Noise and instabilities in generalised spin-boson systems

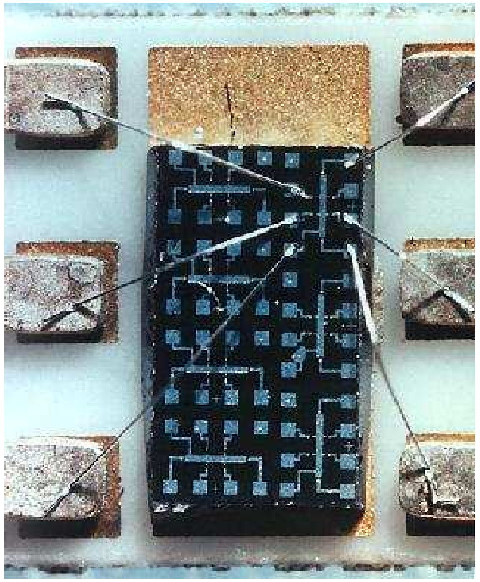
T. Brandes

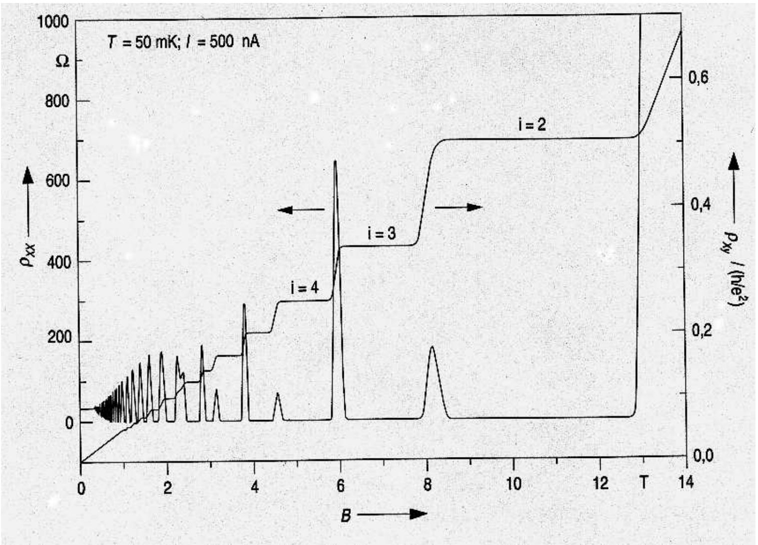
- Quantum Mechanical Transport ($N = 1$ qubit)
- Quantum Noise
- Transport, noise, entanglement ($N = 2$ qubits)
- Instabilities: Quantum Phase Transitions ($N \rightarrow \infty$ qubits)

Co-workers: R. Aguado (Madrid), C. Emary (San Diego), N. Lambert (Manchester),

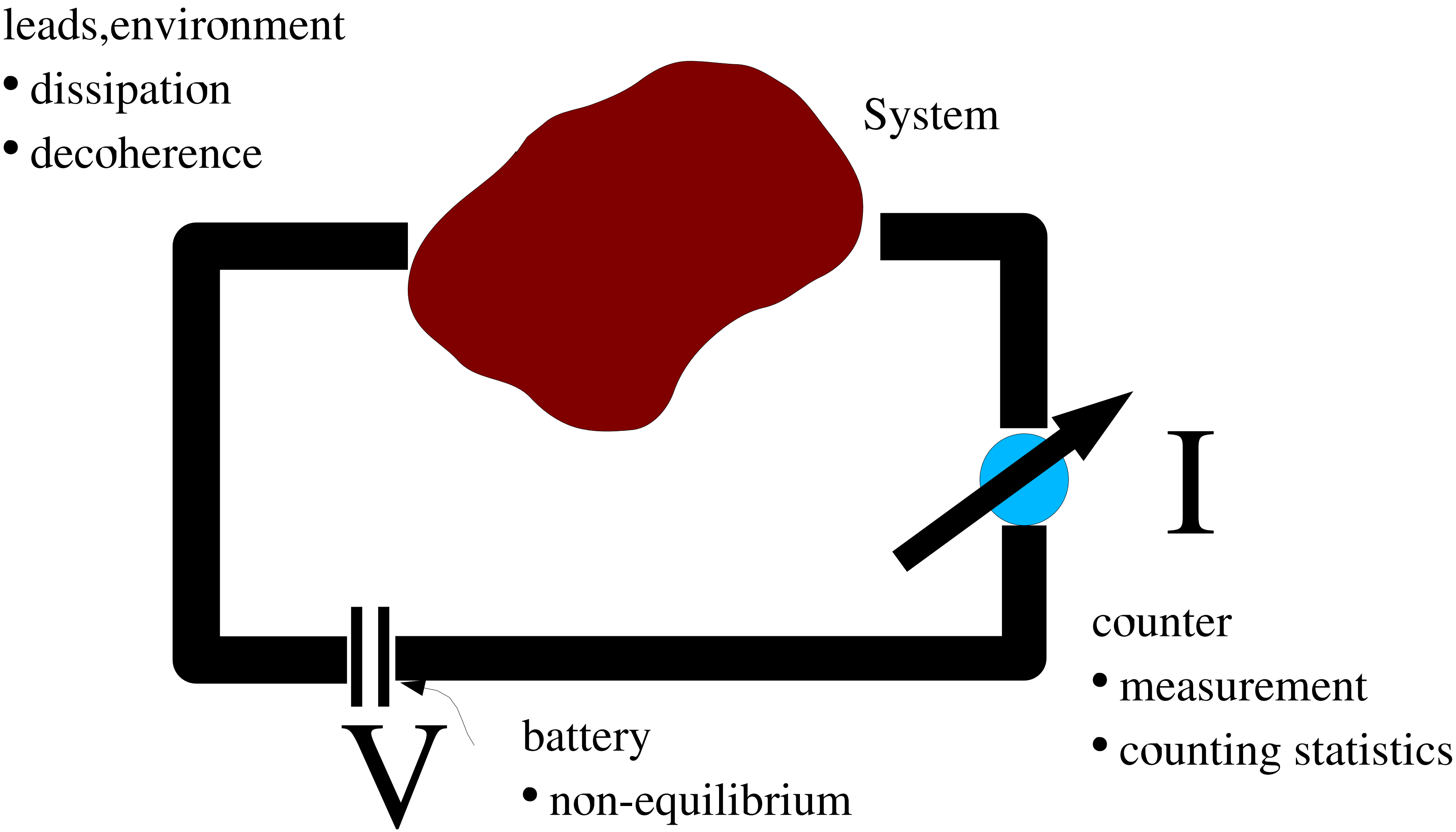
MANCHESTER 1824 The University of Manchester

Electronic Transport





$$R_K = \frac{h}{e^2} = 25812,807\Omega.$$



TRANSPORT = system + non-equilibrium + external world

Electronic Transport

Things are difficult. Start from something simple?

SMALL STUFF:

- Dimension 2 (2DEG), 1 (wires), 0 (few-level quantum systems).
- Single Electron Transistor.
- charge/flux/spin qubits (controllable two-level systems)

- tunneling $\rightsquigarrow$ quantum superpositions
- interactions $\rightsquigarrow$ entanglement
- environment $\rightsquigarrow$ decoherence

$\rightsquigarrow$ arena of *Mesoscopic Physics*.

VOLUME 91, NUMBER 22 PHYSICAL REVIEW LETTERS week ending 28 NOVEMBER 2003

Coherent Manipulation of Electronic States in a Double Quantum Dot

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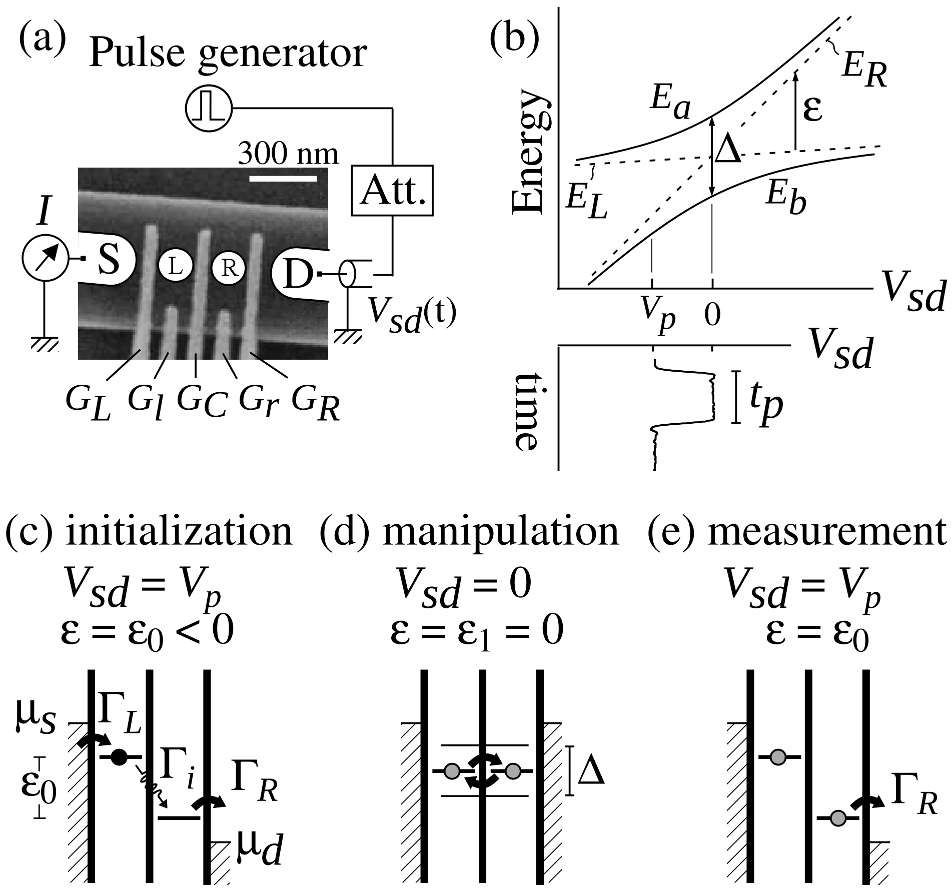
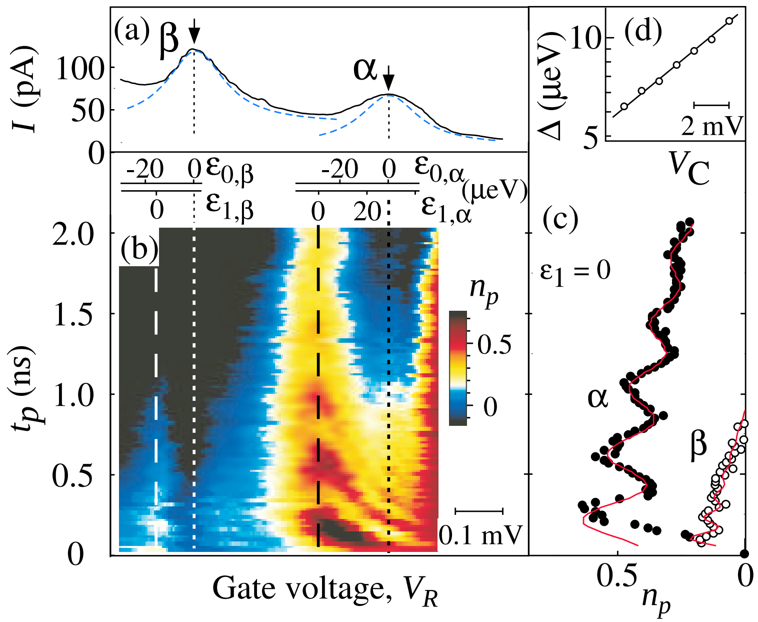


