Work and entropy production in Generalised Gibbs ensembles

Martí Perarnau Llobet

Max-Planck-Institute for Quantum Optics, Munich

Non-Markovianity and Strong Coupling Effects in Thermodynamics
Bad Honnef, April 2017

Collaboration Berlin&Barcelona

Arnau Riera
(ICFO)

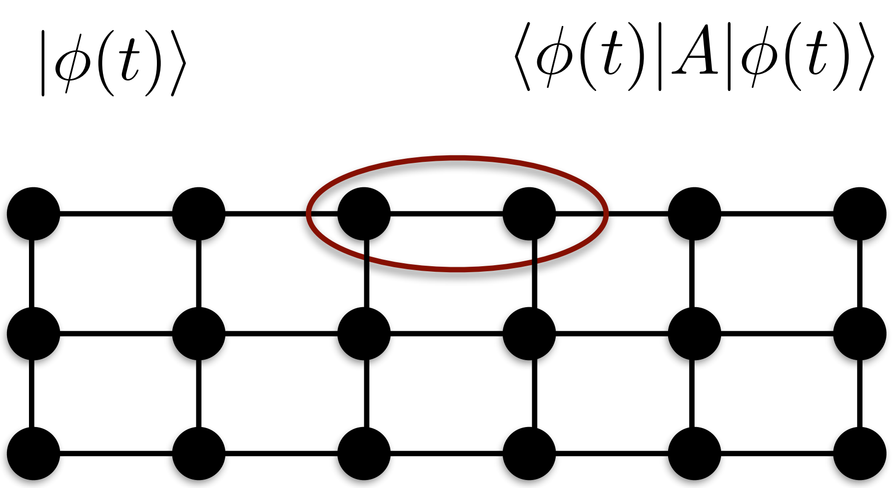
Henrik Wilming
(FU Berlin)

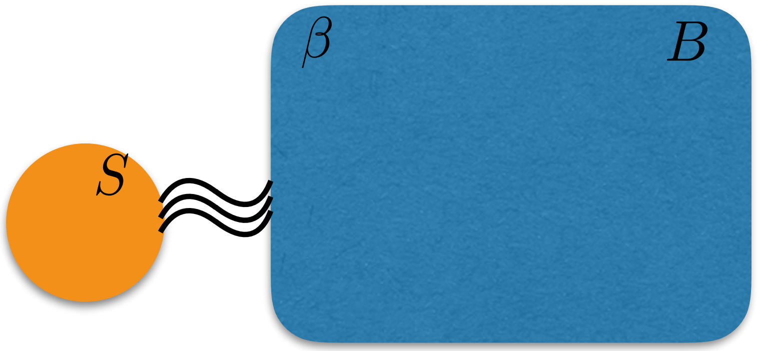
Rodrigo Gallego
(FU Berlin)

Jens Eisert
(FU Berlin)

