

# Adiabatic Steering of Quantum Dot Based Qubits

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**Abstract.** We discuss a scheme for performing simple adiabatic state vector rotations in coupled semiconductor quantum dots, including a current ‘read-out’ mechanism that enables one to visualise quantum coherent oscillations and the effect of dissipation in an electronic ‘pump’ current.

## 1. Introduction

Time-dependent Hamiltonians are models for quantum mechanical systems with external control parameters. They are of particular interest if one wishes to utilize structures with few degrees of freedoms in order to create and steer, e.g., superpositions of wavefunctions. A simple example is the adiabatic following of a spin  $1/2$  in a slowly varying magnetic field. The probably first quantitative analysis goes back to Landau, Zener and Rosen [1]: ‘Landau-Zener tunneling’ between adiabatic levels occurs if the parameter are changed too fast.

In quantum optics, ‘adiabatic following’, i.e. the rotation of a ground state, is performed by taking use of the coherent population trapping effect in three-level systems which usually requires two laser pulses [2]. Semiconductor double quantum dots [3–7] have many similarities with real atoms, but the transfer of quantum optics concepts [8, 9] has to be done with care: interaction effects and dissipation are different in a semiconductor environment.

We have previously suggested a scheme of how to realize adiabatic following and to extract relevant dephasing times in coupled semiconductor quantum dots [10]. In this paper, I discuss the conceptually (and probably experimentally) simpler situation of steering a single electron charge on two electrostatically coupled quantum dots which themselves are coupled to electron reservoirs. This is perhaps the most primitive ‘non-main stream’ solid state realization of a one-qubit operation in the sense that it is neither based on Cooper pairs [11] nor the spin [12], but simply on the electron charge [13], yet it leads to the prediction of controllable ‘charge-pumping’ [14] (read-out) in what could be called a ‘Nakamura-experiment’ [15] in dots.

Adiabatic quantum pumps have been realized [16] and discussed [17–21] in a regime of weak Coulomb correlations, where mesoscopic scattering theory of non-interacting systems can be applied. We have recently suggested adiabatic control of the wave function in the opposite strong Coulomb blockade regime of a triple quantum dot [14]. Furthermore, adiabatic quantum computation with Cooper pairs [22] and adiabatic controlled-NOT gates for quantum computation have been proposed by Averin [23].

On the theoretical side, the physics discussed below is essentially the adiabatic Landau-Zener-Rosen problem [1] in presence of dissipation [24], modified by the fact that the two-level system (double quantum dot groundstates) is connected to external electron reservoirs, although the latter are treated in a rather elementary way here. Real non-equilibrium or many-body-effects due to Kondo physics still have to be explored. More details on the calculations related to the results presented here can be found in [25].

## 2. Model

In our model, the ‘charge qubit’ is realized in two coupled quantum dots defined by two electron states  $|L\rangle$  and  $|R\rangle$  with a time-dependent energy difference  $\varepsilon(t)$  and coupled by a tunnel matrix element  $T_c(t)$ , as described by the time-dependent Hamiltonian

$$H_0(t) = \frac{\varepsilon(t)}{2}\sigma_z + T_c(t)\sigma_x, \quad (1)$$

with  $\sigma_z := |L\rangle\langle L| - |R\rangle\langle R|$  and  $\sigma_x := |L\rangle\langle R| + |R\rangle\langle L|$ . This system is coupled to two external leads (source and drain) with fixed chemical potentials, cf. Fig. (1) right.

The initial state  $|\text{in}\rangle$  is an additional electron in the left dot with an energy  $\varepsilon_L$  well below the chemical potential of the left lead. Naively, one would guess that transfer to the right dot is possible by slowly increasing and then decreasing the tunnel coupling  $T_c$ , but quantum mechanics requires the simultaneous change of at least two parameters, i.e.,  $T_c(t)$  and  $\varepsilon(t)$ .

The instantaneous, hybridized eigenstates of the isolated coupled quantum dot are

$$|\pm\rangle = \frac{1}{N_\pm} [\pm 2T_c|L\rangle + (\Delta \mp \varepsilon)|R\rangle] \quad (2)$$

$$N_\pm := \sqrt{4|T_c|^2 + (\Delta \mp \varepsilon)^2}, \quad \Delta := \sqrt{\varepsilon^2 + 4|T_c|^2}.$$

The eigenvalues  $\varepsilon_\pm = \pm \frac{1}{2}\Delta$  of the coupled system represent two energy surfaces over the  $T_c$ - $\varepsilon$  plane the lower of which is shown in Fig. (1), left.

