NON-MARKOVIANITY AND STRONG COUPLING EFFECTS IN OPTIMAL CONTROL THEORY

1 8 0 3

Leonardo A. Pachón

Instituto de Física
Universidad de Antioquia

http://gfam.udea.edu.co/~lpachon/

Bad Honnef, Germany
April 11th , 2017

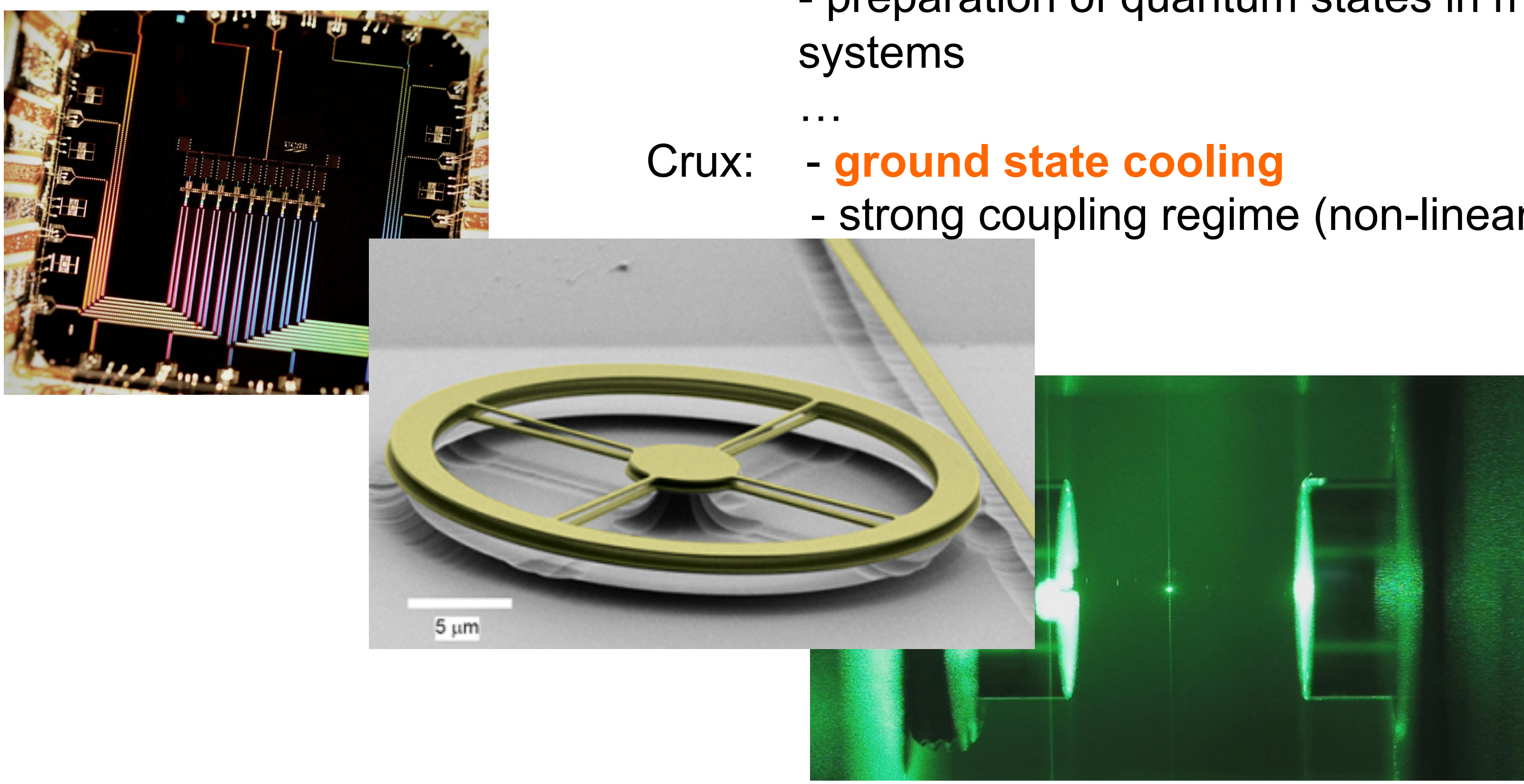
Johan F. Triana, Andres F. Estrada

Grupo de Física Atómica y Molecular, Instituto de Física, Universidad de Antioquia.

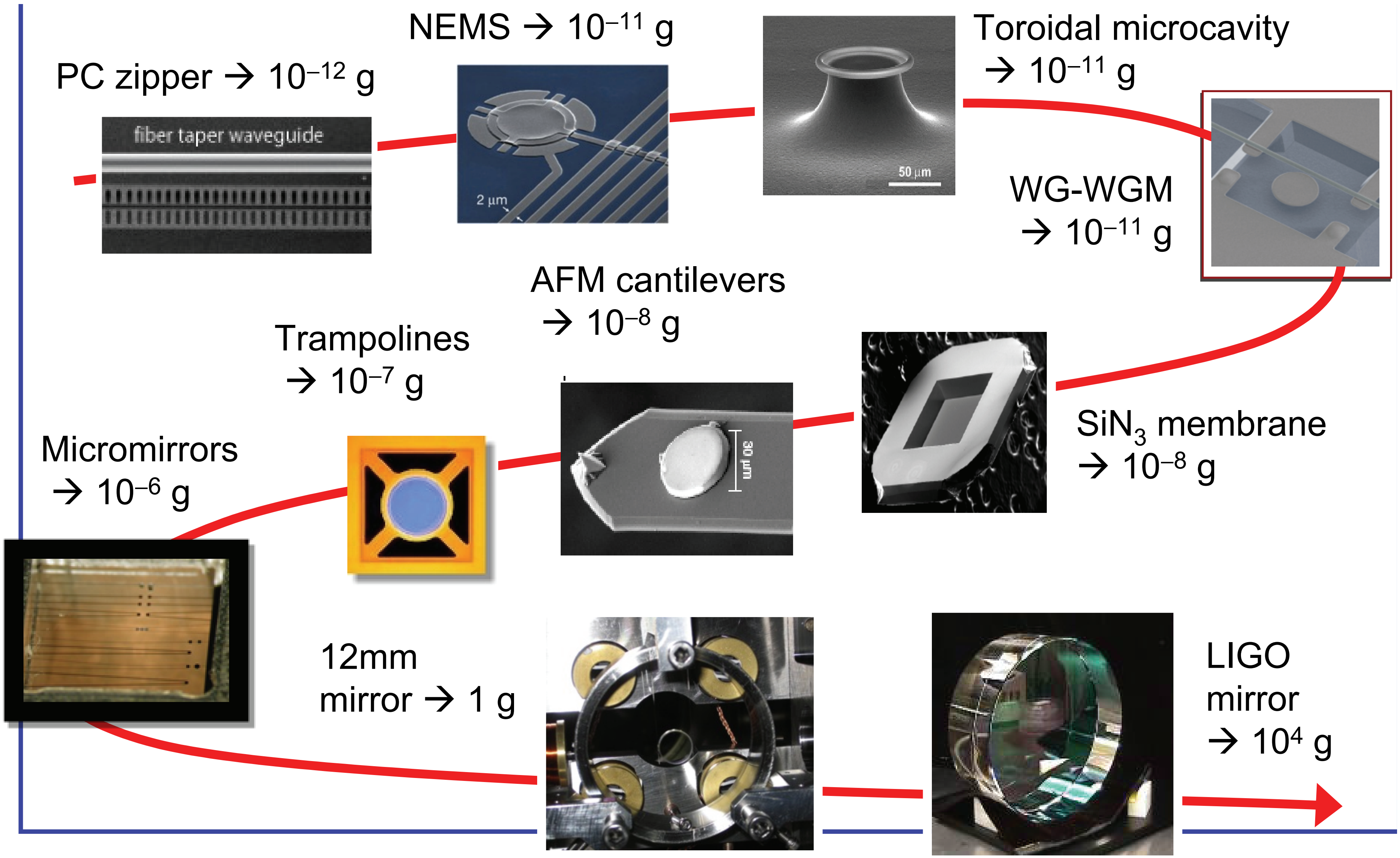
Optomechanics: third technology in the quantum regime