Equilibration of closed quantum systems

Quantum Thermodynamics



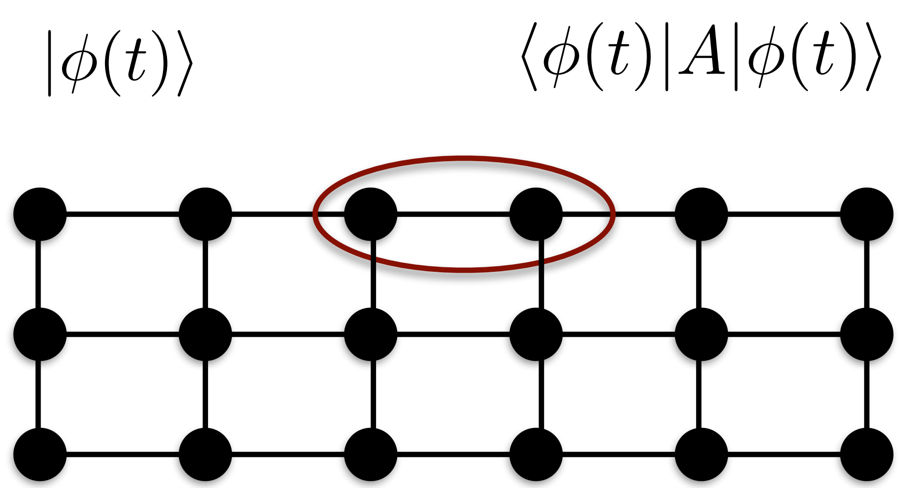


Equilibration of closed quantum systems

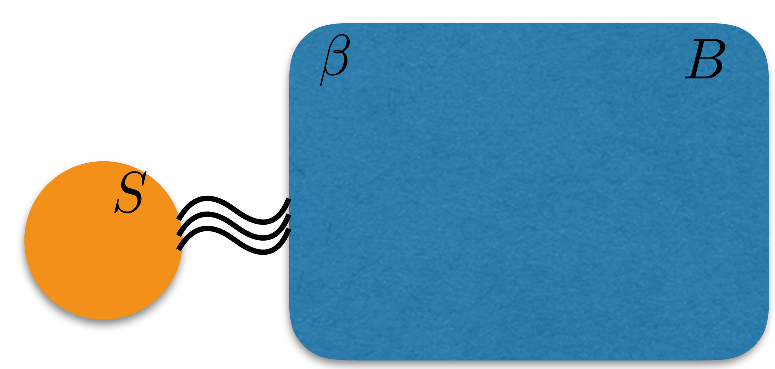
Pure states quantum statistical mechanics

Do finite quantum systems equilibrate?

If so, do they thermalise?



Quantum Thermodynamics

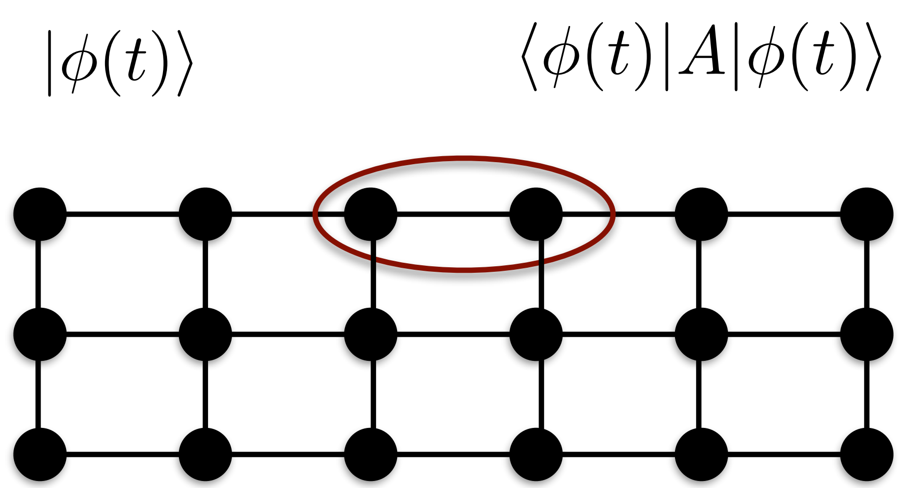


Equilibration of closed quantum systems

Pure states quantum statistical mechanics

Do finite quantum systems equilibrate?

If so, do they thermalise?

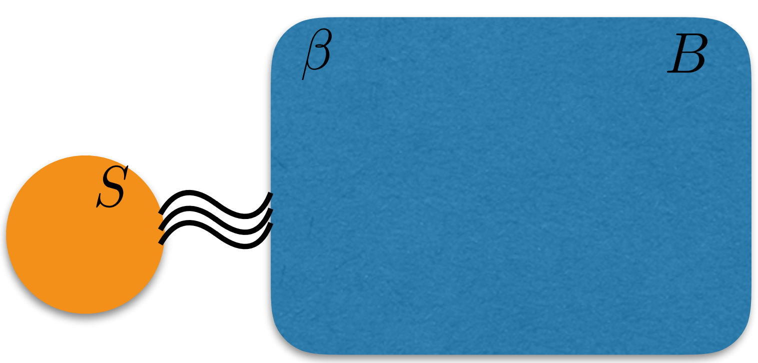


Quantum Thermodynamics

Second law: Heat engines, fluctuation theorems, resource-theoretic considerations…

Gibbs states and/or thermalisation are taken for granted.

However, when dealing with small quantum systems, this basic assumption is often not quite true (strong coupling, integrable systems…)



Equilibration of closed quantum systems

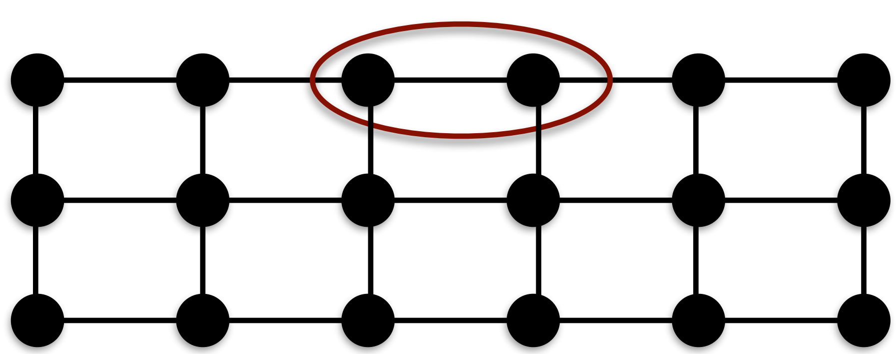
Pure states quantum statistical mechanics

Do finite quantum systems equilibrate?