Due to the adiabatic theorem, the initial groundstate  $|L\rangle$  of the system is rotated into the instantaneous eigenstate  $|-\rangle$ , Eq. (2), if the speed (frequency) of the rotation is much smaller than  $\Delta/\hbar$ . In this case, the time enters as a parameter into the state  $|-\rangle$  via  $T_c(t)$  and  $\varepsilon(t)$ , and it can be used to calculate approximate expectation values such as

$$\langle \sigma_z \rangle_{\text{ad}} = -\varepsilon(t)/\Delta(t), \quad (3)$$

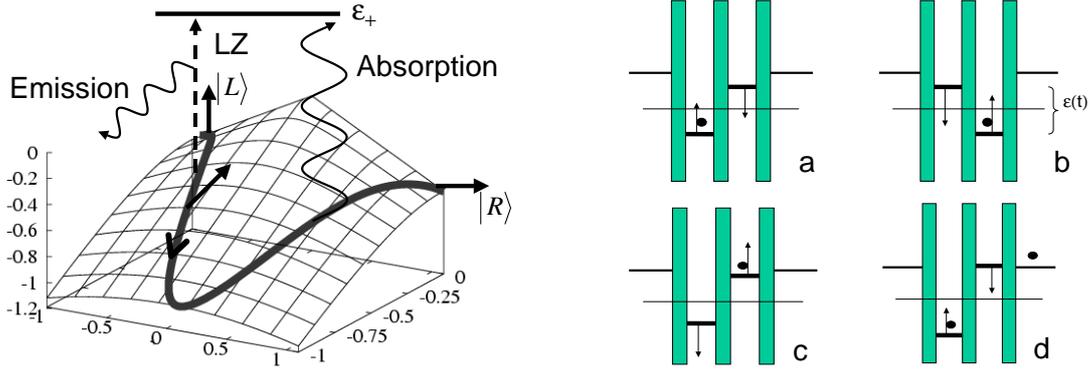
which excellently reproduces the overall form of the numerically obtained  $\langle \sigma_z \rangle_t$ . The exact solution exhibits the expected quantum mechanical oscillations with frequency  $\Delta(t)/\hbar$  around the adiabatic value, which are strongest when the tunnel coupling is fully switched on. Due to Landau–Zener tunneling from the adiabatic ground state  $|-\rangle$  to the excited state  $|+\rangle$ , there is always a finite albeit small probability  $P_L$  for the electron to remain in the left dot, i.e., the excited state after the rotation.  $P_L$  can be made exponentially small for large enough level splitting  $\Delta$  and slow pulses, but depends on the exact pulse shape  $\varepsilon(t), T_c(t)$ .

## 3. Dissipation and Fidelity

The adiabatic transfer remains incomplete in the presence of dissipation, i.e.  $\langle \sigma_z \rangle$  can considerably deviate from its non-dissipative ‘ideal’ value  $-1$  corresponding to the electron in the right dot. Clearly, for too long swap duration, inelastic transitions to the excited level will have sufficient time to destroy the coherent transfer. On the other hand, if the swap operation proceeds too fast, Landau–Zener transitions become stronger and again lead to deviations  $\delta\langle \sigma_z \rangle > 0$ . This means that for a given inelastic scattering rate  $\Gamma_{\text{in}}$  there should be an optimal swap duration such that these two competing processes balance each other. One problem in quantifying this argument [26] is that  $\Gamma_{\text{in}}$  naturally is time-dependent itself.

Instead, one can use time-dependent perturbation theory [24] to find the probability for a transition from  $-$  to  $+$  due to the interaction with environmental bosons (phonons, electron-hole excitations in the leads)

$$P_{+-}(t) = \int_0^\infty d\omega \rho(\omega) \{ n_B(\omega) f(\omega, t) + [1 + n_B(\omega)] f(-\omega, t) \}$$



**Figure 1.** LEFT: lower adiabatic energy surface of double quantum dot with energy difference  $\varepsilon$  and tunnel coupling  $T_c$ . The curve on the surface corresponds to a time-dependent  $\varepsilon - T_c$ -pulse that adiabatically rotates the ground state from the left into the right dot. Competing processes are Landau-Zener (LZ) tunneling into the excited state and absorption and emissions of bosons. RIGHT: Scheme for adiabatic charge pumping.