Goals: - interference between quantum technologies
- preparation of quantum states in macroscopic systems
…

Crux: - **ground state cooling**
- strong coupling regime (non-linear effects)



Rev. Mod. Phys. 86, 1391 (2014)

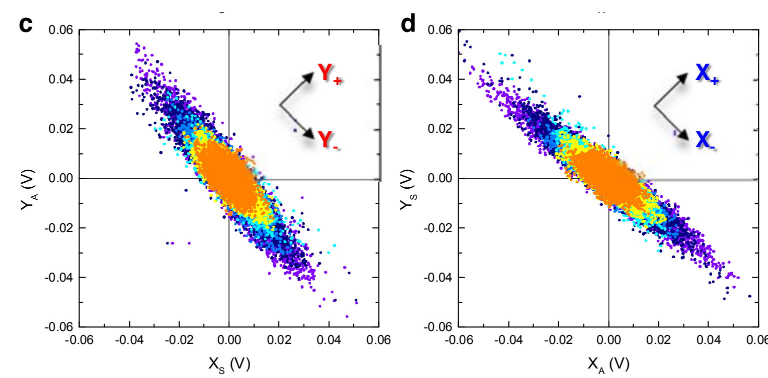


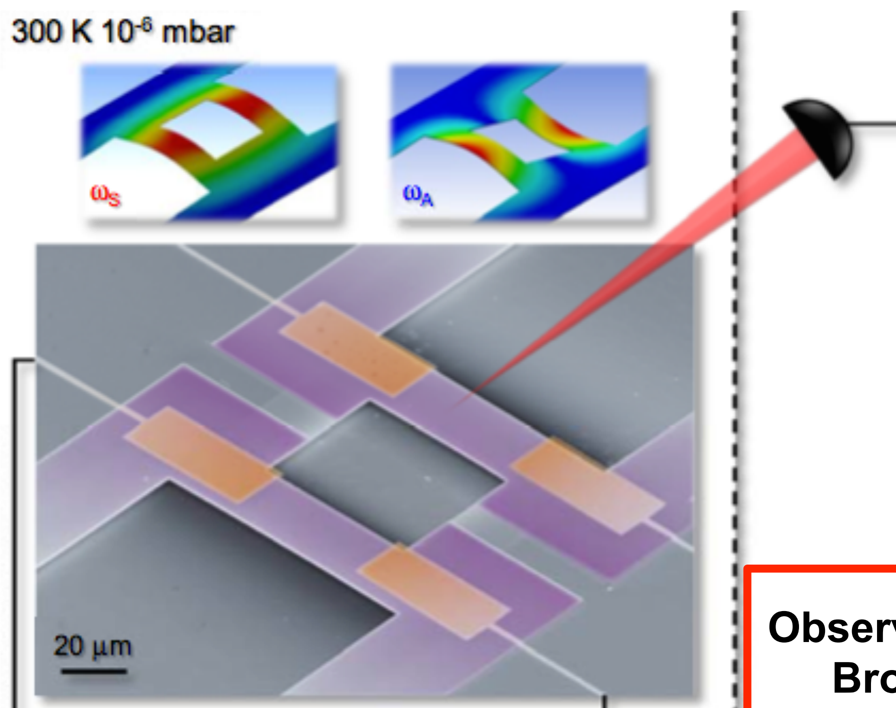
Rev. Mod. Phys. 86, 1391 (2014)

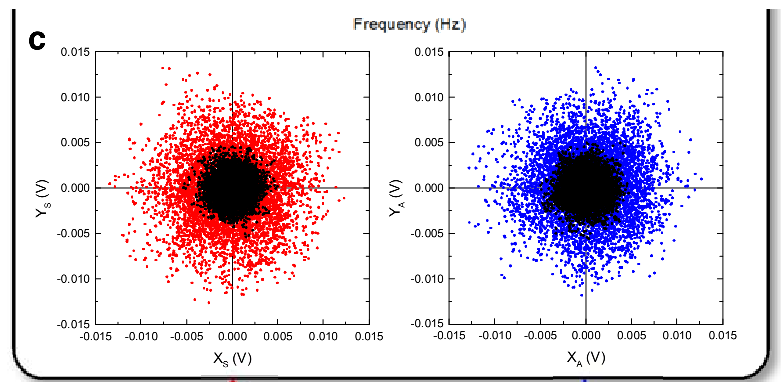
Two-mode squeezing in an electromechanical resonator

Mahboob et al. Phys. Rev. Lett. 113, 167203 (2014)

$$H_S = \sum_{\alpha=1}^{2}\left(\frac{p_\alpha^2}{2m_\alpha} + \frac{1}{2}m_\alpha\omega_\alpha^2 q_\alpha^2\right) + c(t)q_1q_2$$





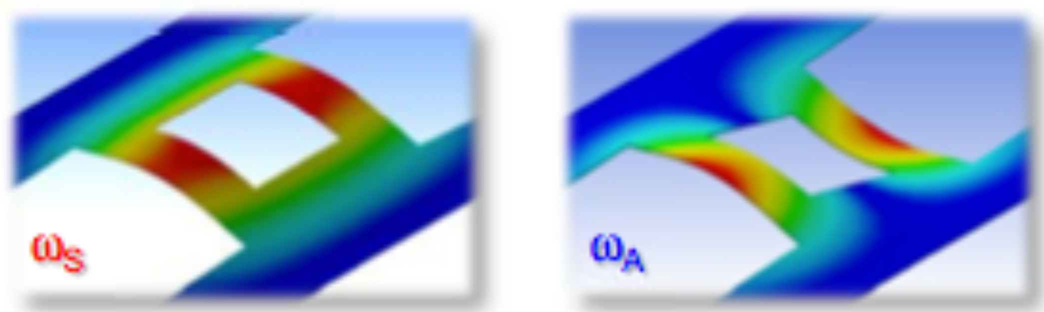


Observation of non-Markovian micro-mechanical Brownian motion Nature Comm. 6, 7606 (2015)