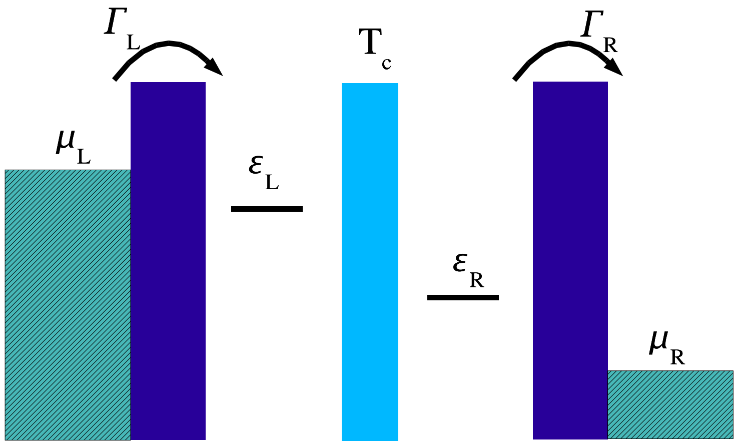


cf. TB, T. Vorrath, PRB **66**, 075341 (2003); U. Hartmann, F. K. Wilhelm, PRB **67**, 161307 (2003), M. Thorwart, J. Eckel, E.R. Mucciolo cond-mat/0505621 (2005).

These are also useful in order to understand transport ‘from scratch’.

Three-State Transport Model

- Transport model for the smallest quantum system: $SU(2)$ plus one empty state.
- $|L\rangle = |N_L+1, N_R\rangle$ 'left', $|R\rangle = |N_L, N_R+1\rangle$ 'right', $|0\rangle = |N_L, N_R\rangle$ 'empty'.



- internal bias $\varepsilon = \varepsilon_L - \varepsilon_R$, tunnel coupling T_c.

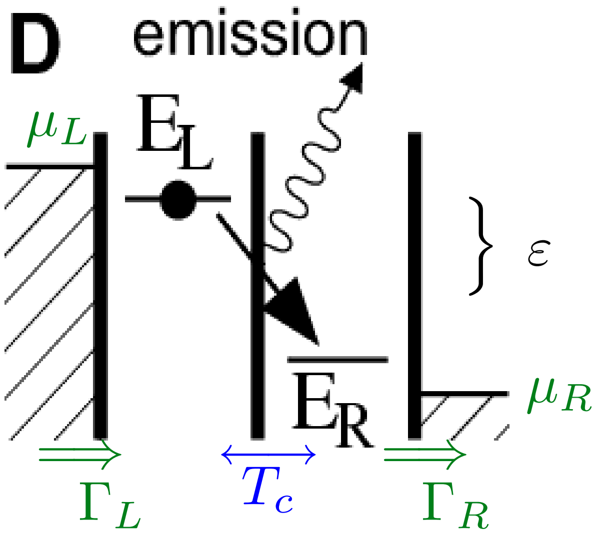
$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_{res} + \mathcal{H}_T, \quad \mathcal{H}_S = \frac{\varepsilon}{2}\hat{\sigma}_z + T_c\hat{\sigma}_x$$

$$\mathcal{H}_T = \sum_{k_i}(V_k^i c_{k_i}^\dagger |0\rangle\langle i| + H.c.), i = L, R.$$

One goal: calculate density operator ρ for $t \to \infty$. ρ has 4 (not 3) real parameters,

$$\rho = \begin{pmatrix} \rho_{00} & 0 & 0 \\ 0 & \rho_{LL} & \rho_{LR} \\ 0 & \rho_{RL} & \rho_{RR} \end{pmatrix}, \quad \rho_{00} = 1 - \rho_{LL} - \rho_{RR}.$$

Double Quantum Dots



3 states $|L\rangle$, $|R\rangle$, $|0\rangle$

$\hat{\sigma}_z \equiv |L\rangle\langle L| - |R\rangle\langle R|, \quad \hat{\sigma}_x \equiv |L\rangle\langle R| + |R\rangle\langle L|.$

$$\mathcal{H} = \mathcal{H}_{SB} + \mathcal{H}_{res} + \mathcal{H}_T$$

$$\mathcal{H}_T = \sum_{k_\alpha} (V_k^\alpha c_{k_\alpha}^\dagger |0\rangle\langle\alpha| + H.c.), \alpha = L, R$$

$$\mathcal{H}_{SB} = \left[\frac{\varepsilon}{2} + \sum_{\mathbf{Q}} \frac{g_Q}{2}\left(a_{-\mathbf{Q}} + a_{\mathbf{Q}}^\dagger\right)\right]\hat{\sigma}_z + T_c\hat{\sigma}_x + \mathcal{H}_B.$$

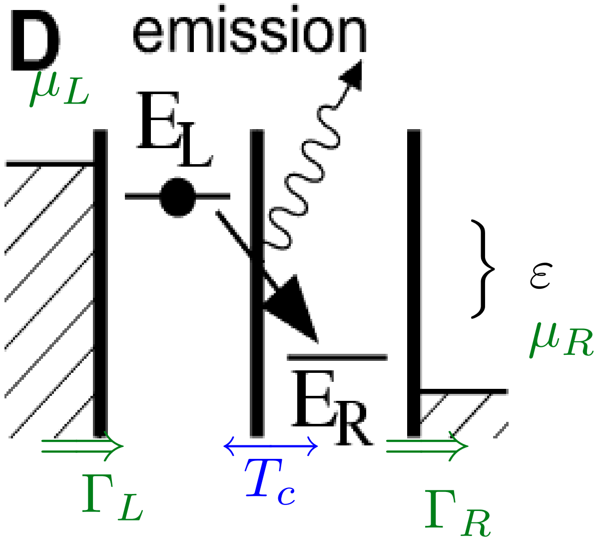
Loc-Deloc Transition at $\alpha = 1$, *Leggett et al 87*

- 'Internal' Parameter ε, T_c;

$$J(\omega) \equiv \sum_{\mathbf{Q}} |g_Q|^2 \delta(\omega - \omega_Q) = \begin{cases} 2\alpha\omega_{\rm ph}^{1-s}\omega^s e^{-\frac{\omega}{\omega_c}} \\ \text{microscopic model: } \textit{Phonons...} \end{cases}$$

- 'External' parameters $\mu_L, \mu_R, \Gamma_\alpha(\varepsilon) = 2\pi \sum_{k_\alpha} |V_k^\alpha|^2 \delta(\varepsilon - \varepsilon_{k_\alpha}), \alpha = L/R.$

Formulation



- EOM for reduced density operator

$$\langle \mathbf{A}(t)\rangle = \langle \mathbf{A}(0)\rangle + \int_0^t dt' \left\{ M(t,t')\langle \mathbf{A}(t')\rangle + \Gamma_L \mathbf{e}_1 \right\}.$$

- $\mu_L - \mu_R \to \infty$ (Gurvitz, Prager 1996, Stoof, Nazarov 1996, Gurvitz 1998.)