If so, do they thermalise?

Review: Gogolin, Eisert, Rep. Prog. Phys. 79, 056001 (2016)

$|\phi(t)\rangle$ $\langle\phi(t)|A|\phi(t)\rangle$

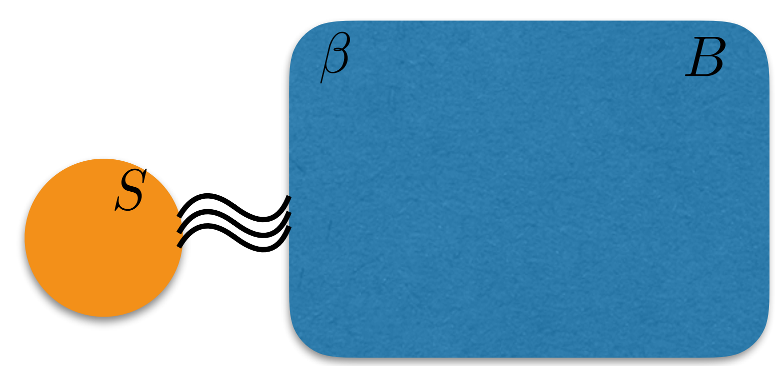


Quantum Thermodynamics

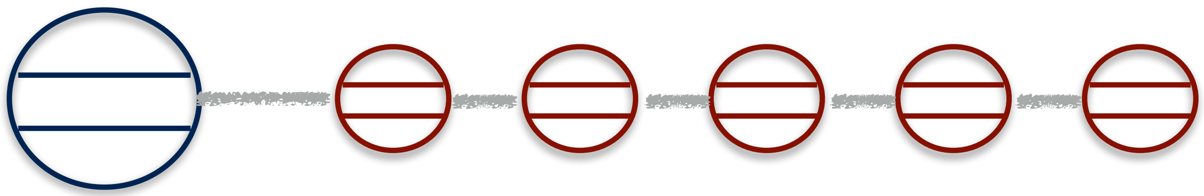
Second law: Heat engines, fluctuation theorems, resource-theoretic considerations…

Gibbs states and/or thermalisation are taken for granted.

However, when dealing with small quantum systems, this basic assumption is often not quite true (strong coupling, integrable systems…)



A chain of fermions: An example of many-body equilibration

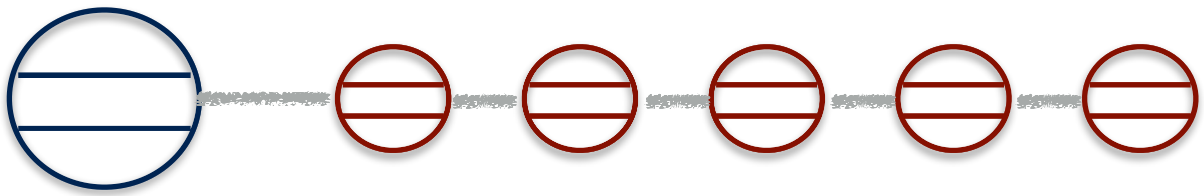


$$H_S = \epsilon_S a^\dagger a$$

$$H_B = \sum_i \epsilon_B b_k^\dagger b_k + g(b_k^\dagger b_{k+1} + b_{k+1}^\dagger b_k)$$

$$V = g'(b_1^\dagger a + a^\dagger b_1)$$

A chain of fermions: An example of many-body equilibration



$$H_S = \epsilon_S a^\dagger a$$

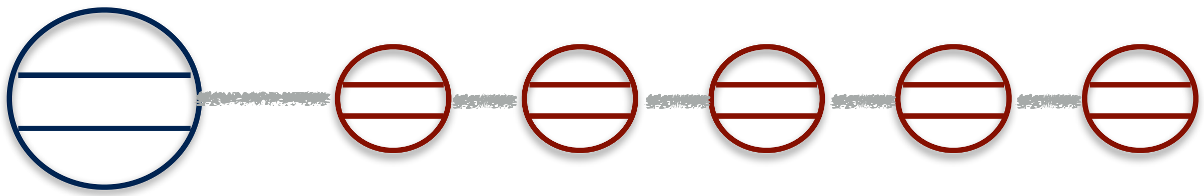
$$H_B = \sum_i \epsilon_B b_k^\dagger b_k + g(b_k^\dagger b_{k+1} + b_{k+1}^\dagger b_k)$$