$$f(\omega, t) := \left| \int_0^t dt' \frac{T_c(t')}{\Delta(t')} e^{-i \int_0^{t'} ds [\Delta(s) - \omega]} \right|^2. \quad (4)$$

Here,  $n_B(\omega)$  is the equilibrium distribution of bosons at temperature  $T$  with an effective spectral density  $\rho(\omega)$  [25]. The main idea now is to proceed by introducing ‘elliptic’ pulses  $(T_c(t), \varepsilon(t))$  that are defined by curves on the  $\varepsilon_-$ -surface with *constant* energy difference  $\Delta$  to the excited state  $\varepsilon_+$ . For the particular sinusoidal form

$$T_c(t) = -\frac{\Delta}{2} \sin \Omega t, \quad \varepsilon(t) = -\Delta \cos \Omega t \quad (5)$$

of  $T_c(t)$  and  $\varepsilon(t)$ , the time-dependent Hamiltonian

$$H_0(t) = -\frac{\Delta}{2} [\cos(\Omega t) \sigma_z + \sin(\Omega t) \sigma_x] \quad (6)$$

becomes exactly integrable: it corresponds to a spin  $\frac{1}{2}$  in a magnetic field that rotates within the  $x$ - $z$ -plane around the  $y$ -axis with frequency  $\Omega$ . The solution for the inversion  $\langle \sigma_z \rangle_t$  is easily obtained by transforming the Schrödinger equation into a rotating frame (Rabi solution). One obtains

$$\begin{aligned} \langle \sigma_z \rangle_t^{\text{Rabi}} &= \left[ \left( \frac{\Delta}{\omega_R} \right)^2 + \left( \frac{\Omega}{\omega_R} \right)^2 \cos(\omega_R t) \right] \cos(\Omega t) + \frac{\Omega}{\omega_R} \sin(\omega_R t) \sin(\Omega t) \\ \omega_R &:= \sqrt{\Omega^2 + \Delta^2}. \end{aligned} \quad (7)$$

Here, the Rabi frequency  $\omega_R$  corresponds to maximal ‘detuning’  $\delta = \Omega - 0$  since there is no static ‘magnetic field’  $\propto \sigma_y$  in  $y$ -direction.

The use of the harmonic pulse, Eq. (5), has the further advantage that the quantity  $f(\omega, t)$ , Eq. (4), can be evaluated analytically. The swap operation requires a pulse acting half a period from the initial time  $t = 0$  to the final time  $t_f = \pi/\Omega$ . Using  $t = t_f$  in  $f(\omega, t)$ , we obtain

$$f\left(\omega, \frac{\pi}{\Omega}\right) = \frac{1}{\Omega^2} \left[ \frac{\cos\left(\frac{\pi}{2}x\right)}{x^2 - 1} \right]^2, \quad x := (\Delta - \omega)/\Omega. \quad (8)$$

In the adiabatic limit  $\Omega/\Delta \rightarrow 0$ , we find an approximation to the integrals Eq. (4). In that limit, one has

$$f\left(\omega, \frac{\pi}{\Omega}\right) \rightarrow \frac{c}{\Omega} \delta(\Delta - \omega), \quad c = \frac{\pi^3 J_{3/2}(\pi)}{4\sqrt{2}} \quad (9)$$

whence the dissipation induced change of the inversion becomes

$$\delta\langle\sigma_z\rangle_f^{\text{diss}} = 2P_{+-}(t_f) = 2\frac{c}{\Omega}\rho(\Delta)n_B(\Delta). \quad (10)$$

The sum of the coherent contribution  $\delta\langle\sigma_z\rangle_t$  from the Rabi solution at  $t = t_f$ , Eq. (7), and the perturbative dissipative contribution  $\delta\langle\sigma_z\rangle_f^{\text{diss}}$  leads to

$$\begin{aligned} \delta\langle\sigma_z\rangle_f \approx & 1 - \left[ \left(\frac{\Delta}{\omega_R}\right)^2 + \left(\frac{\Omega}{\omega_R}\right)^2 \cos\left(\frac{\pi\omega_R}{\Omega}\right) \right] \\ & + 2\frac{c}{\Omega} \frac{\rho(\Delta)}{\exp(\Delta/k_B T) - 1}, \quad \Omega \ll \Delta, \quad c = 2.4674. \end{aligned} \quad (11)$$

The  $1/\Omega$  dependence at small  $\Omega$  of the dissipative contribution to  $\delta\langle\sigma_z\rangle_f$  indicates that for too long pulse duration, the electron swap remains incomplete due to incoherent dissipation. On the other hand, if the pulse duration is too short (larger  $\Omega$ ), the oscillatory coherent contribution from  $\langle\sigma_z\rangle_f^{\text{Rabi}}$  dominates.