- 'Memory Kernel'

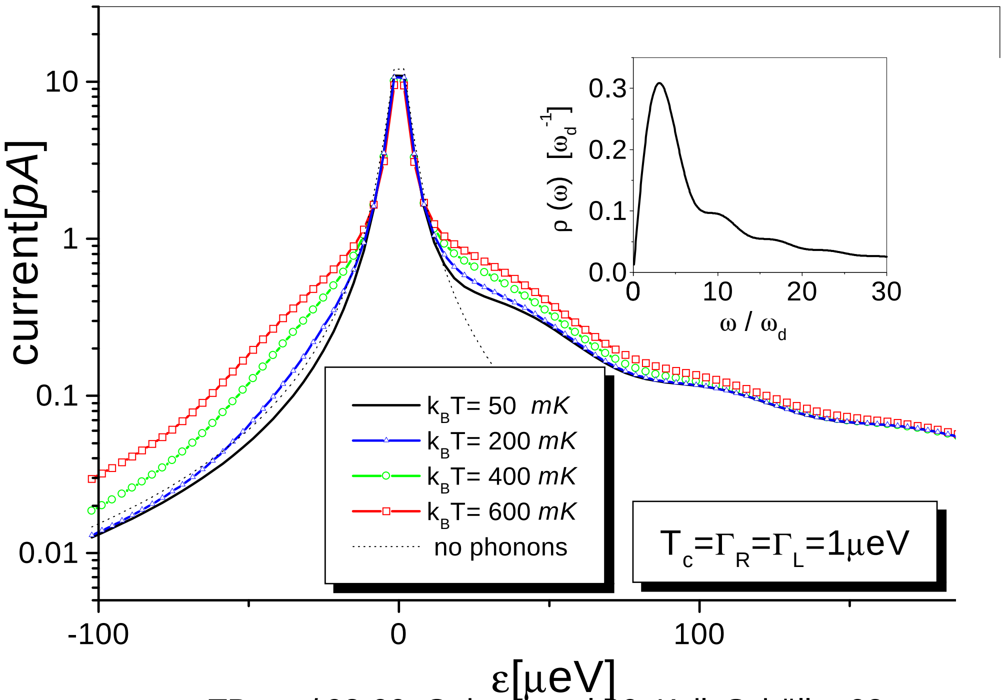
$$z\hat{M}(z) = \begin{bmatrix} -\hat{G} & \hat{T}_c \\ \hat{D}_z & \hat{\Sigma}_z \end{bmatrix}, \quad \hat{G} \equiv \begin{pmatrix} \Gamma_L & \Gamma_L \\ 0 & \Gamma_R \end{pmatrix}, \quad \hat{T}_c \equiv iT_c(\sigma_x - 1)$$

- Blocks $\hat{D}_z$, $\hat{\Sigma}_z$: Dephasing, Relaxation $\begin{cases} \text{PER} \\ \text{POL} \end{cases}$

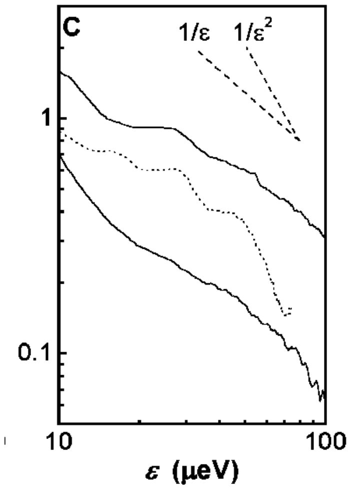
- Polaron-Transformation (POL) $\equiv$ NIBA (non-interacting blib approximation): calculate $\hat{D}_z$ and $\hat{\Sigma}_z$ using bosonic correlation function

$$C_\varepsilon^{[*]}(z) \equiv \int_0^\infty dt e^{-zt} e^{[-]i\varepsilon t} \exp(-\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\left[(1-\cos\omega t)\coth\left(\frac{\beta\omega}{2}\right) \pm i\sin\omega t\right]).$$

- Polaron tunneling $\rightsquigarrow$ 'boson shake-up' effect
- Re$[C_\varepsilon(z)]|_{z=\pm i\omega} = \pi P(\varepsilon \mp \omega)$: P(E)-Theory.



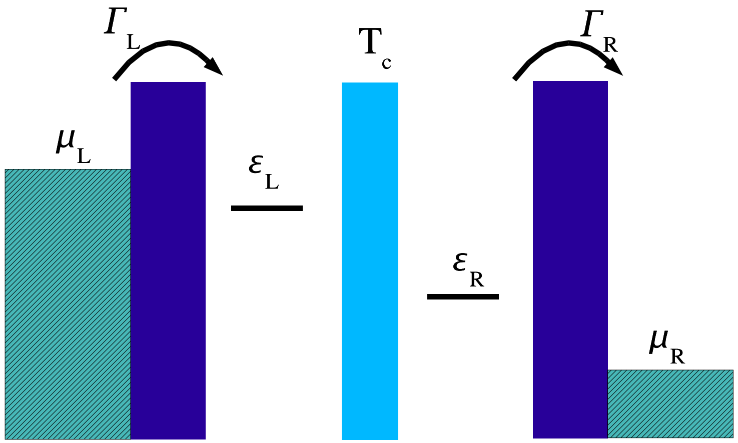
TB *et al* 98-00; Guinea et al 00; Keil, Schöller 02



T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tarucha, and L. P. Kouwenhoven, Science **282**, 932 (1998)

$\propto \varepsilon^{1+2\alpha}, \alpha \approx 0.1$

- Double quantum dots, strong Coulomb blockade $U \to \infty$.



- Complicated problem for any bias $|\mu_L - \mu_R| < \infty$.
- Only $\mu_L - \mu_R \to \infty$ relatively easy. Then, exact (?) solution in Markovian limit (flat tunneling DOS, no memory).
- External tunnel rates; $\Gamma_i(\varepsilon) = 2\pi \sum_{k_i} |V_k^i|^2 \delta(\varepsilon - \varepsilon_{k_i})$.

- Solve Liouville-von-Neumann eq. $\rightsquigarrow$ stationary current (Stoof-Nazarov 1996, Gurvitz 1996)

$$\langle \hat{I} \rangle_{t\to\infty}^{\mathrm{SN}} = -e \frac{T_c^2 \Gamma_R}{\Gamma_R^2/4 + \varepsilon^2 + T_c^2(2 + \Gamma_R/\Gamma_L)},$$

- Just Breit-Wigner. Nothing on spectrum, $\pm\frac{1}{2}\sqrt{\varepsilon^2 + 4T_c^2}$.
- Pure state for $\Gamma_R \to \infty$ (no current): quantum Zeno effect (continuous measurement version): right lead as detector with ∞ bandwidth.

PHYSICAL REVIEW B VOLUME 46, NUMBER 19 15 NOVEMBER 1992-I

Scattering theory of current and intensity noise correlations in conductors and wave guides

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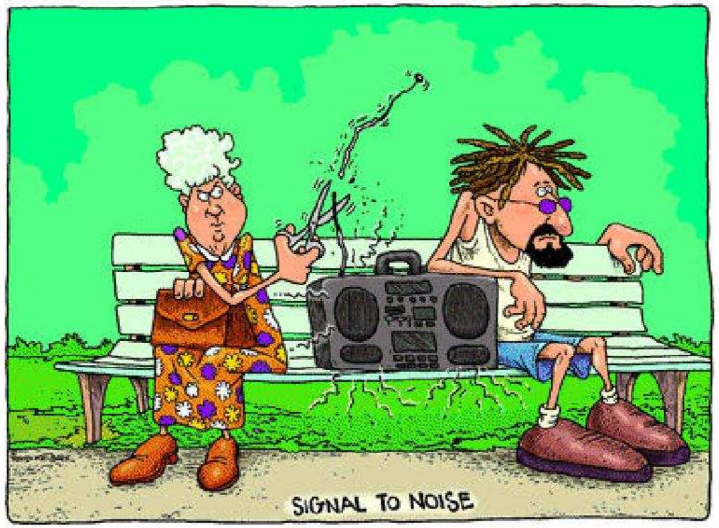
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 16 June 1992)

- Quantum Mechanical Transport
- Quantum Noise
- Entanglement
- Quantum Phase Transitions

R. Landauer : ‘the noise is the signal’.



Quantum Noise: particle statistics, quantum coherence, dissipation, entanglement.

- Noise-Spectrum with current conservation $I_L - I_R = \dot{Q}$, $I = aI_L + bI_R$,

$$\mathcal{S}_{\mathcal{I}}(\omega) \equiv \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle \{\Delta\hat{I}(\tau), \Delta\hat{I}(0)\}\rangle = \underline{aS_{I_L}(\omega) + bS_{I_R}(\omega)} - ab\omega^2 \underline{S_Q(\omega)}$$

- $\underline{S_{I_R}(\omega)}$ using 'Full Counting Statistics',

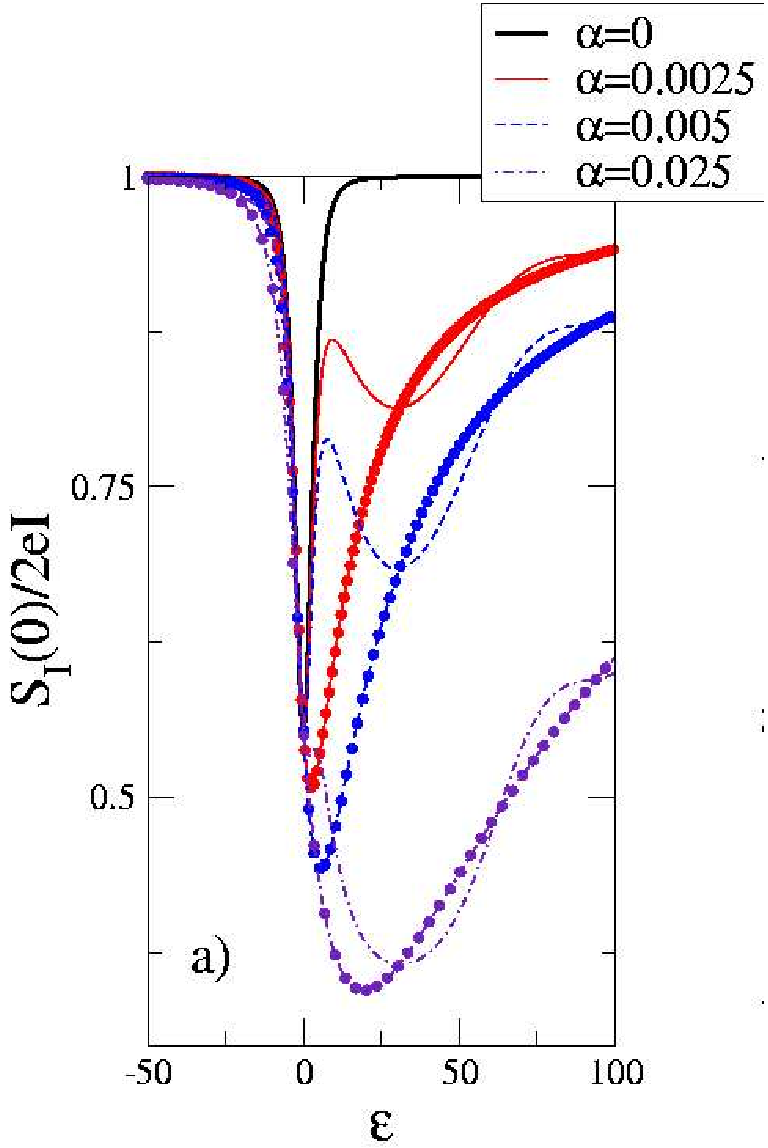
$$\dot{n}_0^{(n)} = -\Gamma_L n_0^{(n)} + \Gamma_R n_R^{(n-1)}, \quad \dot{n}_{L/R}^{(n)} = \pm\Gamma_{L/R} n_0^{(n)} \pm iT_c \left(p^{(n)} - [p^{(n)}]^\dagger\right), \quad \text{etc.}$$

- Quantum Jump Approach $\rho(t) = \sum_n \rho^{(n)}(t)$, generating function

$$G(s,t) \equiv \sum_n s^n \rho^{(n)}(t) = e^{(t-t_0)\Gamma(s)} G(s,t_0)$$

Eigenvalues of $\Gamma(s) \to$ full ω-dependence.

