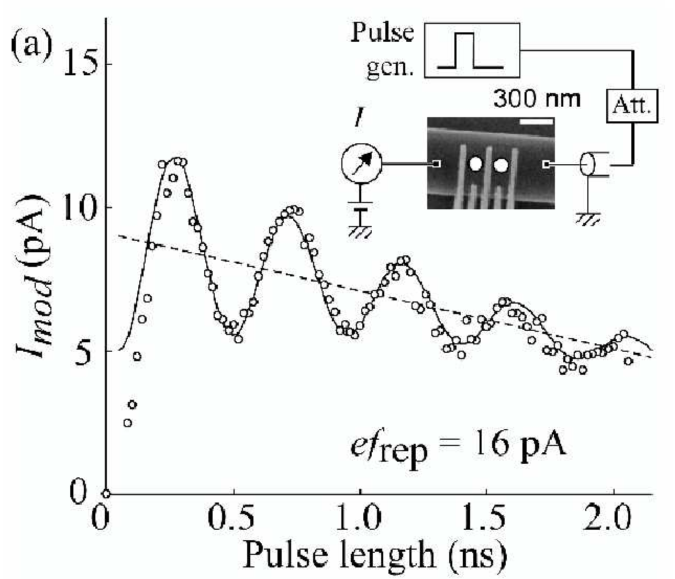
- Non-Markovian corrections (lowest order/ 4th order): D. Loss and D. P. DiVincenzo, cond-mat/0304118 (2003), D. P. DiVincenzo, D. Loss, cond-mat/0405525.

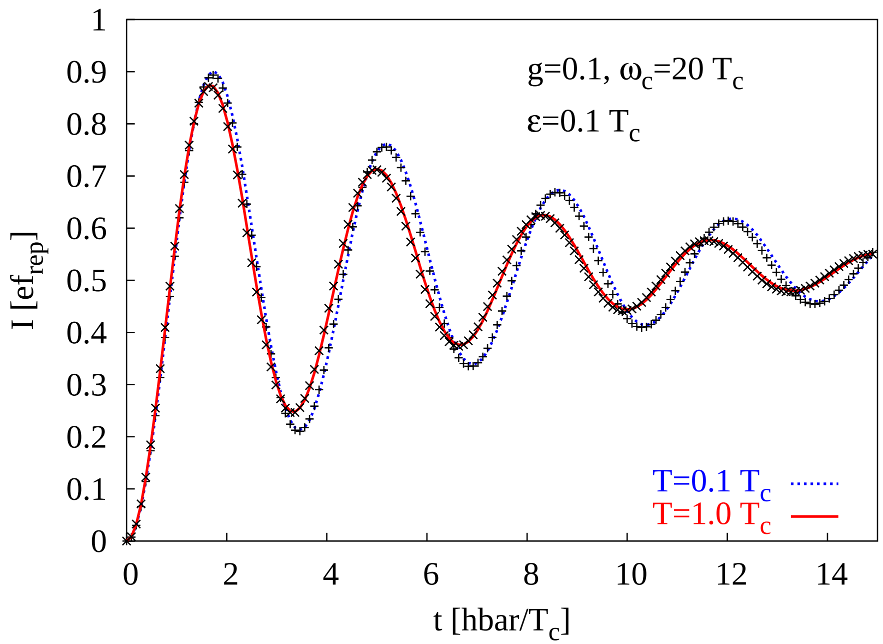




(Experiment) Double dot qubit; T. Hayashi, T. Fujisawa, H. D. Cheong, Y. H. Jeong, and Y. Hirayama, Phys. Rev. Lett. **91**, 226804 (2003).

Left: Sample and pulse sequence. **Right:** Non-linear current profile near two resonance peaks, α and β. (b) Mean dot occupancy $n_p \equiv I_p/ef_{\rm rep}$; V_R (inter-dot bias ε); pulse duration t_p. (c) Coherent oscillations for α and β. (d) Central gate voltage dependence of tunnel coupling $\Delta = 2T_c$.





Perturbation in $\alpha = 2g$, TB, T. Vorrath Phys. Rev. B **66**, 075341 (2002).

$$
\begin{aligned}
\langle n_R \rangle_t \quad &\approx \quad \frac{2T_c^2}{E^2}\left\{ \kappa + \left[\kappa \frac{\gamma_1}{\Gamma_p} - \frac{\varepsilon \mathrm{Re}\gamma_+}{T_c \Gamma_p} \right] \left(1 - e^{-\Gamma_p t}\right) \right. \\
&- \quad \left. e^{-(\frac{\Gamma_p}{2}+\gamma_1)t} \left[\left(\frac{\kappa \Gamma_p}{2E} - \frac{\varepsilon \mathrm{Re}\gamma_+}{E T_c} \right) \sin Et + \kappa \cos Et \right] \right\}, \quad \kappa \equiv 1 - \frac{\mathrm{Im}\gamma_+}{T_c}.
\end{aligned}
$$