$$V = g'(b_1^\dagger a + a^\dagger b_1)$$

$$\rho_0 = \rho_S \otimes \frac{e^{-\beta H_B}}{\mathcal{Z}_B}$$

$$\rho(t) = U \rho_0 U^\dagger, \qquad U = e^{-iHt}$$

A chain of fermions: An example of many-body equilibration



$$H_S = \epsilon_S a^\dagger a$$

$$H_B = \sum_i \epsilon_B b_k^\dagger b_k + g(b_k^\dagger b_{k+1} + b_{k+1}^\dagger b_k)$$

$$V = g'(b_1^\dagger a + a^\dagger b_1)$$

$$\rho_0 = \rho_S \otimes \frac{e^{-\beta H_B}}{\mathcal{Z}_B}$$

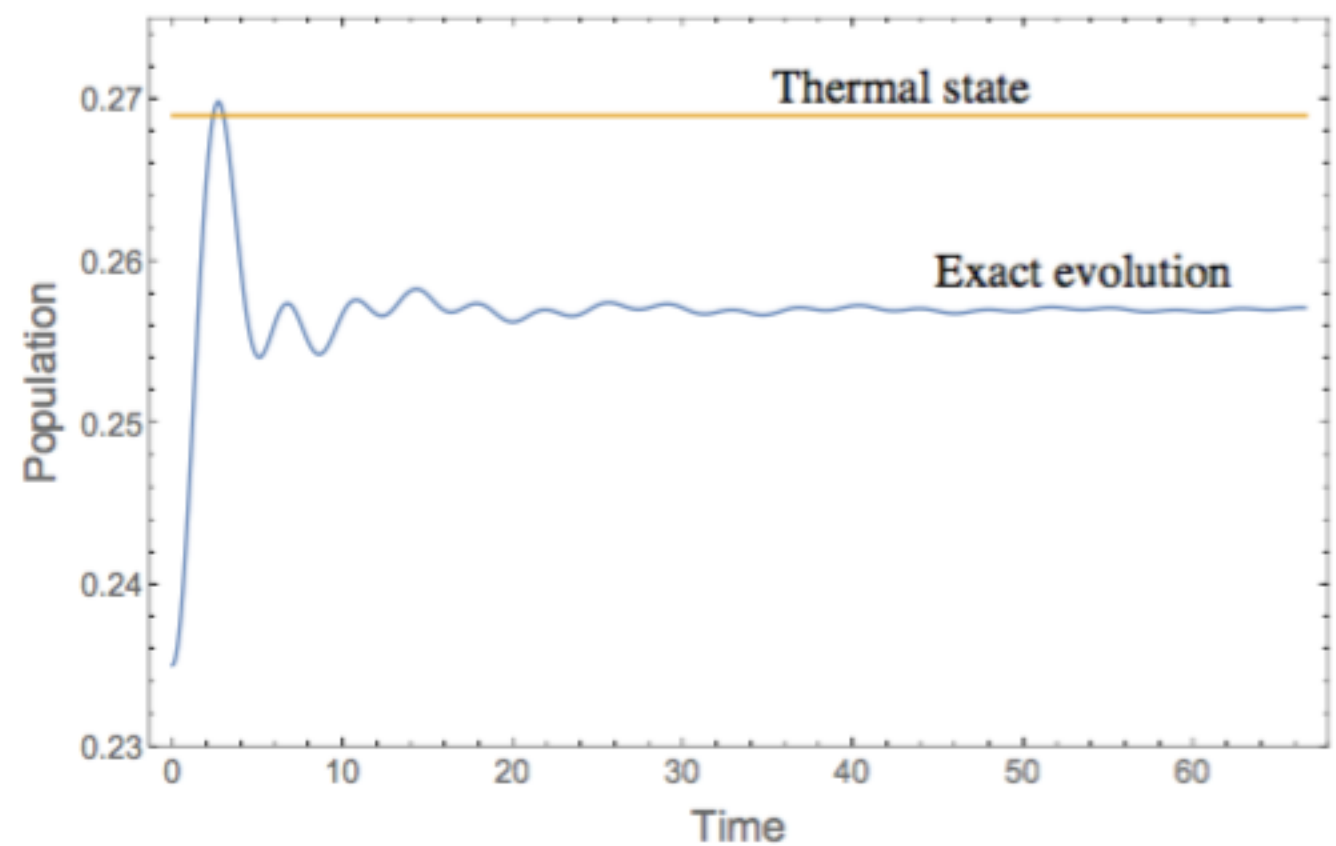
$$\rho(t) = U \rho_0 U^\dagger, \qquad U = e^{-iHt}$$