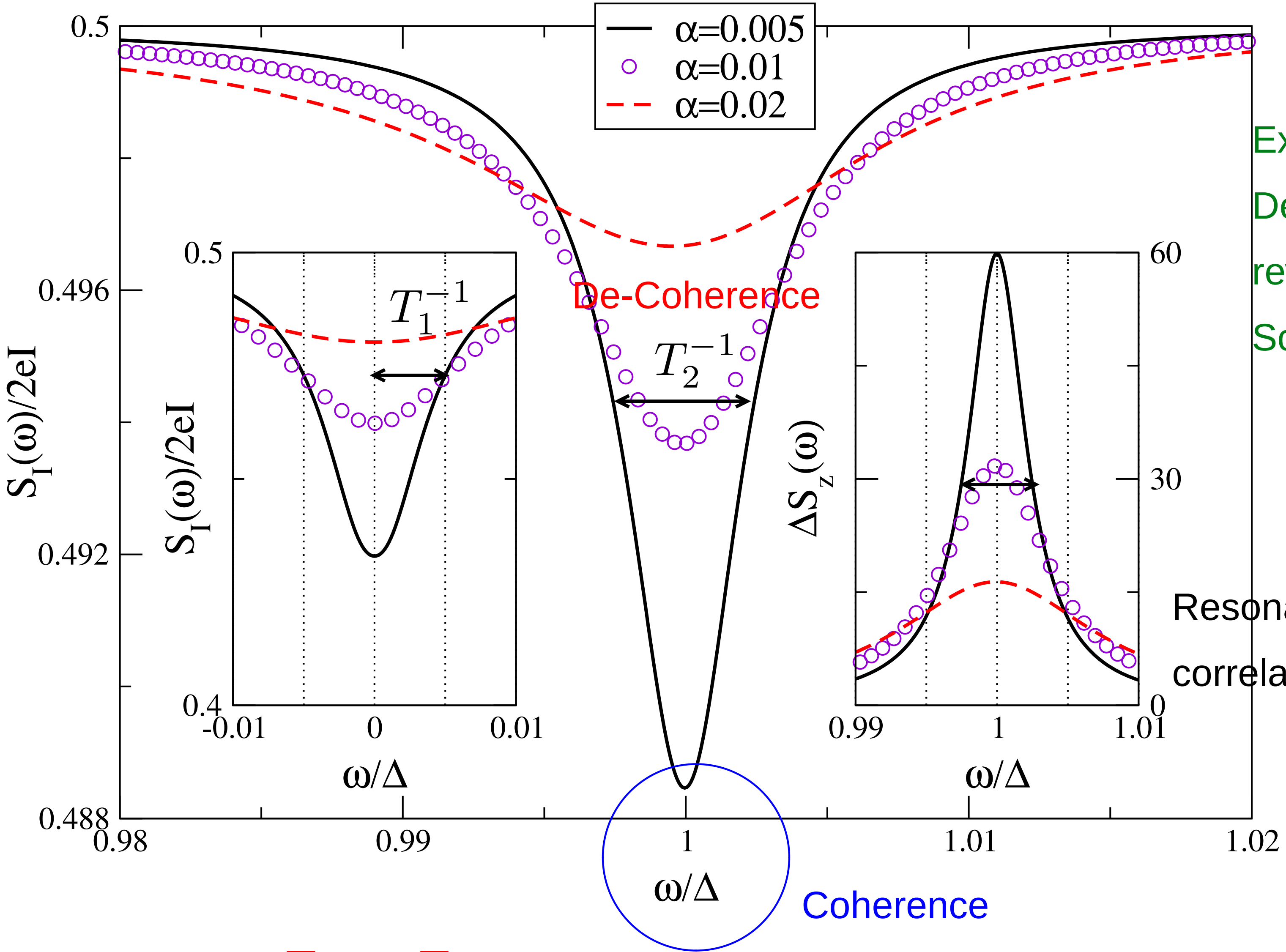
Fano Factor



- Interaction $(U \to \infty) \rightsquigarrow$ no Khlus-Lesovik form '$T(1-T)$'.
- ($\alpha = 0$) coherence suppresses noise: minimum at $\varepsilon = 0$.
- ($\alpha = 0$) large $|\varepsilon|$ 'localises' charge.
- ($\alpha \neq 0$) for $\varepsilon > 0$: dissipation suppresses noise.
- Maximal for $\gamma_p = \Gamma_R$.

frequency dependent noise spectrum

R. Aguado, TB, Phys. Rev. Lett. **92**, 206601 (2004), Eur. Phys. J. B **40**, 357 (2004).



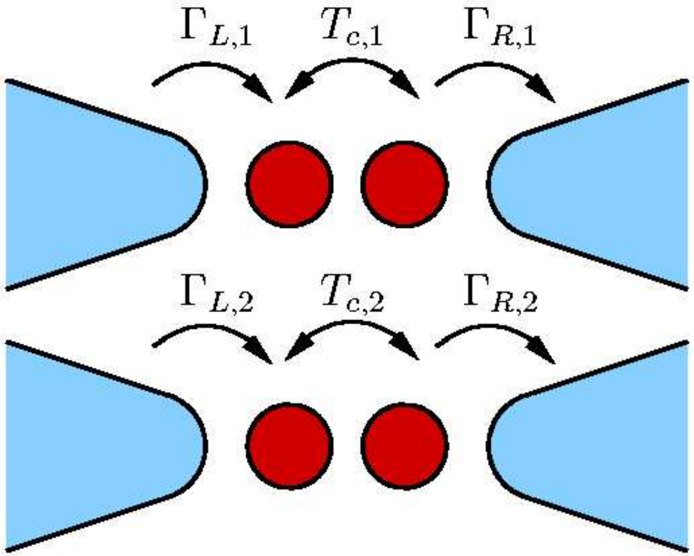
Exp. Cooper-Pair Box: R. Deblock, E. Onac, L. Gurevich, L. P. Kouwenhoven, Science **301**, 203 (2003)

Resonance as Pseudo-Spin-correlation function

contains T_1 und T_2 (PER) !

- Quantum Mechanical Transport
- Quantum Noise
- Entanglement
- Quantum Phase Transitions

Transport through coupled 2-Qubits



- *Phonon* coupling: effective interaction, Dicke effect T. Vorrath, TB, PRB 2003.
- *Coulomb* coupling: two-site Hubbard with (pseudo) spin N. Lambert, TB 2005.

Two Double Quantum Dots: Coulomb-Coupling U

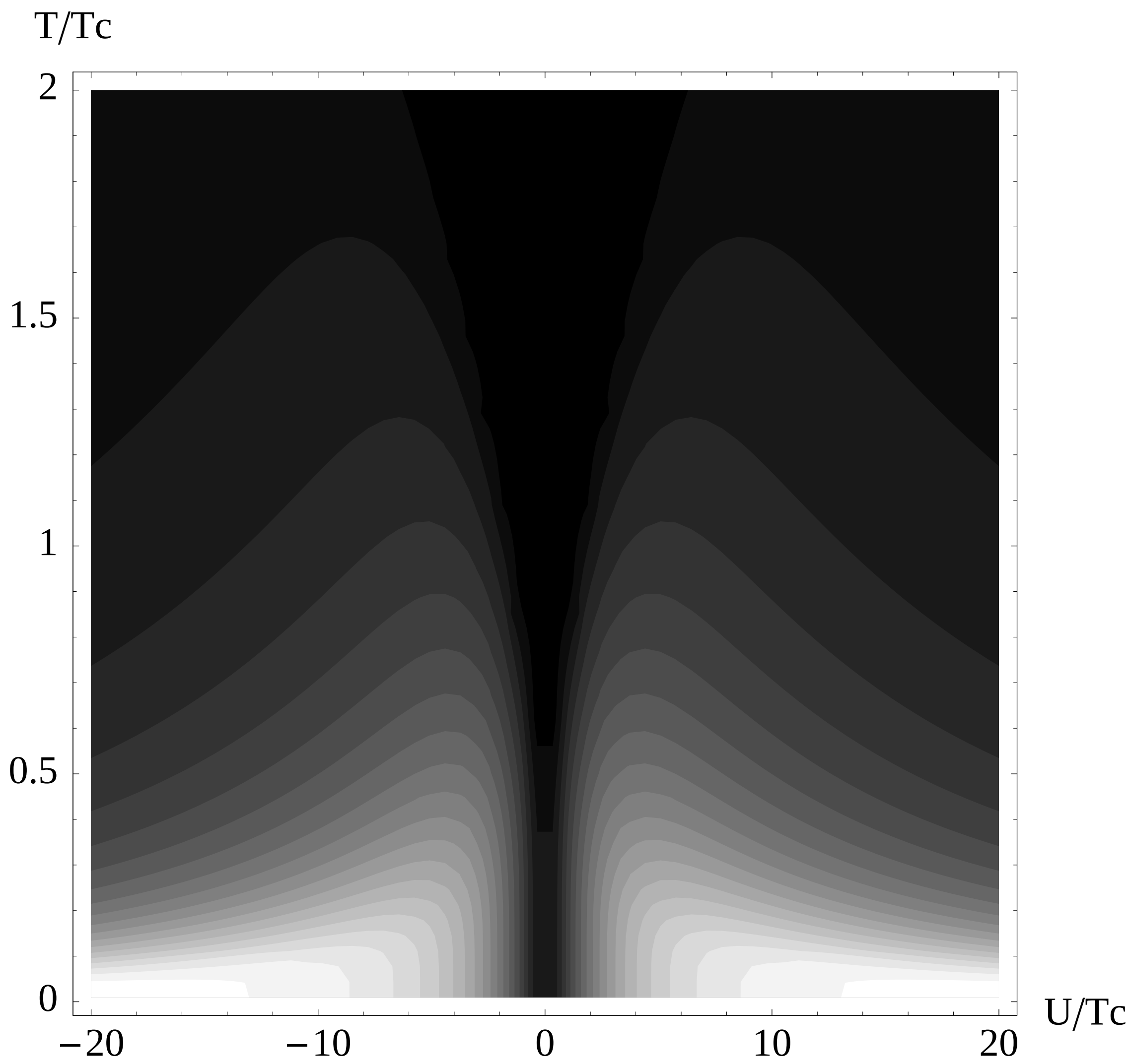
- Total Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_T + \mathcal{H}_{\rm res}$.
- Double qubit

$$\mathcal{H}_0 = \sum_{i=1,2} \left(\varepsilon_i \hat{\sigma}_z^{(i)} + T_i \hat{\sigma}_x^{(i)}\right) + \frac{U}{2}\left(\hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)} + 1\right).$$