POL approach

- Polaron trafo (POL) for strong electron-boson hybridisation, leads to integral equation. Boson correlation function

$$C(t) \equiv \exp\left(-\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\left[(1-\cos\omega t)\coth\left(\frac{\beta\omega}{2}\right) + i\sin\omega t\right]\right);$$

- Introduce $C_\varepsilon^{[*]}(z) \equiv \int_0^\infty dt e^{-zt} e^{[-]i\varepsilon t} C^{[*]}(t)$ to obtain

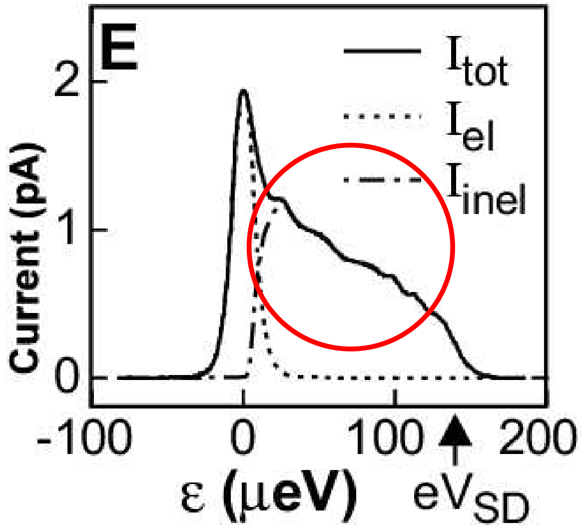
$$\hat{D}_z^{\mathrm{POL}} = iT_c \begin{pmatrix} -1 & \hat{C}^*_{-\varepsilon}(z)/\hat{C}_\varepsilon(z) \\ 1 & -\hat{C}_{-\varepsilon}(z)/\hat{C}^*_\varepsilon(z) \end{pmatrix}, \quad \hat{\Sigma}_z^{\mathrm{POL}} = \begin{pmatrix} \tilde{E} & 0 \\ 0 & \tilde{E}^* \end{pmatrix}, \text{ where}$$

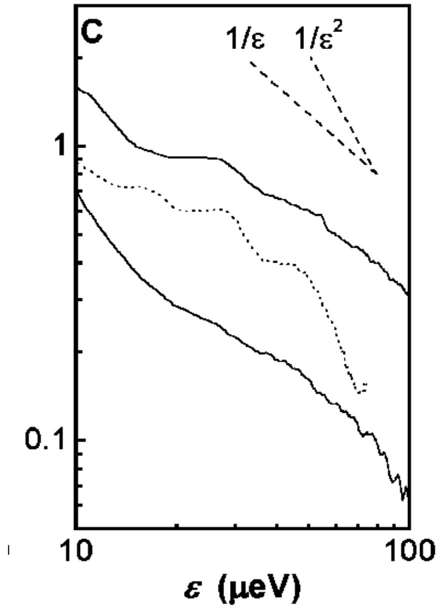
$\tilde{E}^{[*]} \equiv z - 1/C_\varepsilon^{[*]}(z) - \Gamma_R/2$, $C_\varepsilon^{[*]}(z) \equiv \int_0^\infty dt e^{-zt} e^{[-]i\varepsilon t} C^{[*]}(t)$,

$$\boxed{C_\varepsilon^{[*]}(z) \equiv \int_0^\infty dt e^{-zt} e^{[-]i\varepsilon t} \exp\left(-\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\left[(1-\cos\omega t)\coth\left(\frac{\beta\omega}{2}\right) \pm i\sin\omega t\right]\right).}$$