A suppression of the phonon spectral density  $\rho(\omega)$  at certain frequencies  $\omega = \omega_0$  has been predicted for double dots in free-standing phonon cavities (thin plate model) due to symmetry and geometrical confinement [27]. If the Rabi rotation pulse runs at a constant energy difference  $\Delta(t) = \hbar\omega_0$ , it effectively ‘switches off’ the decoherence, at least within second order in the (small) electron-boson coupling constant. Consequently, this defines a one-dimensional ‘decoherence-free manifold’ (curve) on the adiabatic groundstate energy surface of the system.

Note that Eq. (11) is an approximation that only holds in the limit of an infinitely slow adiabatic change, i.e.  $t_f = \pi/\Omega \rightarrow \infty$ . In fact, for any finite pulse duration  $t_f < \infty$ , even in the limit of zero temperature  $T = 0$ , Eq. (4) yields

$$P_{+-}(t_f) = \int_0^\infty d\omega \rho(\omega) f(-\omega, t_f), \quad (12)$$

which shows that there is a small, but finite probability for inelastic transitions from  $|-\rangle$  to  $|+\rangle$  even at zero temperature. These transitions are due to the spontaneous emission of bosons which occur during Landau–Zener transitions from  $|-\rangle$  to  $|+\rangle$  with a finite probability as long as  $t_f$  is finite. The interaction with the bosonic environment therefore changes the results even at zero temperature, in agreement with Grifoni and Hänggi [24].

## 4. Discussion

We now discuss the electron pump that combines Coulomb blockade and quantum adiabaticity in a double dot coupled to external leads. The main idea is to apply time-dependent pulses such that the quantum mechanical time evolution of the two-level system is well separated from a merely ‘classical’ decharging and charging process. One complete cycle of such an operation is sketched in Fig. 1. The cycle starts with an additional electron in the left dot and an adiabatic rotation of the parameters  $(\varepsilon(t), T_c(t))$  such as, e.g., in Eq. (5). This completely quantum-mechanical part of the cycle is performed in the ‘safe haven’ of the Coulomb- and the Pauli-blockade, i.e., with the left and right energy levels of the two dots well below the chemical potentials  $\mu$  of the leads (which are assumed identical here for simplicity). The cycle

continues with leaving the coupling at  $T_c = 0$ ; the two dots then are still in a superposition of the left and the right state. The subsequent lifting of the right level above the chemical potential of the right lead constitutes a measurement of that superposition (collapse of the wave-function): the electron is either in the right dot (with a high probability  $1 - \frac{1}{2}\langle\sigma_z\rangle_f$ ) and tunnels out, or the electron is in the left dot (and nothing happens because the left level is still below  $\mu$  and the system is Coulomb blocked).

We assume that the tunnel rates  $\Gamma_R, \Gamma_L$  to the right and left leads are sufficiently larger than the inverse of the cycle duration  $t_{\text{cycle}}$ ,

$$\Gamma_R, \Gamma_L \gg t_{\text{cycle}}^{-1}. \quad (13)$$

In this case, the discharging of the right dot and the re-charging of the left dot from the left lead is fast enough to bring the system back into its initial state with one additional electron on the left dot. The precise value of  $\Gamma_R, \Gamma_L$  and the precise shape of the  $\varepsilon(t)$ -pulse for  $t_f < t < t_{\text{cycle}}$  then have no effect on the total charge transferred within one cycle. Then, since the probability to transfer one electron from the left to the right in one cycle is given by  $1 - \frac{1}{2}\langle\sigma_z\rangle_f$ , on the average an electron current

$$\langle I \rangle = -e \frac{1 - \frac{1}{2}\langle\sigma_z\rangle_f}{t_{\text{cycle}}} \quad (14)$$

flows from left to right. If condition Eq.(13) is no longer fulfilled, a full master equation including dissipative terms due to electron reservoir coupling has to be solved [28].