Quantum Limit for Driven Linear Non-Markovian Open Systems

AF Estrada, LA Pachon New J. Phys. **17**, 033038 (2015)

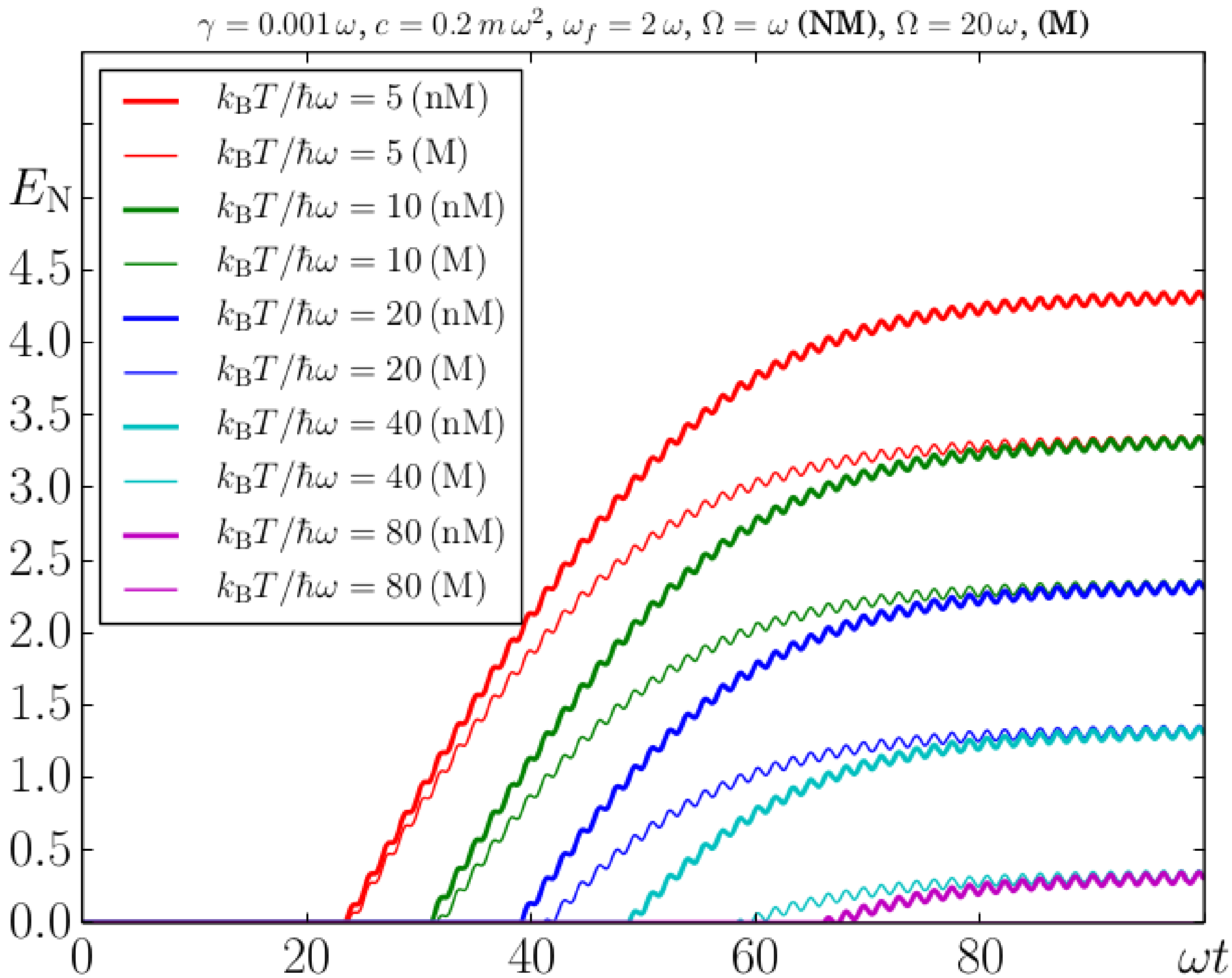
$$\tilde{\rho}(q''_{1+}, q''_{2+}, q''_{1-}, q''_{2-}, t) = \int_{-\infty}^{\infty} dq'_{1+}\, dq'_{2+}\, dq'_{1-}\, dq'_{2-}\; J(q''_{1+}, q''_{2+}, q''_{1-}, q''_{2-}, t; q'_{1+}, q'_{2+}, q'_{1-}, q'_{2-}, 0) \times \rho_S(q'_{1+}, q'_{2+}, q'_{1-}, q'_{2-}, 0),$$



$$J(Q''_1, Q''_2, q''_1, q''_2, t; Q'_1, Q'_2, q'_1, q'_2, 0) = \frac{1}{N(t)} \exp\left\{ \frac{i}{\hbar} \sum_{\alpha=1}^{2} \left[q''_\alpha \dot{Q}_\alpha(t) - q'_\alpha \dot{Q}_\alpha(0) \right] - \frac{1}{\hbar} \int_0^t ds \int_0^s du \sum_{\alpha=1}^{2} K_\alpha(u-s) q_\alpha(s) q_\alpha(u) \right\}$$