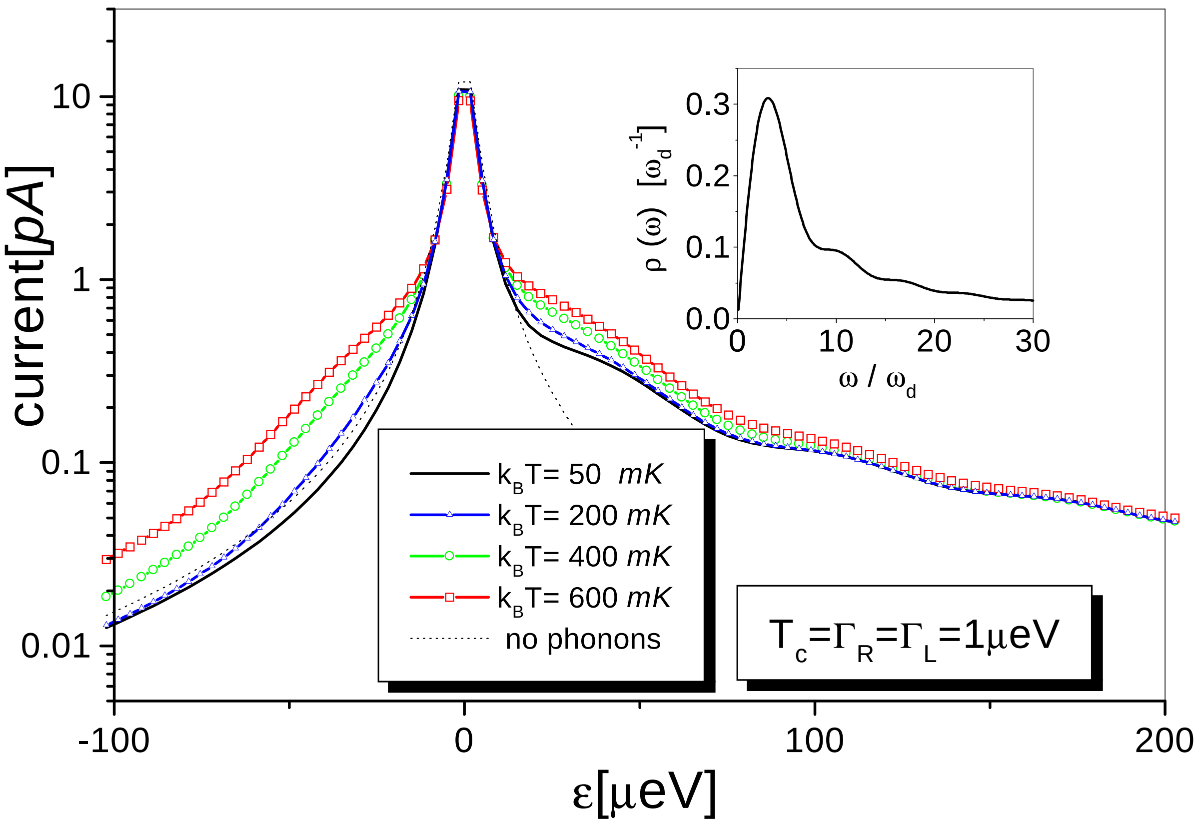
- Relation to $P(E)$-theory, 'Golden-Rule': $\mathrm{Re}[C_\varepsilon(z)]|_{z=\pm i\omega} = \pi P(\varepsilon \mp \omega)$, where $P(\varepsilon)$ is the probability for inelastic tunneling with energy transfer ε.

Experiments on spontaneous emission of phonons, T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tarucha, and L. P. Kouwenhoven, Science **282**, 932 (1998)





Left: Stationary current ($T = 23$mK) as a function of the energy difference ε.
Right: 'Power law dependence'; POL predicts a dependence $\sim 1/\varepsilon^{1+2\alpha}$ (boson shake-up orthogonality catastrophe).



(Calculation) Stationary tunnel current through double quantum dot as a function of the energy difference ε between left and right dot ground state energies. POL approach, dimensionless electron-phonon coupling parameter $\alpha = 0.025$, 3d piezo-electric. Inset: boson spectral density.

Dynamical Fluctuations: Quantum Noise

- Equilibrium (thermal, Nyquist) noise: related to linear conductance by fluctuation-dissipation theorem.
- Shot noise power spectral density

$$\mathcal{S}_{\mathcal{I}}(\omega) \equiv 2\int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\mathcal{S}_I(\tau) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle\{\Delta\hat{I}(\tau), \Delta\hat{I}(0)\}\rangle.$$

- Fano factor $\gamma \equiv \frac{\mathcal{S}_I(0)}{2qI}$ is unity for Poissonian (uncorrelated) noise.
- Current conservation $I_L - I_R = \dot{Q}$, $I = aI_L + bI_R$, $a + b = 1 \longrightarrow$,

$$\boxed{S_I(\omega) = aS_{I_L}(\omega) + bS_{I_R}(\omega) - ab\omega^2 S_Q(\omega).}$$

with capacitance coefficients $a = C_R/(C_L + C_R)$, $b = C_L/(C_L + C_R)$

(D. Mozyrsky, L. Fedichkin, S. A. Gurvitz, G. P. Berman, Phys. Rev. B **66**, 161313 (2002).)

- Charge fluctuations

$$\begin{aligned} S_Q(\omega) &\equiv \lim_{t\to\infty}\int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle\{\hat{Q}(t),\hat{Q}(t+\tau)\}\rangle = 2\mathrm{Re}\left\{\hat{f}(z=i\omega)+\hat{f}(z=-i\omega)\right\} \\ f(\tau) &\equiv \langle\hat{n}_L(t)\hat{n}_L(t+\tau)\rangle + \langle\hat{n}_L(t)\hat{n}_R(t+\tau)\rangle \\ &+ \langle\hat{n}_R(t)\hat{n}_L(t+\tau)\rangle + \langle\hat{n}_R(t)\hat{n}_R(t+\tau)\rangle \end{aligned}$$

- Use quantum regression theorem with resolvent $[z - z\hat{M}(z)]^{-1}$.