- Electron reservoir Hamiltonians $\mathcal{H}_{\rm res} = \sum_{ki\alpha} \epsilon_{ki\alpha} c^\dagger_{ki\alpha} c_{ki\alpha}$.
- Tunnel Hamiltonian

$$\mathcal{H}_T = \sum_k (V_k^{\alpha i} c^\dagger_{ki\alpha} s^i_\alpha + H.c.), \quad \hat{s}^i_\alpha = |0_i\rangle\langle\alpha_i|, \quad \alpha = L, R, \quad i = 1, 2.$$

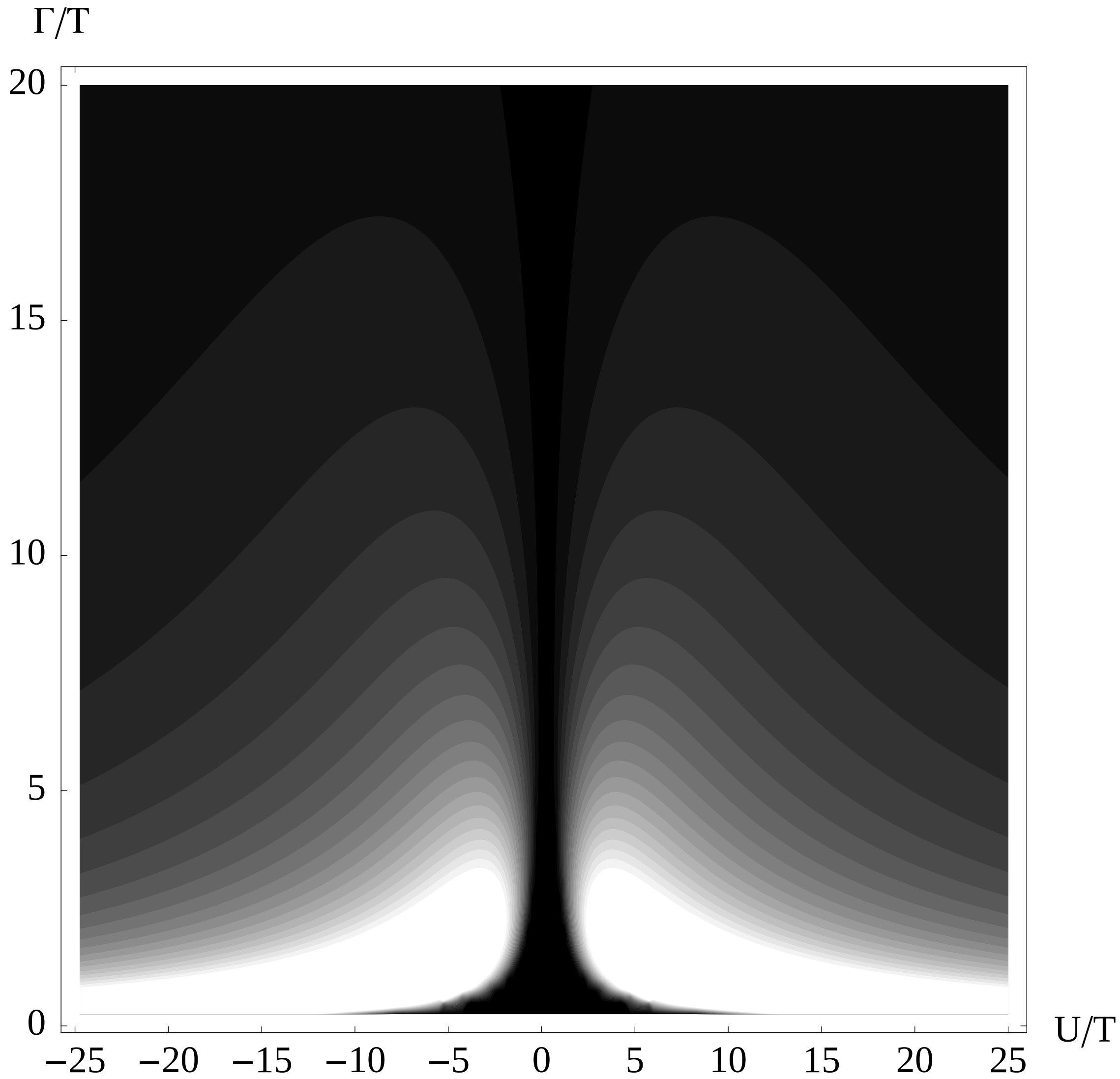
Equilibrium Entanglement ($\mathcal{H}_T = 0$) for $\rho(T) = e^{-\mathcal{H}_0/T}/Z$



- Four eigenvalues of $\mathcal{H}_0$, $E_0 = 0$, $E_1 = U$, and $E_\pm = (U \pm \sqrt{16T_c^2 + U^2})/2$.
- $\rho(T)$ too mixed to be entangled at weak U.
- Entanglement maximum at optimal U-value.

N. Lambert 2005.

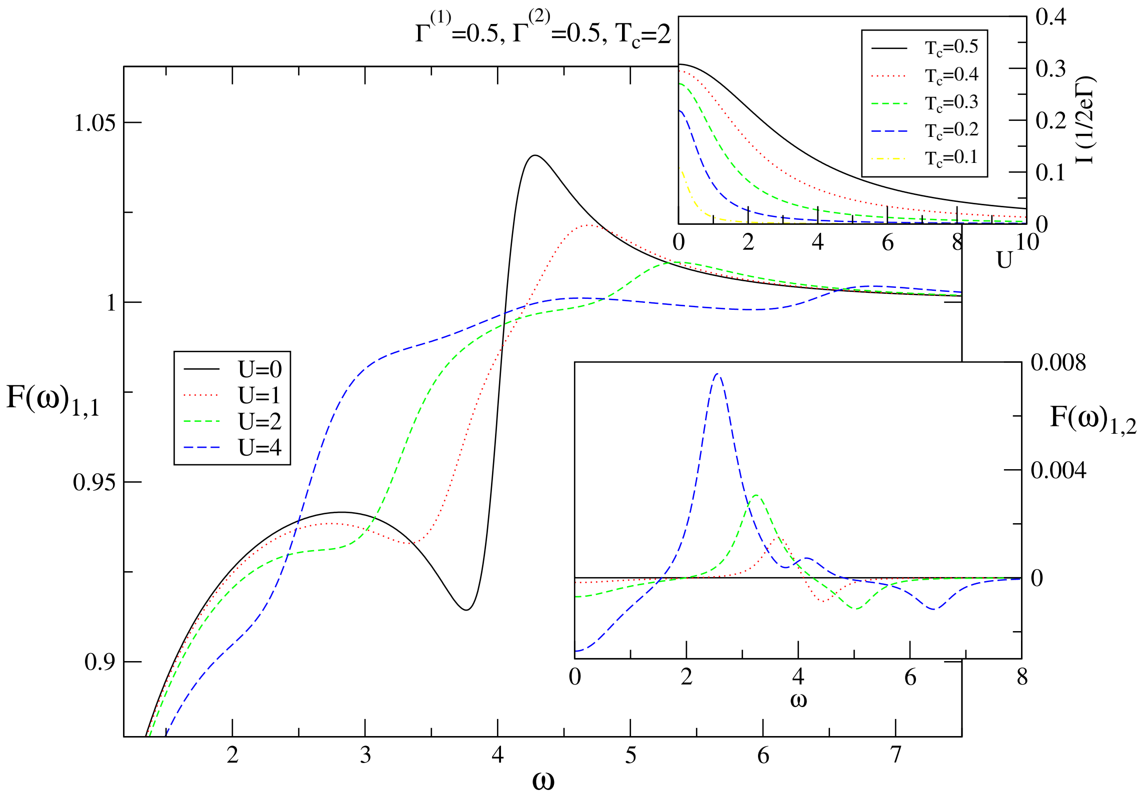
Non-Equilibrium Entanglement ($\mathcal{H}_T \neq 0$).



- Stationary solution ρ_∞ of Master equation.
- Concurrence of two-electron projection $\hat{P}\rho_\infty$.
- State becomes pure for $\Gamma_R \to \infty$ (Zeno), $\Gamma_R \to 0$: only then entanglement $E(U)$ continuous.

