Aalto University
School of Science

Centre for Quantum Engineering

Mesoscopic Heat Engines in Linear Response: A Unifying Perspective

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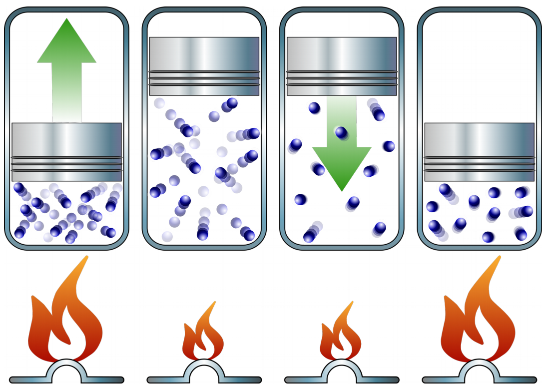
WE-Heraeus-Seminar Non-Markovianity and Strong-Coupling Effects in Thermodynamics

Two Classes

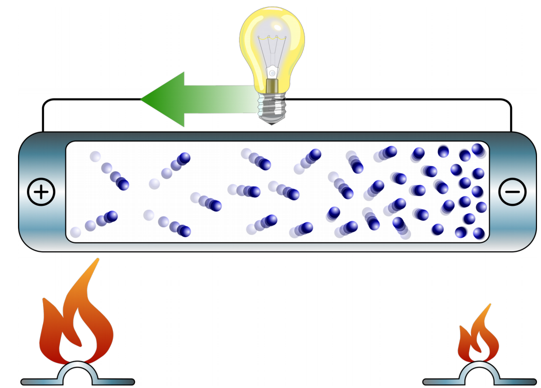
Cyclic Engines



$$\eta \equiv \frac{W^{\mathrm{out}}}{Q^{\mathrm{in}}} \leq \eta_C \equiv 1 - \frac{T_c}{T_h}$$

$$P \equiv \frac{W^{\mathrm{ext}}}{\mathcal{T}} \leq \textbf{?}$$

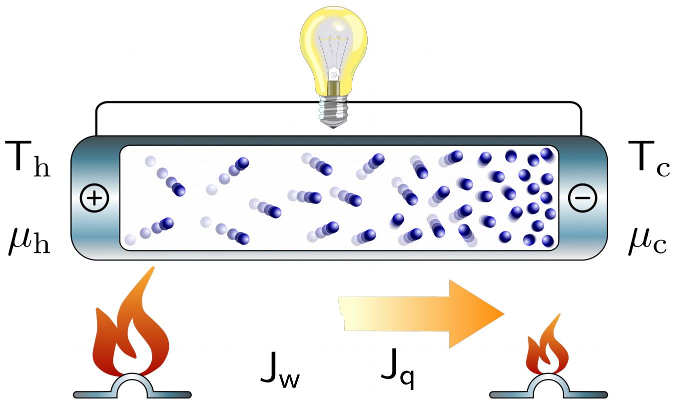
Thermoelectric Engines



I. Unified description of cyclic and thermoelectric engines

II. General trade-off relation between power and efficiency

Irreversible Thermodynamics



Linear response

$$J_w = L_{ww}\mathcal{F}_w + L_{wq}\mathcal{F}_q$$
$$J_q = L_{wq}\mathcal{F}_q + L_{qq}\mathcal{F}_q$$


L. Onsager

Affinities:

$$\mathcal{F}_w \equiv (\mu_h - \mu_c)/T_c$$
$$\mathcal{F}_q \equiv 1/T_c - 1/T_h$$

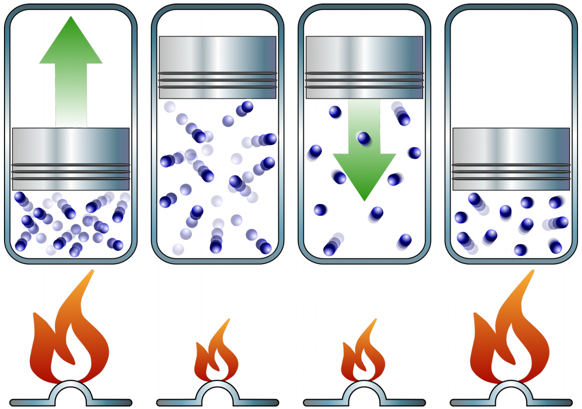
Time-reversal symmetry implies reciprocity relations:

Second law:

$$\dot{S} = \mathcal{F}_w J_w + \mathcal{F}_q J_q \geq 0$$

$$L_{\alpha\beta}[\mathbf{B}] = L_{\beta\alpha}[-\mathbf{B}]$$

Periodic Driving



Working fluid: $H_t(\mathbf{x}) \equiv H^0 + \Delta H g_t^w(\mathbf{x})$

Environment: $T_t \equiv \dfrac{T_c T_h}{T_h + (T_c - T_h)\gamma_t^q}$

Affinities & Generalized Fluxes:

$$\mathcal{F}_w \equiv \Delta H / T_c \qquad \mathcal{F}_q \equiv 1/T_c - 1/T_h$$

$$J_w \equiv \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} dt \int dx\, \dot{g}_t^w(\mathbf{x}) p_t^{cyc}(\mathbf{x})$$

$$J_q \equiv \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} dt \int dx\, \gamma_t^q H_t(\mathbf{x}) \dot{p}_t^{cyc}(\mathbf{x})$$

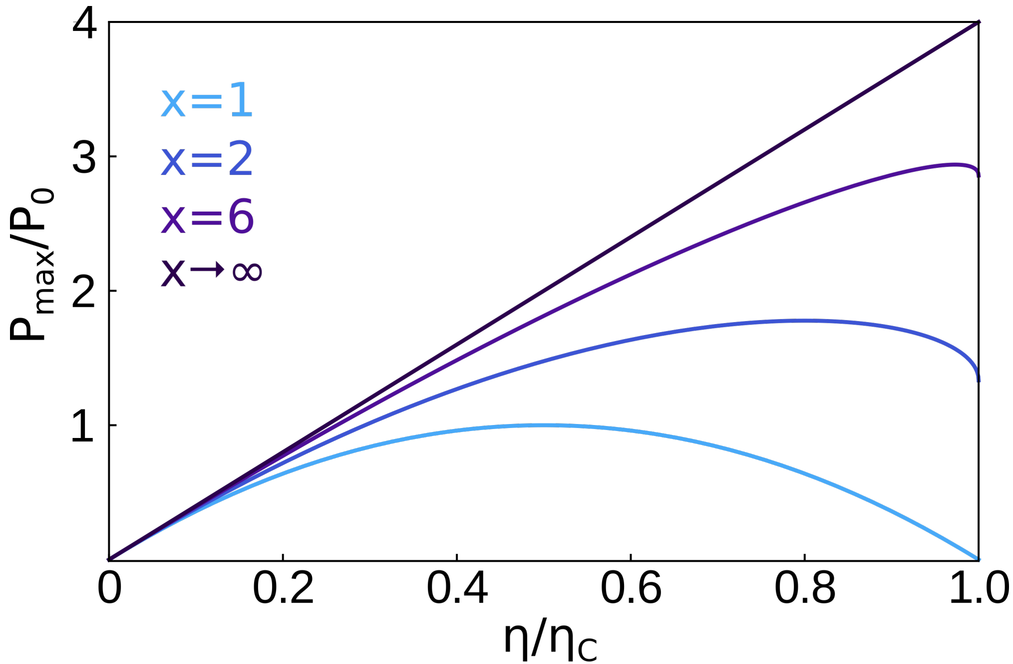
Second law & Reciprocity relations:

$$\dot{S} = \mathcal{F}_w J_w + \mathcal{F}_q J_q \geq 0$$

$$L_{\alpha\beta}[H_t, T_t, \mathbf{B}] = L_{\beta\alpha}[H_{-t}, T_{-t}, -\mathbf{B}]$$

[1] K. Brandner, K. Saito, U. Seifert; Phys. Rev. X **5** 031019 (2015)

Power vs Efficiency



$$P = -T_c \mathcal{F}_w J_w \qquad \eta = P/J_q$$

$$P_0 \equiv T_c \mathcal{F}_q^2 L_{qq} \qquad \eta_C \equiv 1 - T_c/T_h$$

$$x \equiv L_{wq}/L_{qw}$$

Engines with broken time-reversal symmetry can apparently reach Carnot-efficiency at finite power.