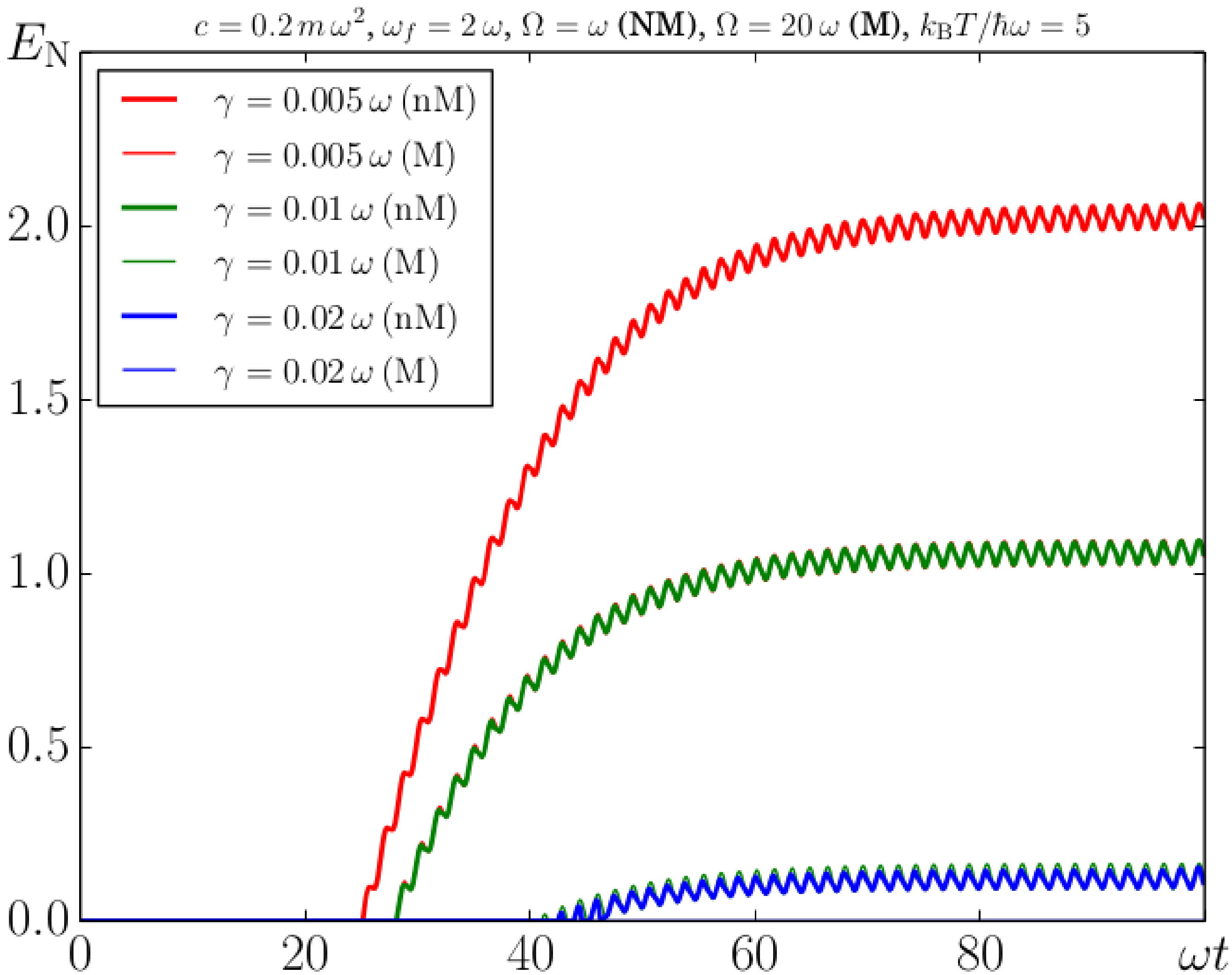
Quantum Limit for Driven Linear Non-Markovian Open Systems

AF Estrada, LA Pachon New J. Phys. **17**, 033038 (2015)



Quantum Limit for Driven Linear Non-Markovian Open Systems

AF Estrada, LA Pachon New J. Phys. **17**, 033038 (2015)



Quantum Limit for Driven Linear Non-Markovian Open Systems

AF Estrada, LA Pachon New J. Phys. **17**, 033038 (2015)

$$S(\omega)=\frac{1}{2m}J(\omega)\coth\left(\frac{1}{2}\hbar\omega\beta\right)\leq\omega\mu$$

$$\frac{k_B T}{\hbar\omega}\gg 1$$

$$\frac{1}{m}k_B T J(\omega)<\hbar\omega^2\mu$$

Non-Markovian scaled parameters (for the spectral density used here at long time)

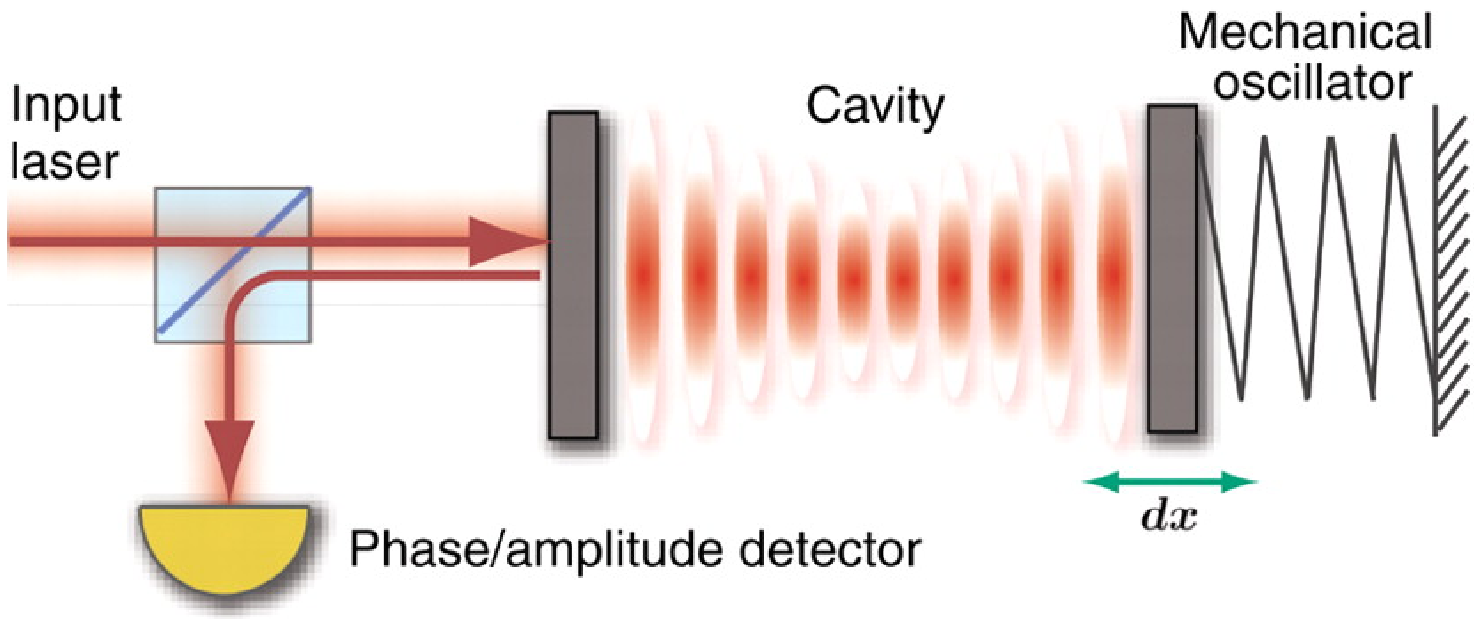
$$T_{NM}=T\left(1-\frac{1}{1+\frac{\Omega^2}{\omega^2}}\right)$$

$$\gamma_{NM}=\gamma\left(1-\frac{1}{1+\frac{\Omega^2}{\omega^2}}\right)$$

$$\mu_{NM}=\mu\left(1-\frac{1}{1+\frac{\Omega^2}{\omega^2}}\right)^{-1}$$

Ultrafast Optimal Sideband Cooling under Non-Markovian Evolution

JF Triana, AF Estrada, LA Pachon Phys. Rev. Lett. **116**, 183602 (2016)