  (M. Lax 1960s, quantum optics books)

Qubit Dynamics and Reservoirs

- Fluctuations of $I_R(t) = e\sum_n n\dot{P}_n(t)$ with prob. of finding n electrons in the collector at time t, $P_n(t) = n_0^{(n)}(t) + n_L^{(n)}(t) + n_R^{(n)}(t)$.
- Introduce additional degree of freedom n, write EOMs

$$\dot{n}_0^{(n)} = -\Gamma_L n_0^{(n)} + \Gamma_R n_R^{(n-1)}, \quad \dot{n}_{L/R}^{(n)} = \pm\Gamma_{L/R} n_0^{(n)} \pm iT_c\left(p^{(n)} - [p^{(n)}]^\dagger\right), \quad \text{etc.}$$

- MacDonalds formula, D. K. C. MacDonald, Rep. Progr. Phys. **12**, 56, 1948; R. Ruskov, A. K. Korotkov, Phys. Rev. B **67**, 075303, 2003

$$S_{I_R}(\omega) = 2\omega e^2 \int_0^\infty dt \sin(\omega t) \frac{d}{dt}\left[\langle n^2(t)\rangle - (t\langle I\rangle)^2\right].$$

- Alternative: electron counting statistics 'counting charge without breaking the circuit', L. S. Levitov, H. Lee, G. D. Lesovik, J. Math. Phys. **37**, 4845 (1996).

- From n-dependent EOM

$$S_{I_R}(\omega) = 2eI\left\{1 + \Gamma_R\left[\hat{n}_R(-i\omega) + \hat{n}_R(i\omega)\right]\right\}$$

$$z\hat{n}_R(z) = \frac{\Gamma_L g_+(z)}{\left\{z + \Gamma_R + g_-(z)\right\}(z + \Gamma_L) + (z + \Gamma_R + \Gamma_L)g_+(z)}$$

$$g_{+[-]}(z) = \pm iT_c(\mathbf{e}_1 - \mathbf{e}_2)\left[z - \hat{\Sigma}_z\right]^{-1}\hat{D}_z\mathbf{e}_{1[2]}.$$

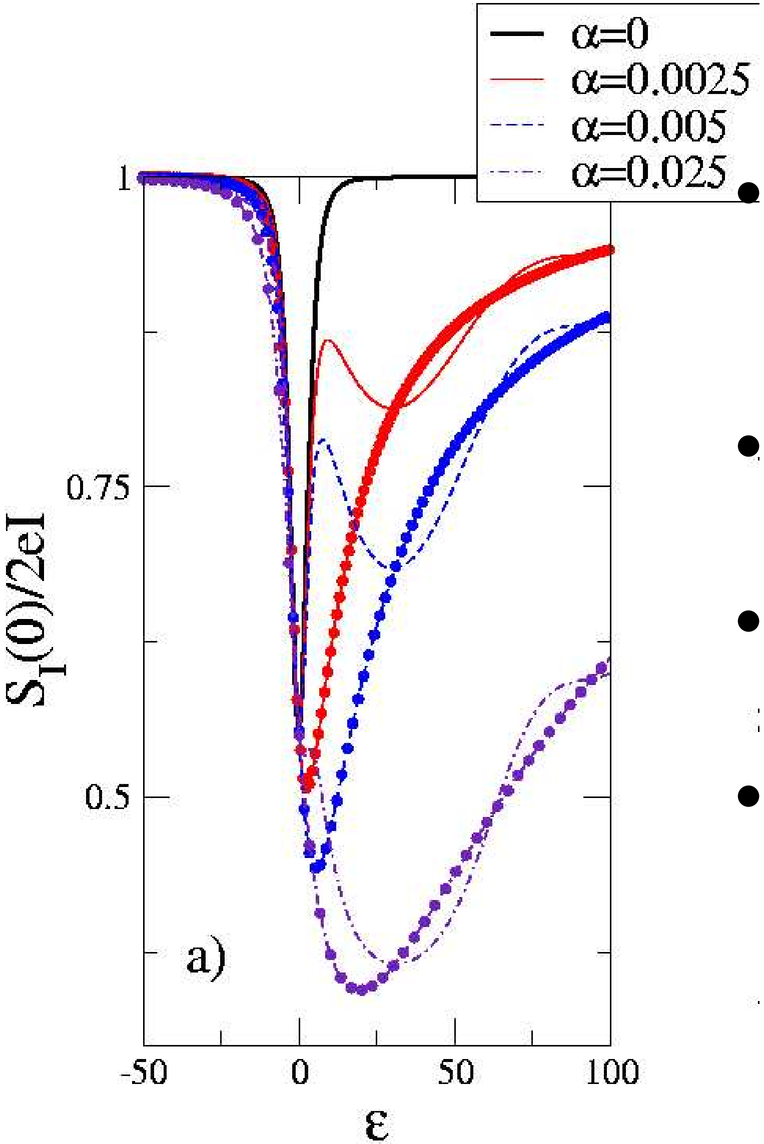
R. Aguado, TB, Phys. Rev. Lett. **92**, 206601 (2004)

Results: Zero Frequency (Shot Noise)