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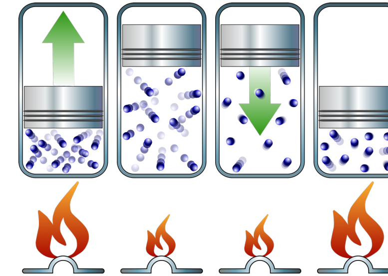
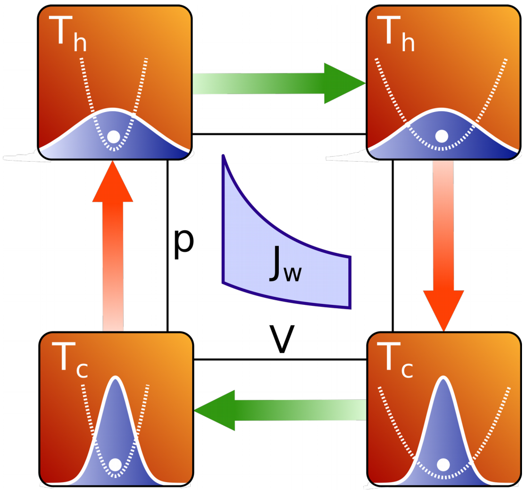
[1] G. Benenti, K. Saito, G. Casati; Phys. Rev. Lett. **106** 230602 (2011)

[2] K. Brandner, K. Saito, U. Seifert; Phys. Rev. X **5** 031019 (2015)

Stochastic Heat Engines





Energy conservation
& Detailed balance:

$$\mathcal{L}_t^{\mathrm{rev}\dagger}(\mathbf{x})\mathrm{H}_t(\mathbf{x}) = 0$$

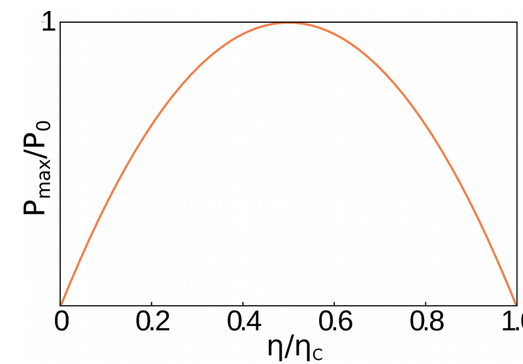
$$\mathcal{L}_t^{\mathrm{irr}}(\mathbf{x})\mathrm{p}_t^{\mathrm{eq}}(\mathbf{x}) = \mathrm{p}_t^{\mathrm{eq}}(\mathbf{x})\mathcal{L}_t^{\mathrm{irr}\dagger}(\mathbf{x})$$

Fokker-Planck dynamics:

$$\partial_t \mathrm{p}_t^{\mathrm{cyc}}(\mathbf{x}) = [\mathcal{L}_t^{\mathrm{rev}}(\mathbf{x}) + \mathcal{L}_t^{\mathrm{irr}}(\mathbf{x})]\mathrm{p}_t^{\mathrm{cyc}}(\mathbf{x})$$

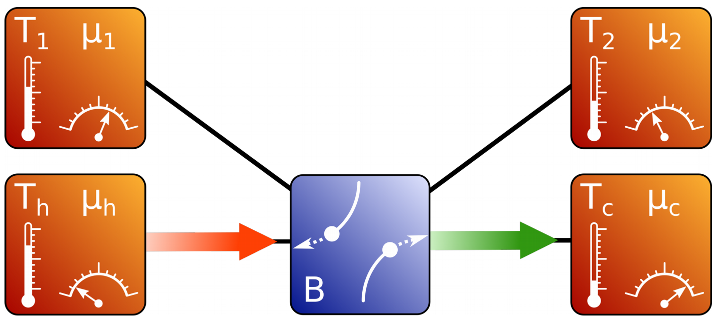
$$\frac{\mathrm{P}}{\mathrm{P}_0} \leq \frac{\eta}{\eta_C}\left(1 - \frac{\eta}{\eta_C}\right)$$