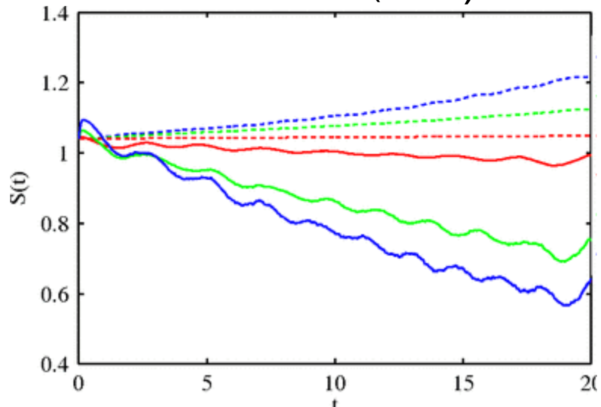
$$\hat{H} = \hbar\omega_C \hat{a}^\dagger \hat{a} + \hbar\Omega_m \hat{b}^\dagger \hat{b} - \hbar g_0(t)\hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \quad \longrightarrow \quad \hat{a} = \sqrt{\bar{n}_{cav}} + \delta\hat{a}$$

$$\hat{H} = \hbar\omega_C \hat{a}^\dagger \hat{a} + \hbar\Omega_m \hat{b}^\dagger \hat{b} - \hbar g(t)(\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

Rev. Mod. Phys. 86, 1391 (2014)

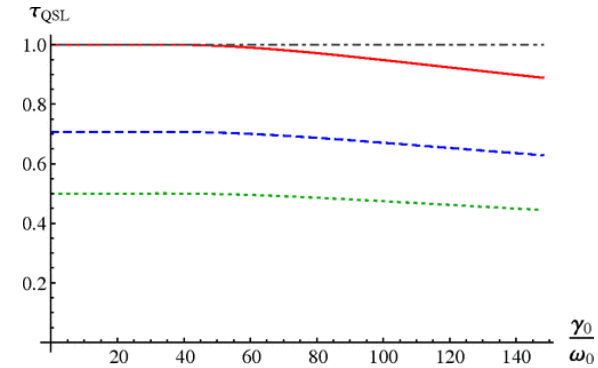
The exploration of non-Markovian dynamics has already leaded to the enhancement of

entropy transfer
Phys. Rev. Lett. 107, 130404 (2011)

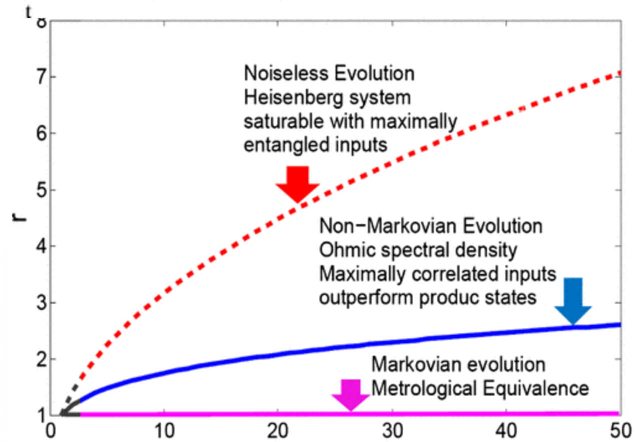


violation of second law of thermodynamics (2012)

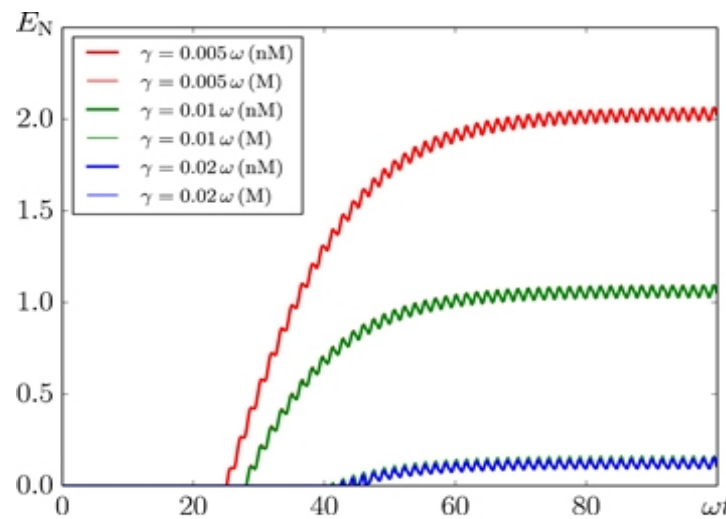
Experimental evidence of non-Markovian dynamics (2015)



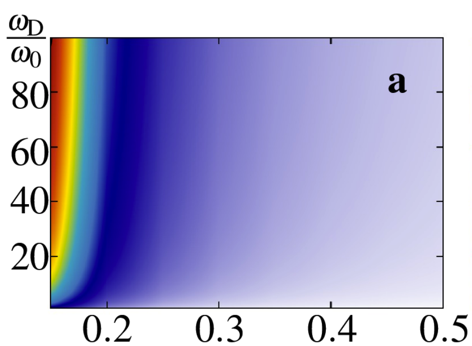
Quantum speed limits
Phys. Rev. Lett. 111, 010402 (2013)



improvements in quantum metrology
Phys. Rev. Lett. 109, 233601 (2012)



Entanglement survival
Phys. Rev. Lett. 105 180501 (2010)
Phys. Rev. Lett. 108, 160402 (2012)
New J. Phys. 17, 033038371 (2015)



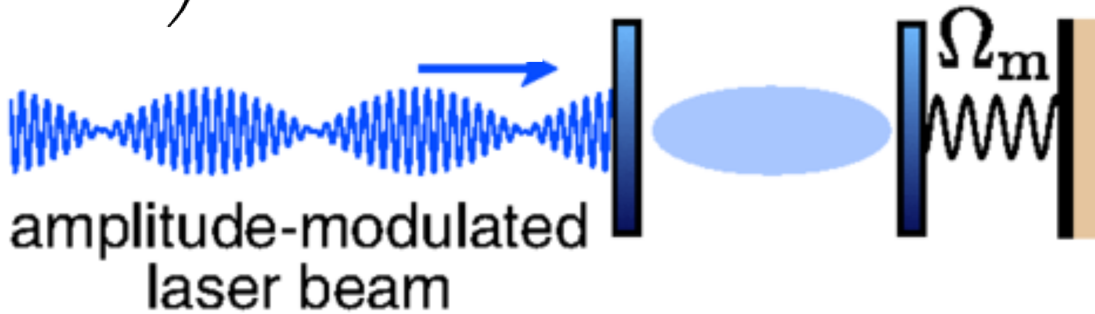
corrections to thermal equilibrium states
arXiv:1401.1418v2

Coherent control Faraday Discuss. 163, 485 (2013), J. Chem. Phys. 139, 164123 (2013)

Ultrafast Optimal Sideband Cooling under Non-Markovian Evolution

JF Triana, AF Estrada, LA Pachon Phys. Rev. Lett. **116**, 183602 (2016)

$$H_S = \sum_{\alpha=1}^{2} \left(\frac{p_\alpha^2}{2m_\alpha} + \frac{1}{2} m_\alpha \omega_\alpha^2 q_\alpha^2 \right) + c(t) q_1 q_2$$



Rev. Mod. Phys. 86, 1391 (2014)