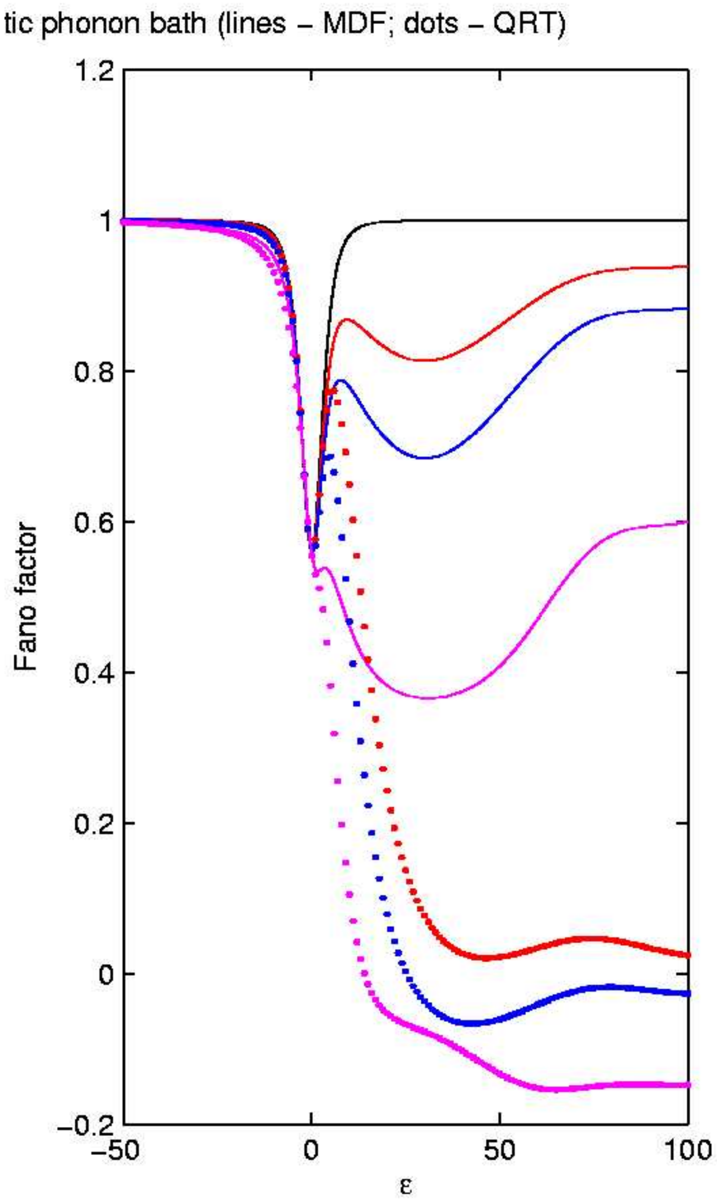
$$\frac{S(\omega=0)}{2eI} = 1 + 2\Gamma_R \frac{d}{dz}\left[z\hat{n}_R(z)\right]_{z=0} (\neq 1 - T_{\rm eff}).$$

- Can not be written in Khlus-Lesovik form $S(\omega=0) = 2e^2 \int \frac{dE}{2\pi} t(E)[1-t(E)]$.
- No dissipation $(\alpha = 0)$: recover Elattari and Gurvitz, Physics Letters A 292, 289 (2002); [M. S. Choi, F. Plastina, R. Fazio, Phys. Rev. B **67**, 045105 (2003) for Cooper Pair Box]