$$\text{population} = \mathrm{Tr}(a^\dagger a \rho(t))$$

$$\rho_S(t) = \mathrm{Tr}_B \rho(t) \approx \omega_{\mathrm{eq}}$$

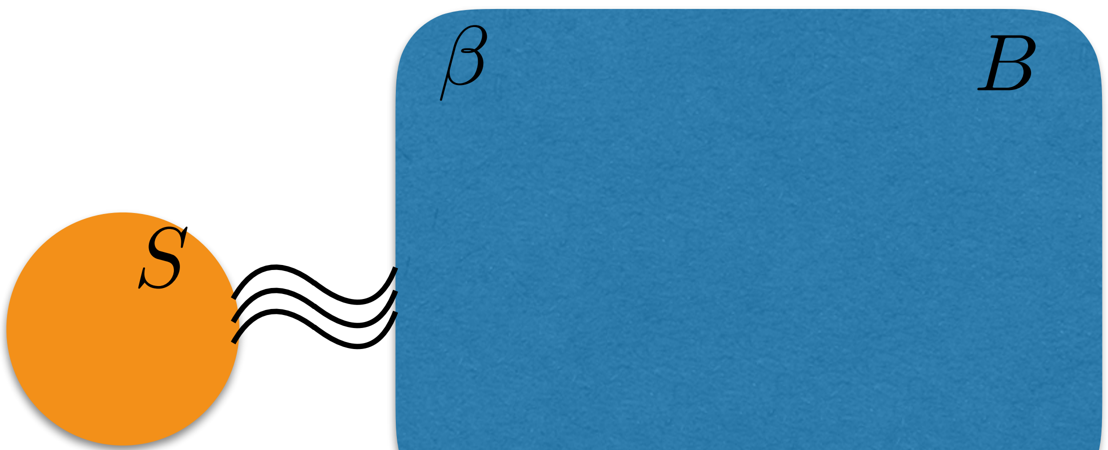
$$\omega_{\mathrm{eq}} \neq \frac{e^{-\beta H_S}}{\mathcal{Z}_S}$$

Bath of 100 fermions



Crash course on equilibration

Review: Gogolin, Eisert, Rep. Prog. Phys. 79, 056001 (2016)



$\dim(H_S) \ll \dim(H_B)$

$$H = H_S + V + H_B$$

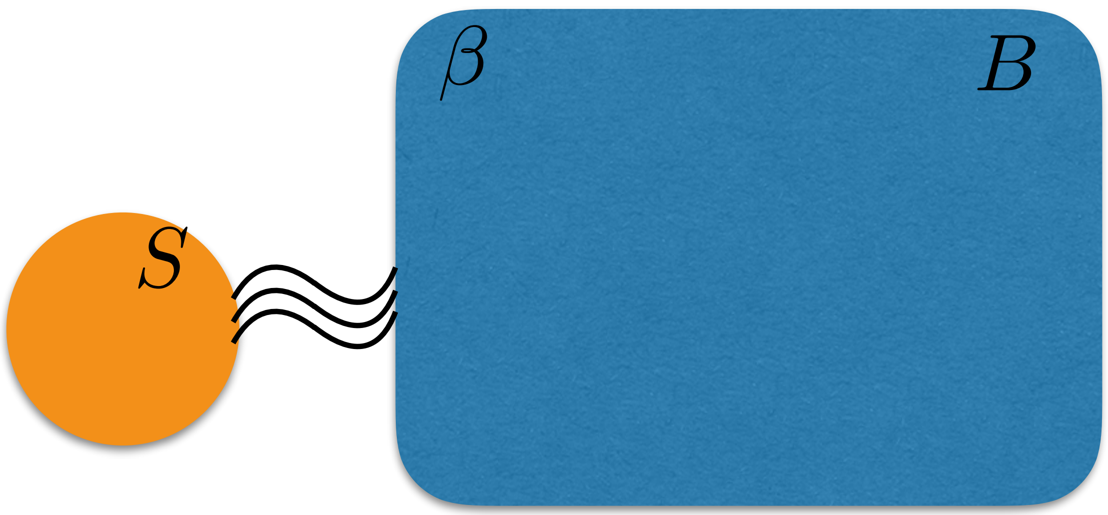
$$U = e^{-iHt} \qquad \rho_S(t) = \mathrm{Tr}_B\left(U\rho_{SB}U^\dagger\right)$$