## 5. Conclusion

In our scheme, the leads essentially act as classical measurement devices of the quantum-mechanical time-evolution between the two dots. Measuring the current  $\langle I \rangle$  as a function of the pulse length  $t_f = \pi/\Omega$  then offers a scheme to make quantum mechanical oscillations such as those predicted in Eq. (11) visible in the electronic current, similar to the recent experiment by Nakamura *et al.* in a superconducting Cooper pair box [15].

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## References

- [1] L. D. Landau, Phys. Z. Sov. **2**, 46 (1932), C. Zener, Proc. R. Soc. A **137**, 696 (1932), N. Rosen, C. Zener, Phys. Rev. **40**, 502 (1932).
- [2] K. Bergmann, H. Theuer, B. W. Shore, Rev. Mod. Phys. **70**, 1003 (1998).
- [3] N. C. van der Vaart, S. F. Godjin, Y. V. Nazarov, C. J. P. M. Harmans, J. E. Mooij, L. W. Molenkamp, and C. T. Foxon, Phys. Rev. Lett. **74**, 4702 (1995).
- [4] R. H. Blick, D. Pfannkuche, R. J. Haug, K. v. Klitzing, and K. Eberl, Phys. Rev. Lett. **80**, 4032 (1998).
- [5] T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tarucha, and L. P. Kouwenhoven, Science **282**, 932 (1998); S. Tarucha, T. Fujisawa, K. Ono, D. G. Austin, T. H. Oosterkamp, and W. G. van der Wiel, Microelectr. Engineer. **47**, 101 (1999).
- [6] T. H. Oosterkamp, T. Fujisawa, W. G. van der Wiel, K. Ishibashi, R. V. Hijman, S. Tarucha, and L. P. Kouwenhoven, Nature **395**, 873 (1998).
- [7] R. H. Blick, D. W. van der Weide, R. J. Haug, and K. Eberl, Phys. Rev. Lett. **81**, 689 (1998).
- [8] T. Brandes and F. Renzoni, Phys. Rev. Lett. **85**, 4148 (2000).
- [9] T. Brandes, in *Interacting Electrons in Nanostructures*, Vol. 579 of *Lecture Notes in Physics*, edited by R. Haug and H. Schoeller (Springer, Heidelberg, 2001).
- [10] T. Brandes, F. Renzoni, and R. H. Blick, Phys. Rev. B **64**, 035319 (2001).
- [11] Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).

- [12] D. Loss, D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
- [13] R. H. Blick and H. Lorenz, Proceedings IEEE International Symposium on Circuits and Systems **II**, 245 (2000).
- [14] F. Renzoni and T. Brandes, Phys. Rev. B **64**, 245301 (2001).
- [15] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature **398**, 786 (1999).
- [16] M. Switkes, C. M. Marcus, K. Campman, and A. C. Gossard, Science **283**, 1905 (1999).
- [17] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
- [18] M. L. Polianski and P. W. Brouwer, Phys. Rev. B **64**, 075304 (2001).
- [19] J. N. H. J. Cremers and P. W. Brouwer, Phys. Rev. B **65**, 115333 (2002).
- [20] M. Moskalets and M. Büttiker, Phys. Rev. B **64**, 201305 (2001).
- [21] E. R. Mucciolo, C. Chamon, and C. M. Marcus, cond-mat/0112157, 2001.
- [22] D. V. Averin, Solid State Comm. **105**, 659 (1998).
- [23] D. V. Averin, in *Quantum computing and quantum communications*, Vol. 1509 of *Lecture Notes in Computer Science* (Springer, Berlin, 1999), p. 413.
- [24] M. Grifoni and P. Hänggi, Phys. Rep. **304**, 229 (1998).
- [25] T. Brandes and T. Vorrath, cond-mat/0207257, to appear in Phys. Rev. B (2002).
- [26] P. Silvestrini and L. Stodolsky, Phys. Lett. A **280**, 17 (2001).
- [27] S. Dehdal, T. Brandes, and B. Kramer, Phys. Rev. B (Rap. Comm.) **66**, 041301(R) (2002).
- [28] T. Brandes and B. Kramer, Phys. Rev. Lett. **83**, 3021 (1999).