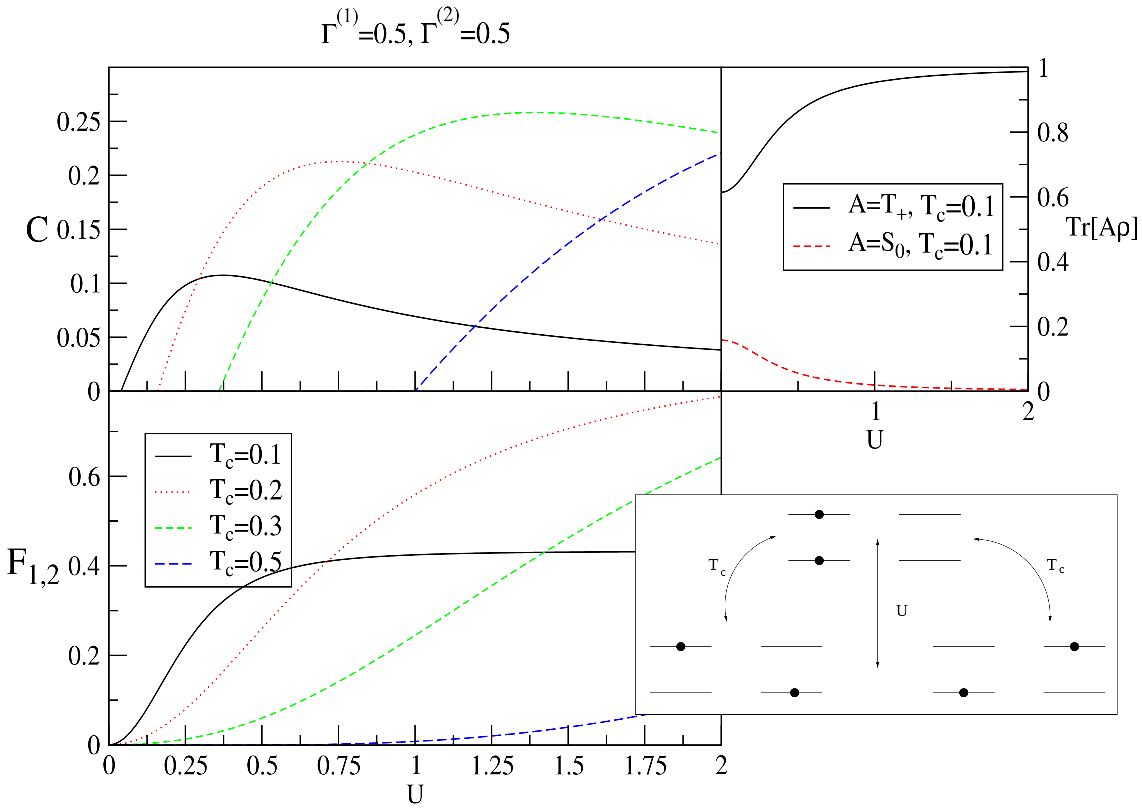
N. Lambert 2005.

Non-Equilibrium Noise.



- 'Diagonal' noise spectrum reveals double qubit *spectrum*.

Non-equilibrium noise and entanglement

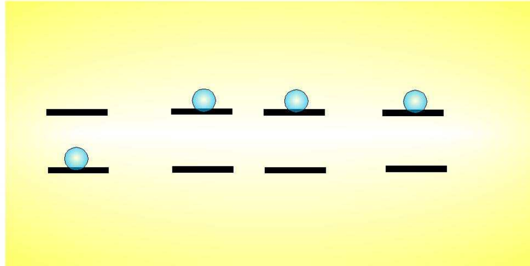


- Concurrence and cross-Fano-factor.

$N \to \infty$ qubits

- Quantum Mechanical Transport
- Quantum Noise
- Entanglement
- Quantum Phase Transitions

Single-mode superradiance (Dicke) model



- N 2-level systems coupled to cavity boson.

$$\mathcal{H}_{\mathrm{Dicke}} = \frac{\omega_0}{2}\sum_{i=1}^{N}\hat{\sigma}_{z,i} + \frac{\lambda}{\sqrt{N}}\sum_{i=1}^{N}\hat{\sigma}_{x,i}\left(a^\dagger + a\right) + \omega a^\dagger a, \quad j = N/2$$

$$= \omega_0 J_z + \frac{\lambda}{\sqrt{2j}}\left(a^\dagger + a\right)\left(J_+ + J_-\right) + \omega a^\dagger a, \quad [J_z, J_\pm] = \pm J_\pm.$$

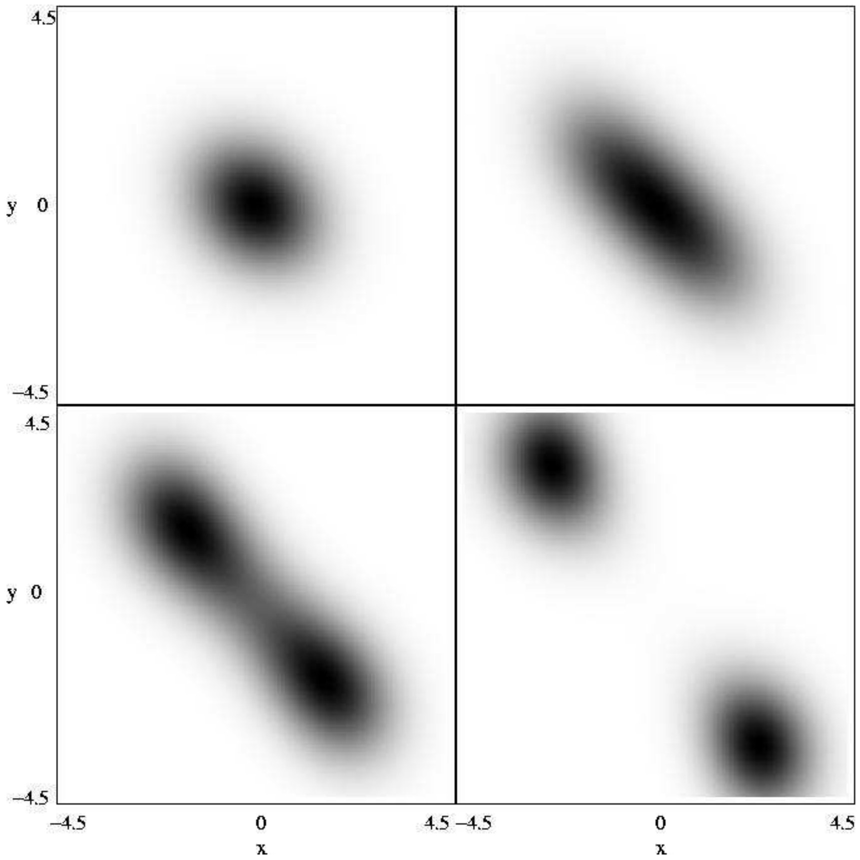
- $N = 1$: Rabi-Hamiltonian: cavity QED, nano-electromechanics...
- $N \to \infty$: $T = 0$-phase transition from $\langle a^\dagger a \rangle = 0$ to $\langle a^\dagger a \rangle \neq 0$ at $\lambda_c = \sqrt{\omega\omega_0}/2$. Exactly solvable K. Hepp and E. Lieb, Ann. Phys. **76**, 360 (1973).
- $N < \infty$: quantum chaos; Kus 85; Graham, Höhnerbach 86; Lewenkopf et al 91; level statistics.

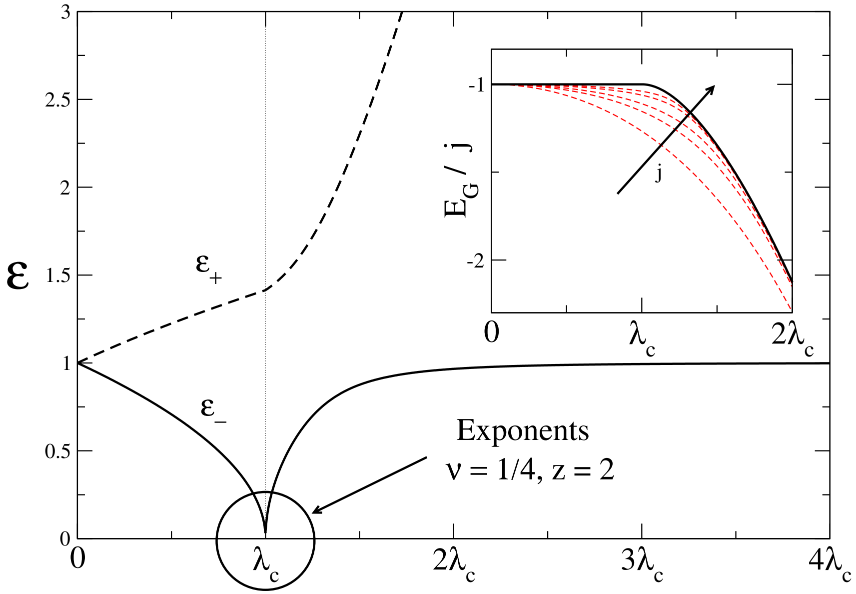
Ground State Wave Function

- Holstein-Primakoff representation $J_z = \left(b^\dagger b - j\right)$, $J_+ = b^\dagger \sqrt{2j - b^\dagger b}$.
- Normal phase $\lambda < \lambda_c$: expand square-roots, **two-mode** effective Hamiltonian