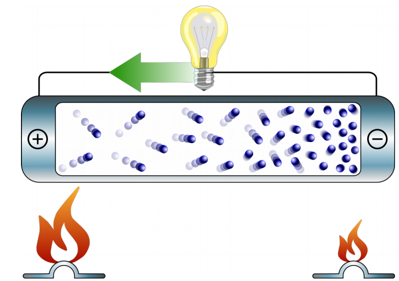
$$\mathrm{P}_0 \sim \langle \mathrm{H}^0 \rangle_{\mathrm{eq}} / \tau_{\mathrm{rel}}$$



[1] K. Brandner, K. Saito, U. Seifert; Phys. Rev. X **5** 031019 (2015)

Multi-Terminal Model





Particle conservation
& Self-consistency:

$$\sum_i |S_E^{ij}|^2 = \sum_j |S_E^{ij}|^2 = 1 \qquad J_\alpha^1 = \cdots = J_\alpha^n = 0$$

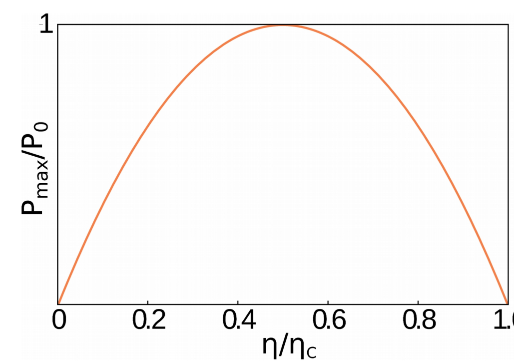
$$J_\alpha^i = \sum_j L_{\alpha w}^{ij} \mathcal{F}_w^j + L_{\alpha q}^{ij} \mathcal{F}_q^j$$

$$L_{\alpha\beta}^{ij} = \int_0^\infty dE\, h_E^\alpha h_E^\beta \left(\delta_{ij} - |S_E^{ij}|^2\right) f_E$$

$$h_E^q = E - \mu_c \quad h_E^w = 1$$

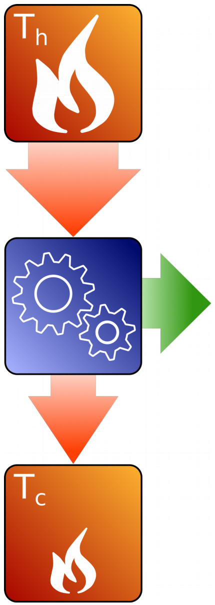
$$\frac{P}{P_0} \leq \frac{\eta}{\eta_C}\left(1 - \frac{\eta}{\eta_C}\right)$$

$$P_0 \sim k_B^2 T_c^2 / \hbar$$



[1] K. Brandner, U. Seifert; Phys. Rev. E **91** 012121 (2015)

Conclusions & Perspectives



I. Unified framework for mesoscopic heat engines

II. General trade-off relation between power and efficiency in linear response

$$\frac{P}{P_0} \leq \frac{\eta}{\eta_C}\left(1 - \frac{\eta}{\eta_C}\right)$$

Extensions & Outlook

- Classical engines in non-linear response [1]
- Quantum engines and the role of coherence [2,3]
- Strong coupling

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[1] N. Shiraishi, K. Saito, H. Tasaki; Phys. Rev. Lett. **117** 190601 (2016)
[2] K. Brandner, U. Seifert; Phys. Rev. E **93** 062134 (2016)
[3] K. Brandner, M. Bauer, U. Seifert; arXiv:1703.02464 (2017)