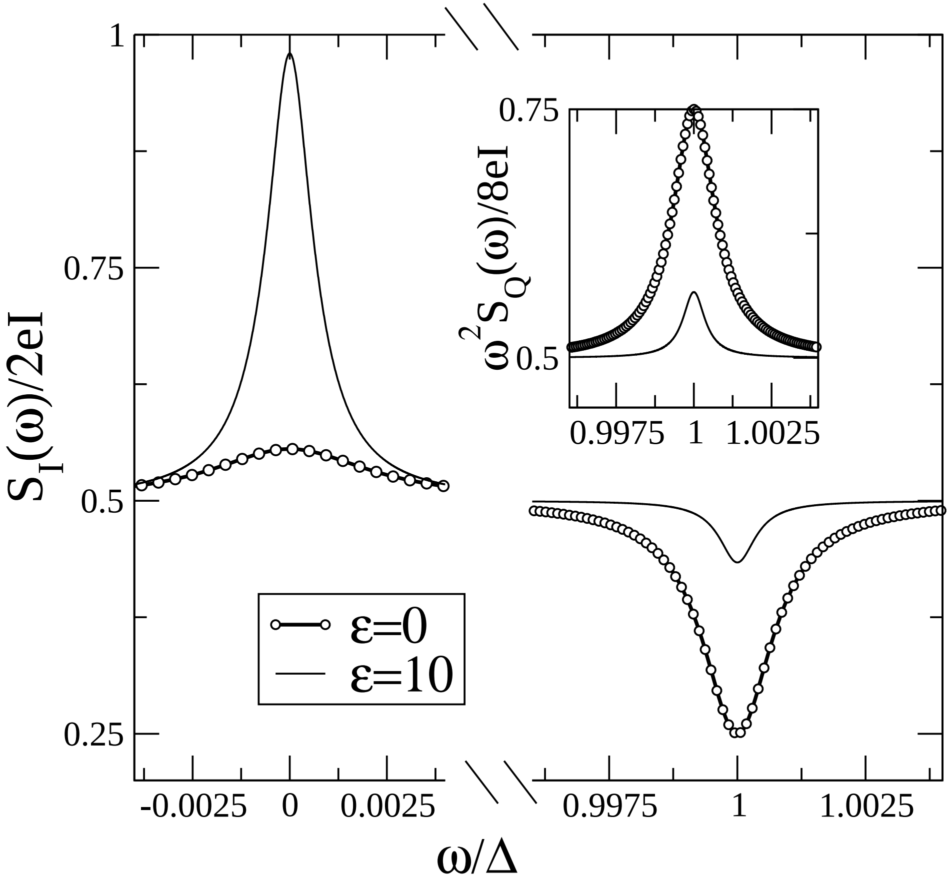
$$\frac{d}{dz}\left[z\hat{n}_R(z)\right]_{z=0} = -\frac{4T_c^2\Gamma_L}{\Gamma_R} \times \frac{4\varepsilon^2(\Gamma_R-\Gamma_L)+3\Gamma_L\Gamma_R^2+\Gamma_R^3+8\Gamma_R T_c^2}{\left[\Gamma_L\Gamma_R^2+4\Gamma_L\varepsilon^2+4T_c^2(\Gamma_R+2\Gamma_L)\right]^2}.$$



- $\alpha = 0$: quantum coherence strongly suppresses noise, minimum for $\varepsilon = 0$ and $\Gamma = \Gamma_R = \Gamma_L = 2\sqrt{2}T_c$.
- Large $|\varepsilon|$: localises charge $\rightarrow$ only one Poisson process $\rightarrow \gamma = 1$.
- Dissipation $\alpha \neq 0 \longrightarrow$ spontaneous emission for $\varepsilon > 0$, noise is reduced below Poisson limit.
- Maximum suppression is reached when the timescales for elastic and inelastic decay coincide, i.e., $\gamma_p = \Gamma_R$.