Goal: Find the c(t) that maximizes entropy transfer under non-Markovian dynamics

Methodology: Use the steepest descent method to find the optimal c(t)

```
sq1q1[t_]:=(sp2p2i (U2p[0] U4[t]-U2[t] U4p[0])^2)/
(4 (2 U2[t] V2[t]-2 U4[t] V4[t])^2
(((-U2p[0] V2[t]+U4p[0] V4[t])
(-m U2[t
(2 U2[t]
V4[t])^2
(V2p[0]
V2[t]-2
U4[t]-U2
V4[t])^2
(-m U2[t
(2 U2[t]
-(m (U2p
V4p[0]))
+((U2p[0
```

```
sq1q1[t_]:=(sp2p2i (U2p[0] U4[t]-U2[t] U4p[0])^2)/
(4 (2 U2[t] V2[t]-2 U4[t] V4[t])^2
(((-U2p[0] V2[t]+U4p[0] V4[t])
(-m U2[t] V2p[0]+m U4[t] V4p[0]))/
(2 U2[t] V2[t]-2 U4[t]
V4[t])^2-(m (U2p[0] U4[t]-U2[t] U4p[0])
(V2p[0] V4[t]-V2[t] V4p[0]))/(2 U2[t]
V2[t]-2 U4[t] V4[t])^2)^2)+(vep2i^2 (U2p[0]
U4[t]-U2[t] U4p[0])^2)/(4 (2 U2[t] V2[t]-2 U4[t]
V4[t])^2 (((-U2p[0] V2[t]+U4p[0] V4[t])
(-m U2[t] V2p[0]+m U4[t] V4p[0]))/
(2 U2[t] V2[t]-2 U4[t] V4[t])^2
-(m (U2p[0] U4[t]-U2[t] U4p[0]) (V2p[0] V4[t]-V2[t]
V4p[0]))/(2 U2[t] V2[t]-2 U4[t] V4[t])^2)^2)
+((U2p[0] U4[t]-U2[t] U4p[0])^2 (u22[t]
```

```
sq1q1[t_]:=(sp2p2i (U2p[0] U4[t]-U2[t] U4p[0])^2)/
[t] V2[t]-2 U4[t] V4[t])^2
0] V2[t]+U4p[0] V4[t])
| V2p[0]+m U4[t] V4p[0]))/
 V2[t]-2 U4[t]
-(m (U2p[0] U4[t]-U2[t] U4p[0])
V4[t]-V2[t] V4p[0]))/(2 U2[t]
^2)/ V4[t])^2)^2)+(vep2i^2 (U2p[0]
4p[0])^2)/(4 (2 U2[t] V2[t]-2 U4[t]
U2p[0] V2[t]+U4p[0] V4[t])
[0]+m U4[t] V4p[0]))/
]-2 U4[t] V4[t])^2
4[t]-U2[t] U4p[0]) (V2p[0] V4[t]-V2[t]
2[t] V2[t]-2 U4[t] V4[t])^2)^2)
t]-U2[t] U4p[0])^2 (u22[t]
t]
```

```
sq1q1[
(4 (2
(((-U2p[0] V4[t])^2 (((-U2p[0] V2[t]+U4p[0] V4[t])
(-m U2[t] \ (-m U2[t] V2p[0]+m U4[t] V4p[0]))/
(2 U2[t] V2 (2 U2[t] V2[t]-2 U4[t] V4[t])^2
```

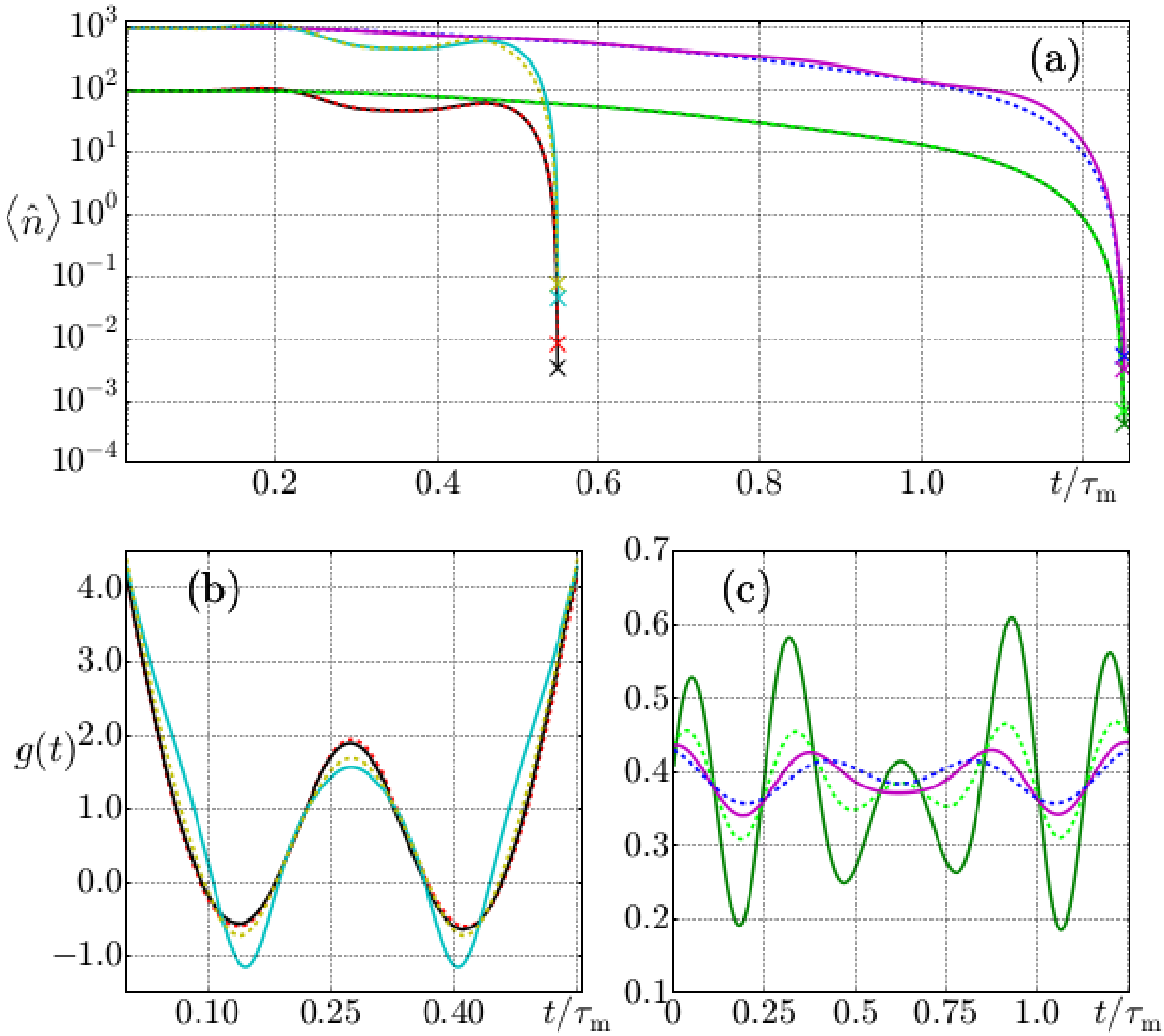
```
 v42[t]+u2[t] (u24[t] v1[t] (-u4[t] v1[t
])+u4[t] (-u42[t] v1[t]^2+u23[t] v1[t]
] v1[t] v2[t]+v1[t] v14[t] v2[t]-v1[t]^2
[t] U4p[0])^2)/ u43[t] v1[t] v4[t]+v1[t]
 v4[t]-2 v11[t] v2[t] v4
v2[t] v41[t]-v1[t]^2 v42[
-u23[t] v2[t]^2-u32[t]
] v2[t] v24[t]-2 u44[t]
[t]+u43[t] v2[t] v4[t]
v21[t] v4[t]-2 v1[t] v22[
 v2[t] v42[t]))+(u4[t] v1
 (u2[t] v2[t]-u4[t] v4[t
```

```
sq1q1[t_]:=(sp2p-(m (U2p[0] U4[t]-U2[t] U4p[0]) (V2p[0] V4[t]-V2[t]
(4 (2 U2[t] V2[t V4p[0]))/(2 U2[t] V2[t]-2 U4[t] V4[t])^2)^2)
(((-U2p[0] V2[t] +((U2p[0] U4[t]-U2[t] U4p[0])^2 (u22[t]
(-m U2[t] V2p[0]+m U4[t] V4p[0]))/
(2 U2[t] V2[t]-2 U4[t]
V4[t])^2-(m (U2p[0] U4[t]-U2[t] U4p[0])
(V2p[0] V4[t]-V2[t] V4p[0]))/(2 U2[t]
V2[t]-2 U4[t] V4[t])^2)^2)+(vep2i^2 (U2p[0]
U4[t]-U2[t] U4p[0])^2)/(4 (2 U2[t] V2[t]-2 U4[t]
V4[t])^2 (((-U2p[0] V2[t]+U4p[0] V4[t])
(-m U2[t] V2p[0]+m U4[t] V4p[0]))/
(2 U2[t] V2[t]-2 U4[t] V4[t])^2
-(m (U2p[0] U4[t]-U2[t] U4p[0]) (V2p[0] V4[t]-V2[t]
V4p[0]))/(2 U2[t] V2[t]-2 U4[t] V4[t])^2)^2)
+((U2p[0] U4[t]-U2[t] U4p[0])^2 (u22[t]
```

```
U4[t] V4p[0]))/
[t]
 U4[t]-U2[t] U4p[0])
 V4p[0]))/(2 U2[t]
)^2)^2)+(vep2i^2 (U2p[0]
^2)/(4 (2 U2[t] V2[t]-2 U4[t]
 V2[t]+U4p[0] V4[t])
U4[t] V4p[0]))/
[t] V4[t])^2
J2[t] U4p[0]) (V2p[0] V4[t]-V2[t]
V2[t]-2 U4[t] V4[t])^2)^2)
t] U4p[0])^2 (u22[t]
```