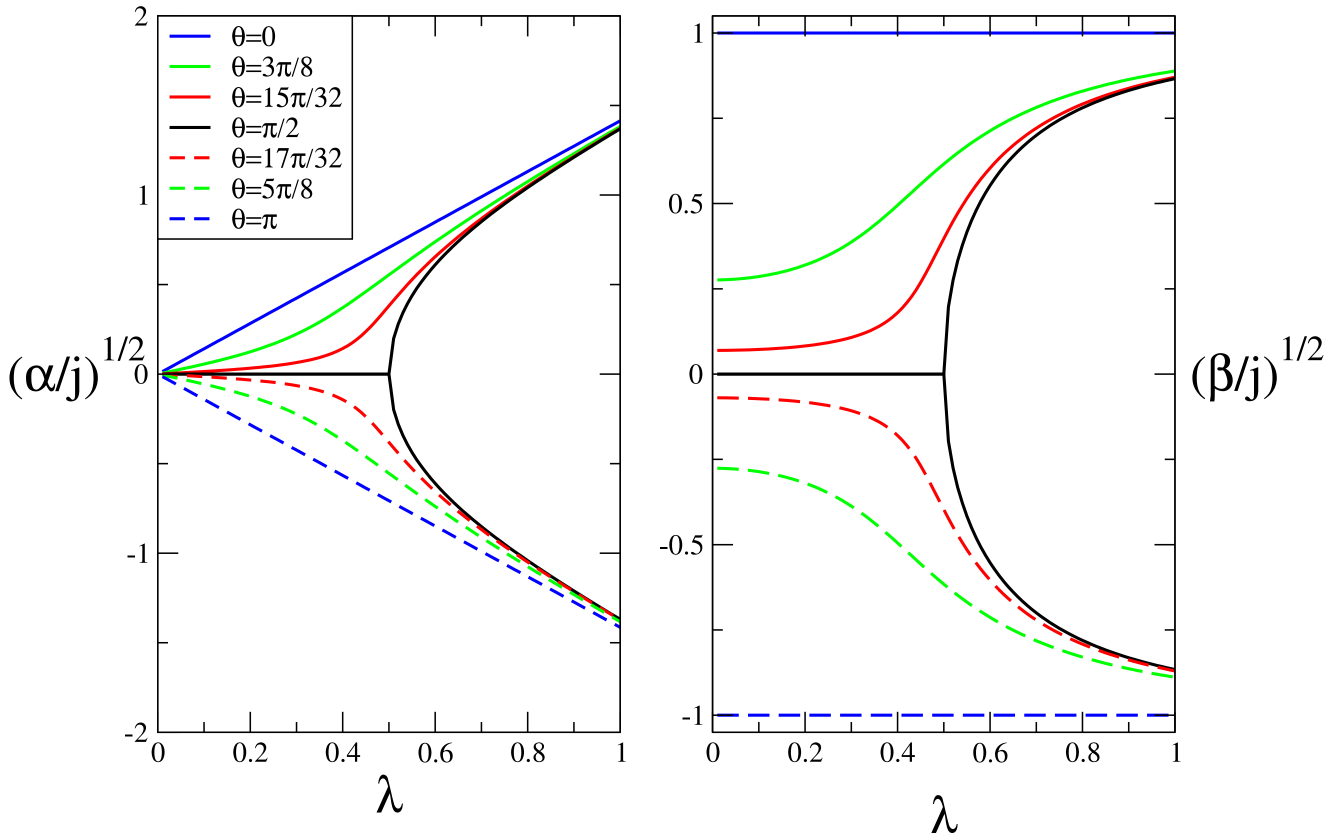
$$\mathcal{H}^{(1)} = \omega_0 b^\dagger b + \omega a^\dagger a + \lambda \left(a^\dagger + a\right)\left(b^\dagger + b\right) - j\omega_0, \quad j \to \infty.$$

- Super-radiant phase $\lambda > \lambda_c$: boson displacement with $\sqrt{\alpha}, \sqrt{\beta} \propto j$, two equivalent effective Hamiltonians (broken parity symmetry).





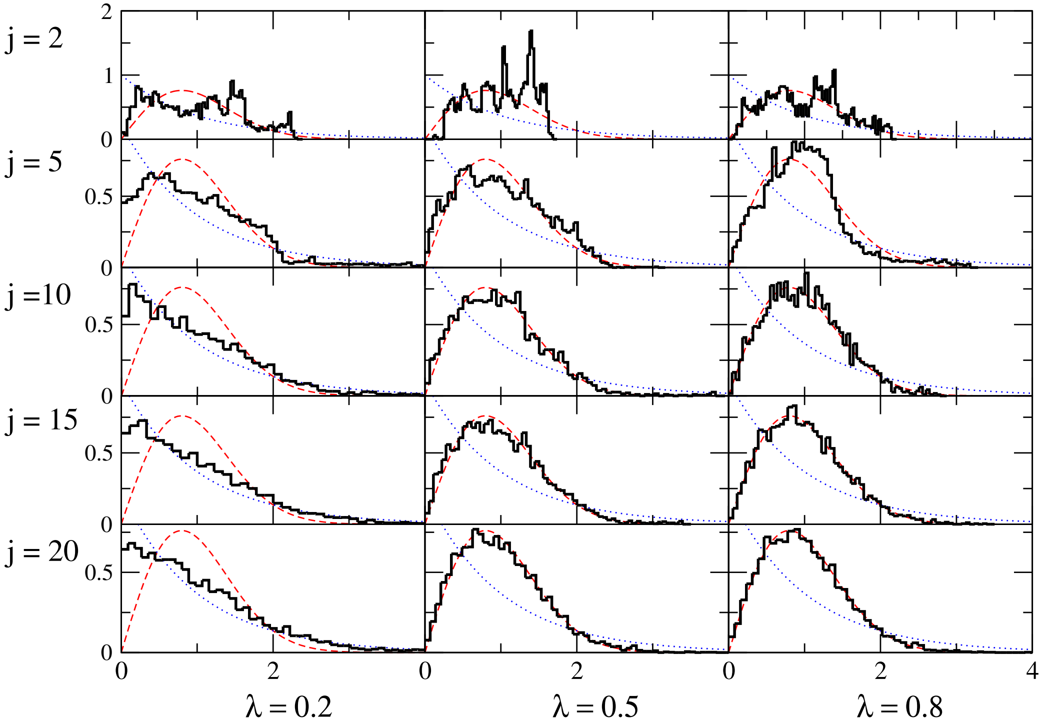
(C. Emary, TB 2003); **Left:** Ground-state $|\psi(x,y)|$ in x-y representation; $x \equiv \frac{1}{\sqrt{2\omega}}\left(a^\dagger + a\right)$ (field mode), $y \equiv \frac{1}{\sqrt{2\omega_0}}\left(b^\dagger + b\right)$ (atom); for $j = 5$ at λ/λ_c = 0.2, 0.5, 0.6, 0.7. **Right:** Excitation energies $\varepsilon_\pm$ for $j \to \infty$. Inset: scaled ground-state energy, E_G/j for $j = 1/2, 1, 3/2, 3, 5, \infty$.

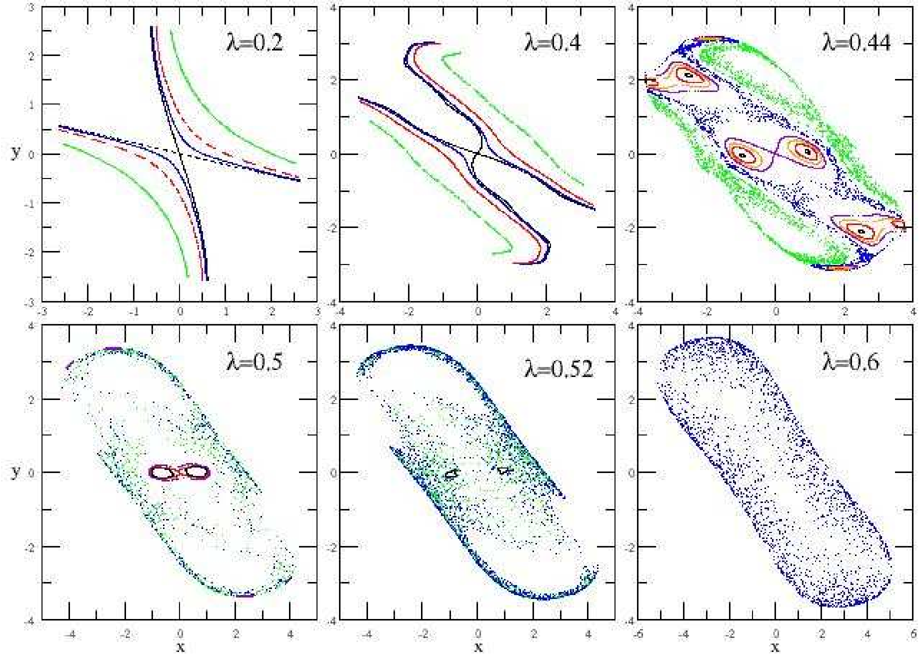


Order parameters $\alpha = \langle a^\dagger a \rangle$ and $\beta = \langle J_z \rangle + N/2$ for generic large spin-boson Hamiltonians (C. Emary, TB; 2004)

$$H_\theta = \omega a^\dagger a + \Omega (J_x \cos\theta + J_z \sin\theta) + \frac{2\lambda}{\sqrt{2j}} \left(a^\dagger + a \right) J_x .$$

Finite N: chaos





- Chaos for finite $N = 2J < \infty$.
- level spacing distribution $P(S)$.
- Transition from Poisson (localised) to Wigner-Dyson (delocalised).
- Classical Hamiltonian via HP trafo and canonical x-y representation.
- 'cat' corresponds to double attractor.

Finite-size corrections: Lipkin-Meshkov-Glick Model Nucl. Phys. **62**, 188 (1965)

- J. Vidal, G. Palacios, and R. Mosseri; Phys. Rev. A **69**, 022107 (2004).

$$H \equiv -\frac{\lambda}{N}\sum_{i<j}^{N}\left(\sigma_x^i\sigma_x^j + \gamma\sigma_y^i\sigma_y^j\right) - \sum_{i=1}^{N}\sigma_z^i$$

$$= -\frac{2\lambda}{N}\left(J_x^2 + \gamma J_y^2\right) - 2J_z + \frac{\lambda}{2}(1+\gamma), \quad J_\alpha \equiv \frac{1}{2}\sum_{i=1}^{N}\sigma_\alpha^i, \quad \alpha = x, y, z.$$

- 2nd order, mean-field type QPT from nondegenerate to doubly degenerate ground state at $\lambda_c = 1$ for any anisotropy parameter $\gamma \neq 1$.