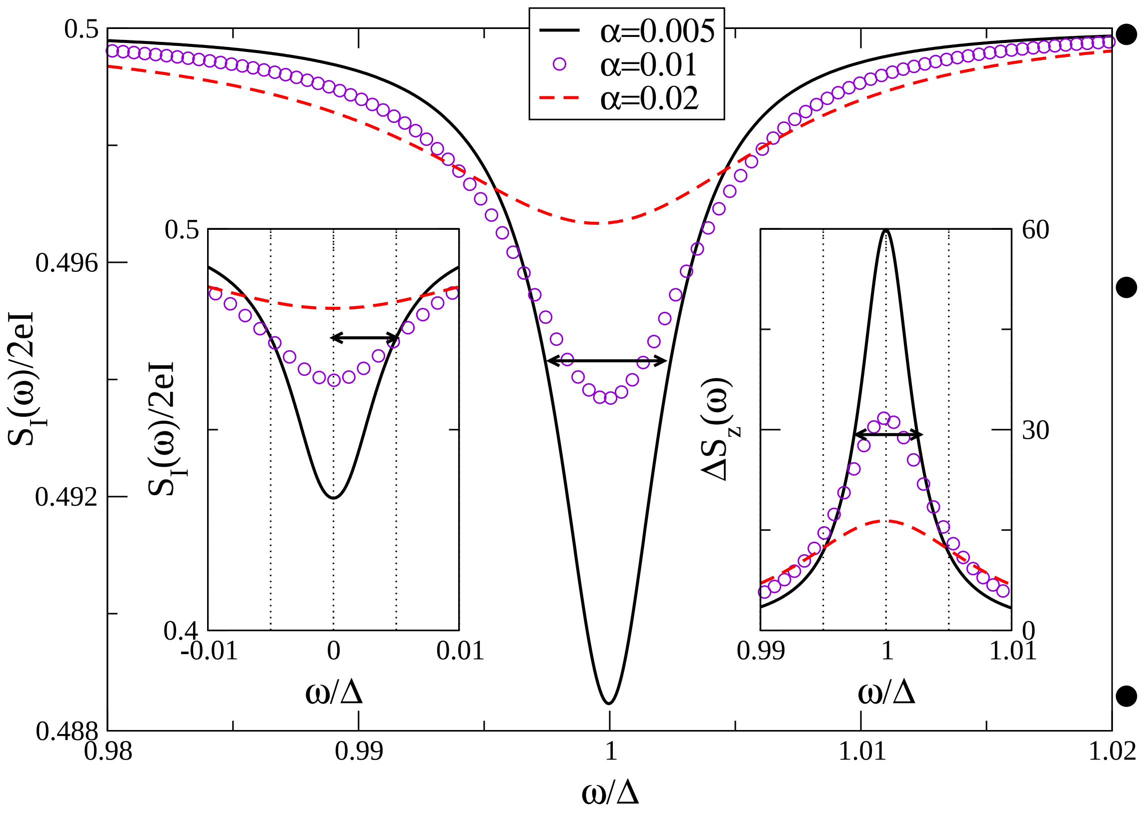


- Comparison with calculation by T. Novotný, A.-P. Jauho (C. Flindt, T. Novotný, A.-P. Jauho,'Current noise in a vibrating quantum dot array', cond-mat/0405512).
- Alternative formalism fully based on quantum regression theorem (QRT).
- Leads to results identical to those from full counting statistics (FCS) for $\alpha = 0$.
- QRT leads to unphysical results for $\alpha > 0$ (dissipative case).
- Why? Does QRT only work for Master equations in Lindblad form ?