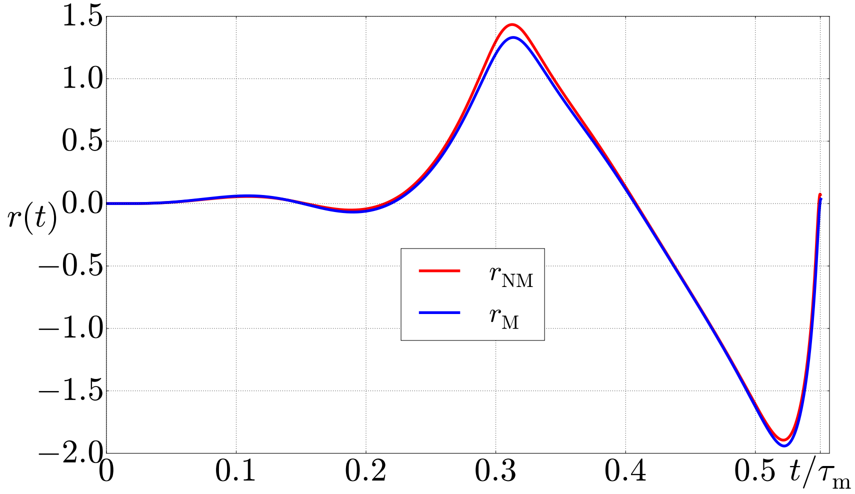
Technical details

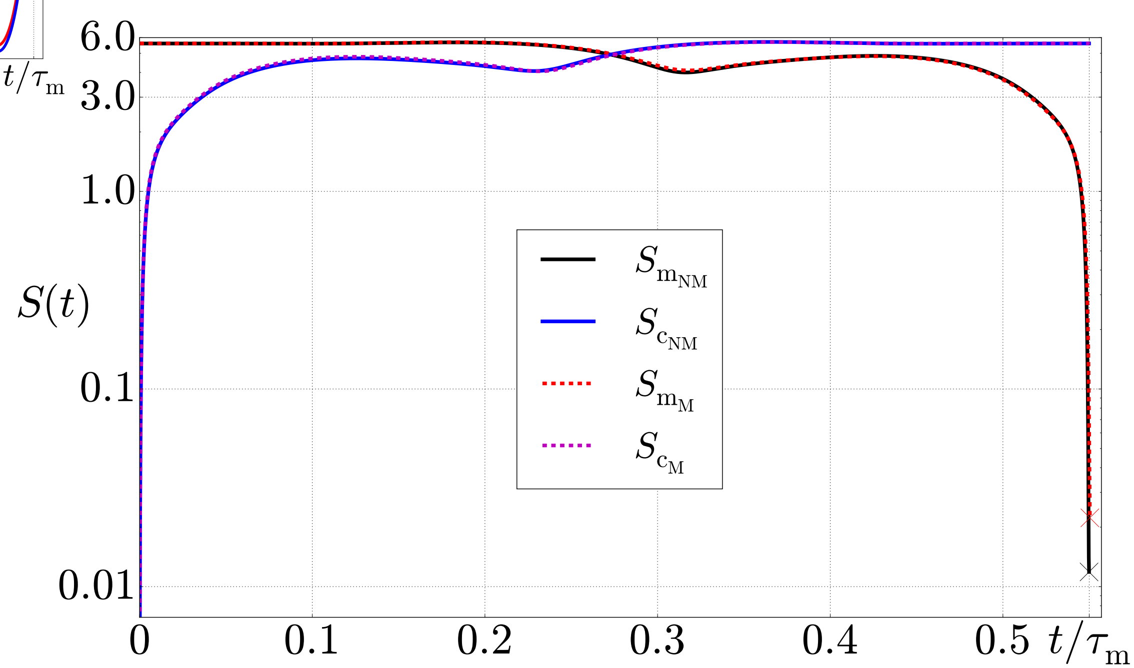
Ultrafast Optimal Sideband Cooling under Non-Markovian Evolution



Ultrafast Optimal Sideband Cooling under Non-Markovian Evolution

JF Triana, AF Estrada, LA Pachon Phys. Rev. Lett. **116**, 183602 (2016)