- Rescaled concurrence $C_N \equiv NC$;

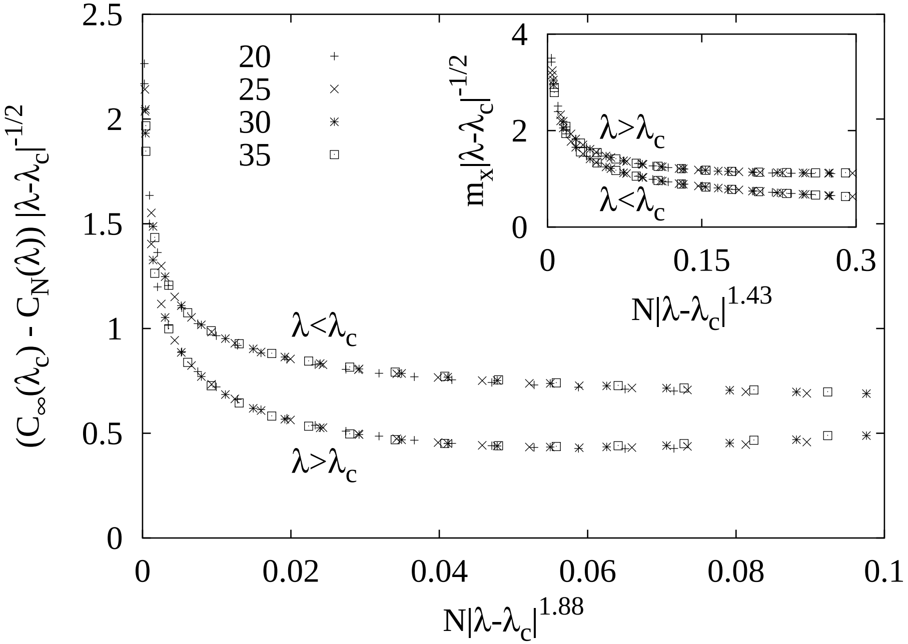
$$1 - C_{N-1}(\lambda_m) \sim N^{-0.33\pm0.01}, \quad \lambda_m - \lambda_c \sim N^{-0.66\pm0.01}, \quad \gamma \neq 1.$$

Finite-Size Scaling in Single-Mode Dicke Model

- Position of entropy maximum $\lambda^{\mathrm{M}} - \lambda_c \propto N^{-0.75\pm0.1}$, concurrence maximum $\lambda^{\mathrm{M}} - \lambda_c \propto N^{-0.68\pm0.1}$, $C_N^{\mathrm{M}}(\lambda_c) - C_N \propto N^{-0.25\pm0.01}$.

More detailed analysis by J. Reslen, L. Quiroga, and N. F. Johnson, cond-mat/0406674

One-parameter scaling analysis



$$C_\infty(\lambda_c) - C_N(\lambda) = |\lambda-\lambda_c|^a f(N|\lambda-\lambda_c|^b)$$

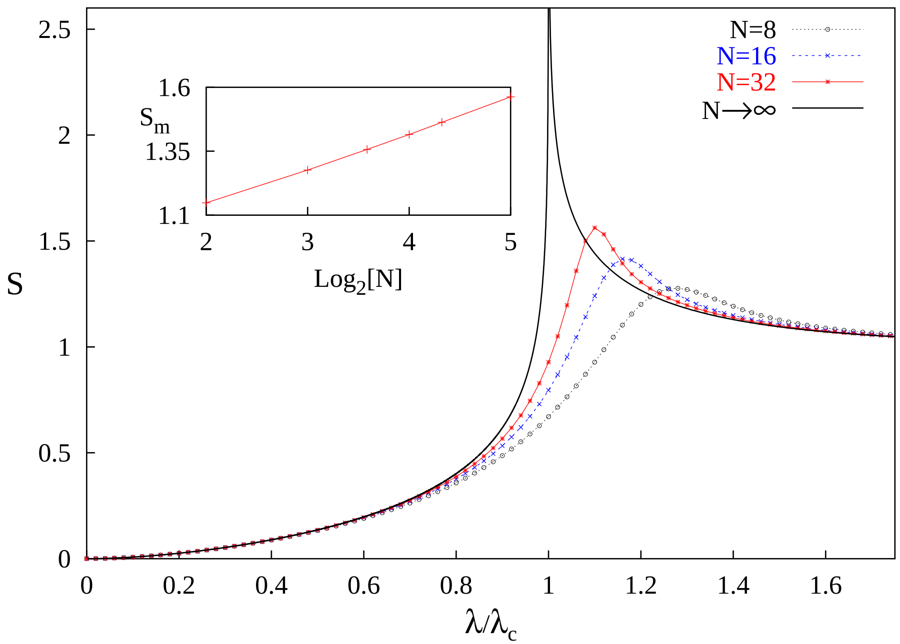
$f(N) \sim N^b$	*Dicke*	*Lipkin*
$C_\infty(\lambda_c) - C_N(\lambda_c)$	-0.26 ± 0.01	-0.30 ± 0.01
$C_\infty^M - C_N^M$	-0.28 ± 0.03	-0.30 ± 0.03
$\lambda_N^M - \lambda_c$	-0.65 ± 0.03	-0.66 ± 0.03

$\rightsquigarrow$ same universality class. Analytical results: J. Vidal *et al.*

MANCHESTER 1824 The University of Manchester

Entanglement between Atoms and Field

- Von-Neumann entropy $S \equiv -\text{tr}\hat{\rho}\log_2\hat{\rho}$ of reduced density matrix (RDM) $\hat{\rho}$ of field-mode.
- Mapped to single harmonic oscillator with frequency Ω_L at temperature $T \equiv 1/\beta$.



$$S = \log_2 \xi + const$$

$$\xi \equiv \varepsilon_-^{-1/2} \propto |\lambda - \lambda_c|^{-z\nu/2}, \nu = \frac{1}{4}, z = 2.$$

For $\lambda \to \lambda_c$, fictitious thermal oscillator parameter $\zeta = \hbar\Omega_\infty/k_B T \to 0$: *classical* limit.

Pairwise Entanglement between Atoms

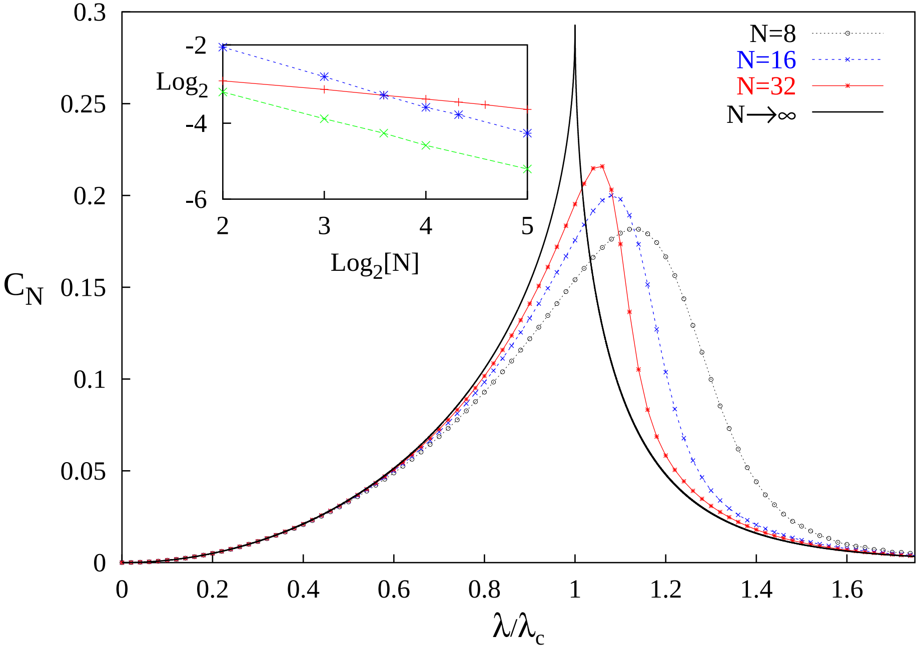
- *Scaled* concurrence $C_N \equiv NC$, S_N symmetry helps (X. Wang and K. Mølmer, Eur. Phys. J. D **18**, 385 (2002).)

- Perturbation theory: $C_N(\lambda \to 0) \sim 2\alpha^2/(1+\alpha^2), \quad \alpha \equiv \lambda/(\omega + \omega_0)$.

- Relation between scaled concurrence and *momentum squeezing*,

$$C_\infty = (1+\mu)\left[\frac{1}{2} - (\Delta p_y)^2/\omega_0\right] + \frac{1}{2}(1-\mu),$$

$\mu = 1$ in normal phase and $\mu = (\lambda_c/\lambda)^2$ in SR phase. Kitagawa-Ueda (Phys. Rev. A **47**, 5138 (1993)) *spin squeezing* for $\xi^2 \equiv \frac{4}{N}(\Delta \vec{S}\vec{n})^2 < 1$. (X. Wang and B. C. Sanders, Phys. Rev. A **68**, 012101 (2003).).



Concurrence assumes its *maximum* $C_\infty = 1-\sqrt{2}/2 \approx 0.293$ *at* the critical point $\lambda = \lambda_c$ (as in Lipkin model, J. Vidal, G. Palacios, and R. Mosseri; Phys. Rev. A **69**, 022107 (2004); J. Reslen, L. Quiroga, and N. F. Johnson, cond-mat/0406674 (2004)).

$$
\begin{aligned}
C_\infty^{x\le 1} &= 1 - \frac{1}{2}\left[\sqrt{1+x} + \sqrt{1-x}\right], \quad x \equiv \lambda/\lambda_c \\
C_\infty^{x\ge 1} &= 1 - \frac{1}{\sqrt{2}x^2}\left[\sin^2\gamma\sqrt{1+x^4-\sqrt{(1-x^4)^2+4}} + \cos^2\gamma\sqrt{1+x^4+\sqrt{(1-x^4)^2+4}}\right] \\
2\gamma &= \arctan[2/(x^2-1)] \quad \text{in SR phase.}
\end{aligned}
$$

N. Lambert, C. Emary, TB, Phys. Rev. Lett. **92**, 073602 (2004).

Summary

- $N = 1, 2$ 'Non-equilibrium qubits'
  - '3 state transport pseudo-spin-boson' model: dissipation, quantum noise.
  - QIP tasks, Q-Optics effects, NEMS stuff (single phonon).
  - So far infinite bias limit. Finite bias: Co-tunneling, Kondo physics ...
- $N \to \infty$ pseudo-spin-boson.
  - Single boson Dicke with chaos ($N < \infty$) and QPT ($N = \infty$).
  - Scaling of finite-N corrections.
  - 'Quantum catastrophes'.

TB, Phys. Rep. **408**, 315 (2005).