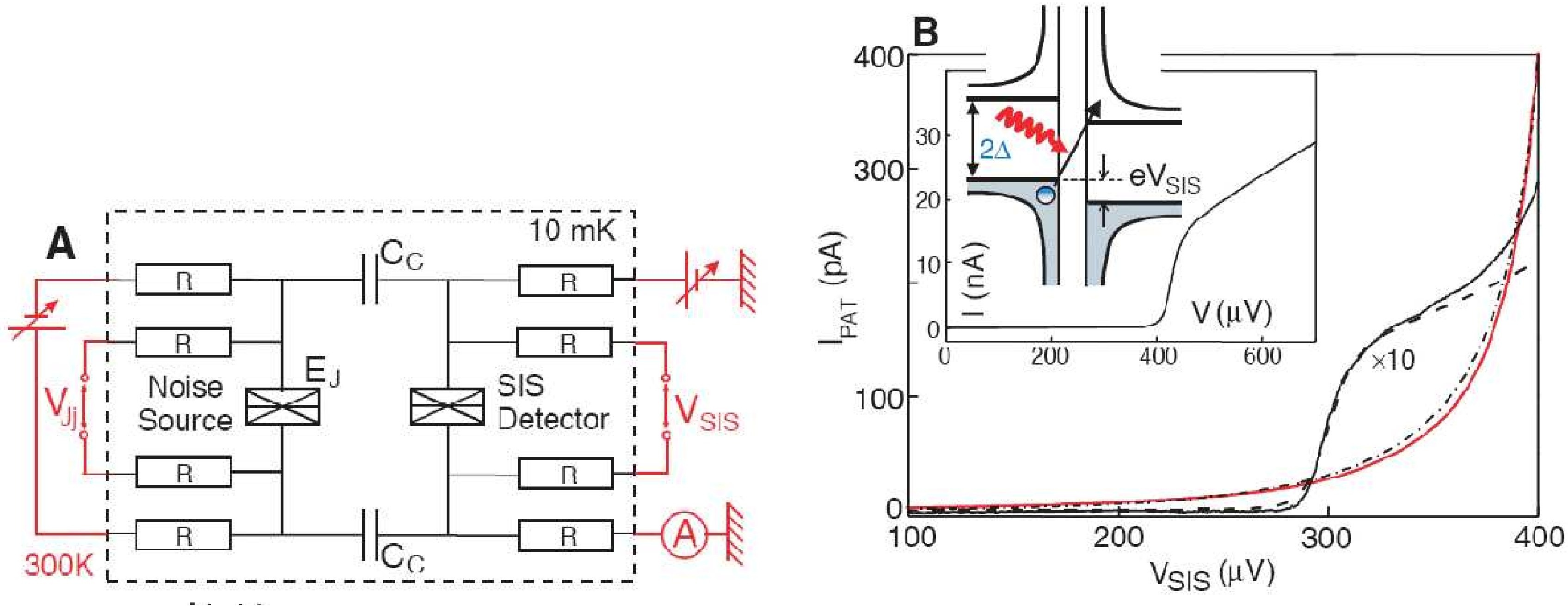


Frequency-dependent noise

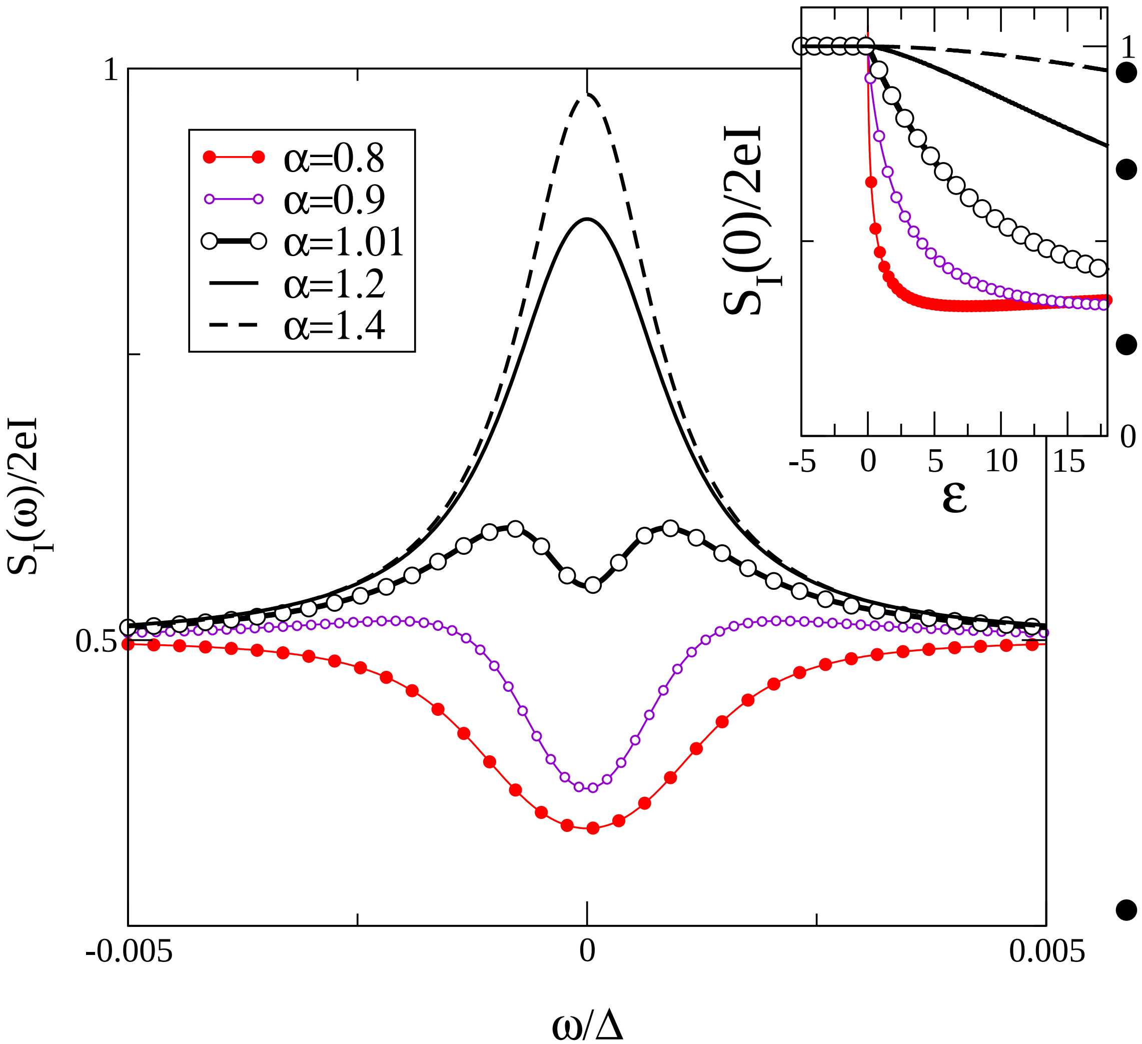
- $\omega = \Delta$: coherent tunneling suppresses noise; dip is destroyed by increasing ε (localising the charge).
- $\omega \to \infty$: Fano factor $\to 1/2$ (symmetric structure).
- resonant shape of (internal) charge noise $S(Q)$.



- Reduction of quantum noise dip around $\omega = \Delta$ yields dephasing rate.
- Noise reflects damping of coherent dynamics, analogous to symmetrized spin correlation function $S_z(\omega) = 1/2 \int_{-\infty}^{\infty} d\omega e^{i\omega\tau} \langle \{\hat{\sigma}_z(\tau), \hat{\sigma}_z\} \rangle$.
- Noise dip around $\omega = 0$ yields relaxation rate (arrows).



Experiment by R. Deblock, E. Onac, L. Gurevich and L. P. Kouwenhoven, Science **301**, 203 (2003). Noise signal at frequency ω converted into a DC, photo-assisted quasi-particle SIS tunnel current. Noise sources: 1) voltage biased (below gap) Josephson junction, 2) current biased Josephson junction in the quasi-particle tunneling regime, 3) Cooper pair box with resonance of $S(\omega)$ around level splitting $\hbar\omega = \sqrt{4E_C(Q/e-1)^2 + E_J^2}$.



- Polaron Noise (large coupling α)
- Near $\omega = 0$, POL and PER coincide at very small α.
- Large α: formation of localised polarons. Localisation/delocalisation transition at $\alpha = 1$ becomes visible in the noise. Suppression of persistent current through strongly dissipative ring, P. Cedraschi, M. Büttiker; Annals of Physics **289**, 1 (2001).
- Poissonian regime with increasing α near $\omega = 0$.

SUMMARY

- Frequency-dependent noise for two-level system with dissipation + transport.
- Charge conservation: QRT and FCS
- Strong and weak dissipation regime.

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