Ultrafast Optimal Sideband Cooling under Non-Markovian Evolution

JF Triana, AF Estrada, LA Pachon Phys. Rev. Lett. **116**, 183602 (2016)

	$\langle \hat{n}(t_{\rm cool}) \rangle$				
$\gamma/\omega_{\rm mm}$	opt.om$_{\rm M}$ + mm$_{\rm M}$	om$_{\rm nM}$ + mm$_{\rm nM}$	opt.om$_{\rm nM}$ + mm$_{\rm nM}$	om$_{\rm M}$ + mm$_{\rm nM}$	opt.om$_{\rm M}$ + mm$_{\rm nM}$
10^{-6}	9.03×10^{-3}	8.86×10^{-3}	3.43×10^{-3}	1.91×10^{-3}	9.99×10^{-5}
10^{-5}	1.04×10^{-2}	1.02×10^{-2}	4.79×10^{-3}	2.60×10^{-3}	7.91×10^{-4}
10^{-4}	3.28×10^{-2}	2.40×10^{-2}	1.99×10^{-2}	1.63×10^{-2}	1.52×10^{-2}
10^{-3}	2.61×10^{-1}	1.61×10^{-1}	1.53×10^{-1}	1.53×10^{-1}	1.51×10^{-1}
10^{-2}	2.45	1.52	1.50	1.50	1.50
10^{-1}	21.12	14.21	13.52	13.52	13.52

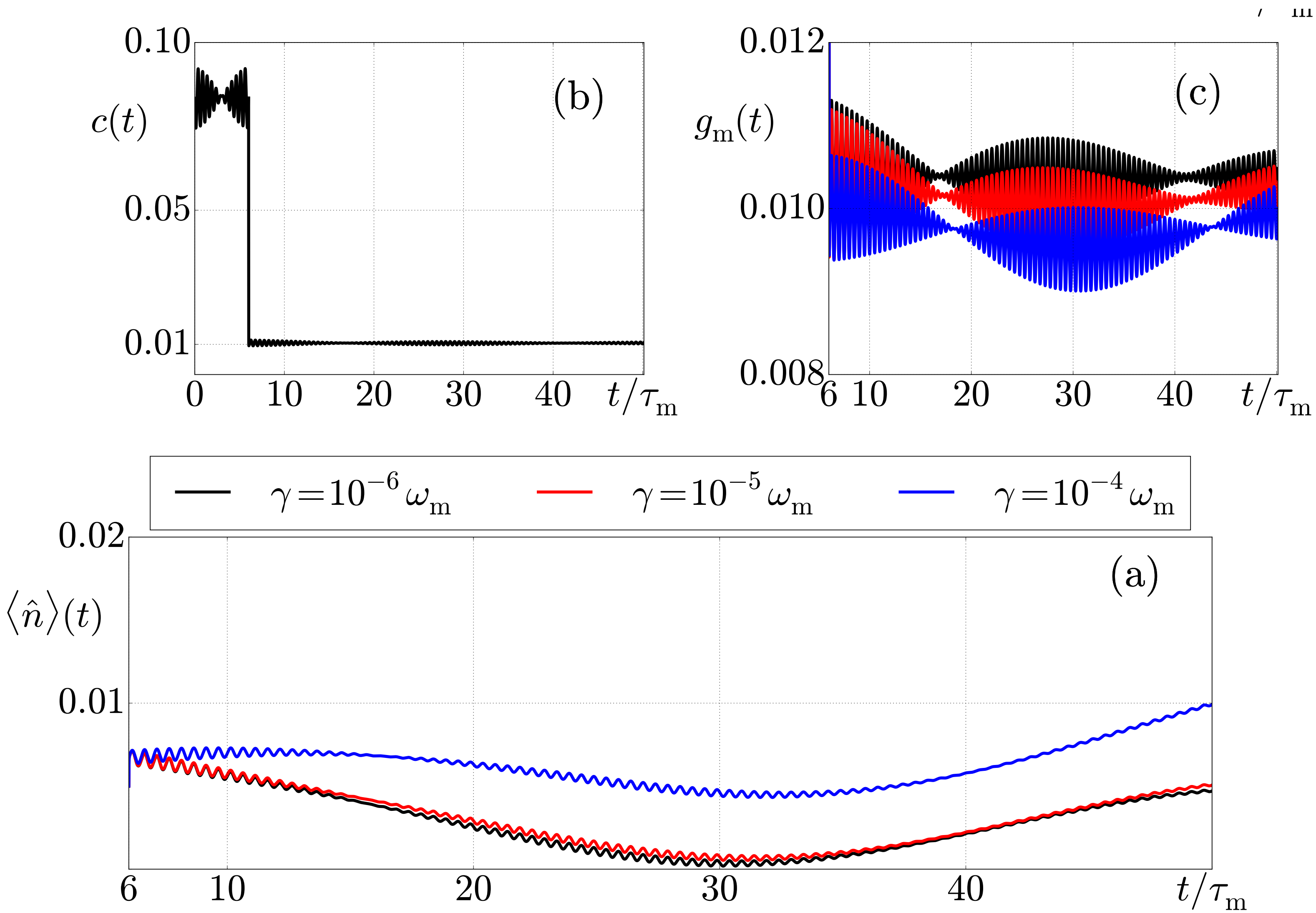
The cavity dissipation rate $\kappa = 10^{-4}\omega_{\rm mm}$

$t_{\rm cool}/\tau_{\rm mm}$	$\kappa/\omega_{\rm om}$	$\langle \hat{n}_{\rm M}(t_{\rm cool}) \rangle$	$\langle \hat{n}_{\rm nM}(t_{\rm cool}) \rangle$	$\langle \hat{n}_{{\rm om}_{\rm M}}(t_{\rm cool}) \rangle$
0.55	1×10^{-3}	0.016	0.015	0.014
0.6	1.5×10^{-2}	0.025	0.019	0.019
0.8	2.5×10^{-2}	0.029	0.030	0.021
0.8	4.5×10^{-2}	0.035	0.060	0.026
0.8	5.5×10^{-2}	0.037	0.086	0.032
1.0	1.25×10^{-1}	0.048	0.356	0.040
1.6	2.15×10^{-1}	0.056	2.34	0.044

The initial parameters are $n_T = 100$, $\gamma = 10^{-4}\omega_{\rm mm}$.

Jltrafast Optimal Sideband Cooling under Non-Markovian Evolution

JF Triana, AF Estrada, LA Pachon Phys. Rev. Lett. **116**, 183602 (2016)



Summary

Surprisingly, significant enhancements are found as the interplay between non-Markovian dynamics in the mechanical mode, Markovian dynamics in optical mode and optimally-designed coupling functions.

Our approach can be readily implemented in semiclassical formulations of quantum mechanics in phase space [J. Chem. Phys. 132 (21), 214102 (2010), Chem. Phys. 375 (2), 209-215 (2010), Phys. Rev. Lett. 102, 150401 (2009)]

Optimal control theory to simulate quantum correlations in quantum 2D spectroscopies.