for most times — time-independent

$$\rho_S(t) \approx \mathrm{Tr}_B(\omega_{SB})$$

Crash course on equilibration

Review: Gogolin, Eisert, Rep. Prog. Phys. 79, 056001 (2016)



$$\dim(H_S) \ll \dim(H_B)$$

$$H = H_S + V + H_B$$

$$U = e^{-iHt} \qquad \rho_S(t) = \mathrm{Tr}_B\left(U \rho_{SB} U^\dagger\right)$$

for most times time-independent

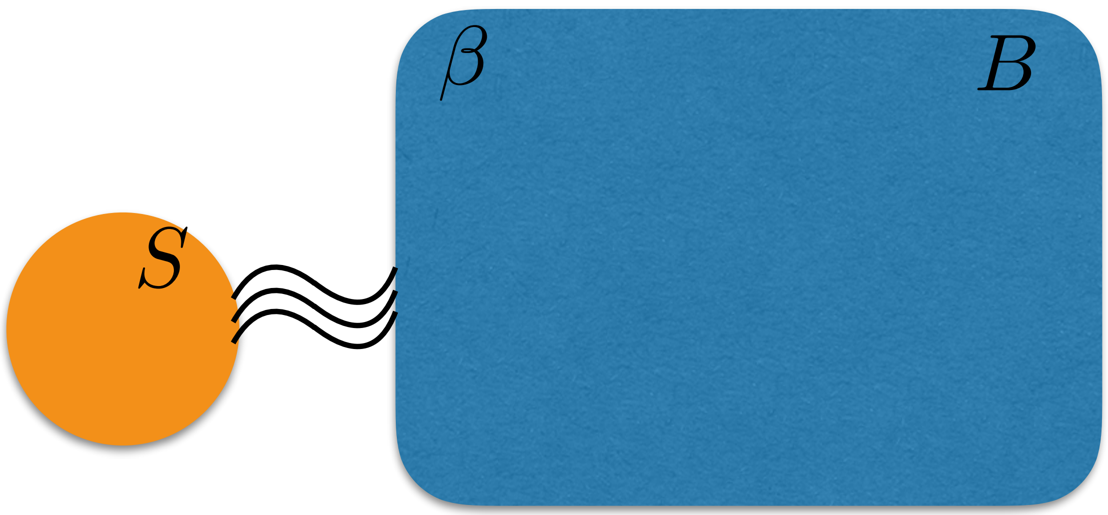
$$\rho_S(t) \approx \mathrm{Tr}_B(\omega_{SB})$$

General result: If a system equilibrates, then, it equilibrates to the state that maximises the entropy given all consents of motion (diagonal ensemble).

$$\omega_{SB} \propto e^{-\sum_i \mu_i \Pi_i} \qquad H = \sum_i e_i \Pi_i \qquad \mathrm{Tr}(\rho_{SB}\Pi_i) = \mathrm{Tr}(\omega_{SB}\Pi_i)$$

Crash course on equilibration

Review: Gogolin, Eisert, Rep. Prog. Phys. 79, 056001 (2016)



$\dim(H_S) \ll \dim(H_B)$

$$H = H_S + V + H_B$$

$$U = e^{-iHt} \qquad \rho_S(t) = \mathrm{Tr}_B\left(U \rho_{SB} U^\dagger\right)$$

for most times — time-independent

$$\rho_S(t) \approx \mathrm{Tr}_B(\omega_{SB})$$

General result: If a system equilibrates, then, it equilibrates to the state that maximises the entropy given all consents of motion (diagonal ensemble).

$$\omega_{SB} \propto e^{-\sum_i \mu_i \Pi_i} \qquad H = \sum_i e_i \Pi_i \qquad \mathrm{Tr}(\rho_{SB}\Pi_i) = \mathrm{Tr}(\omega_{SB}\Pi_i)$$

At the level of S, the diagonal ensemble becomes indistinguishable of the…

Gibbs ensemble

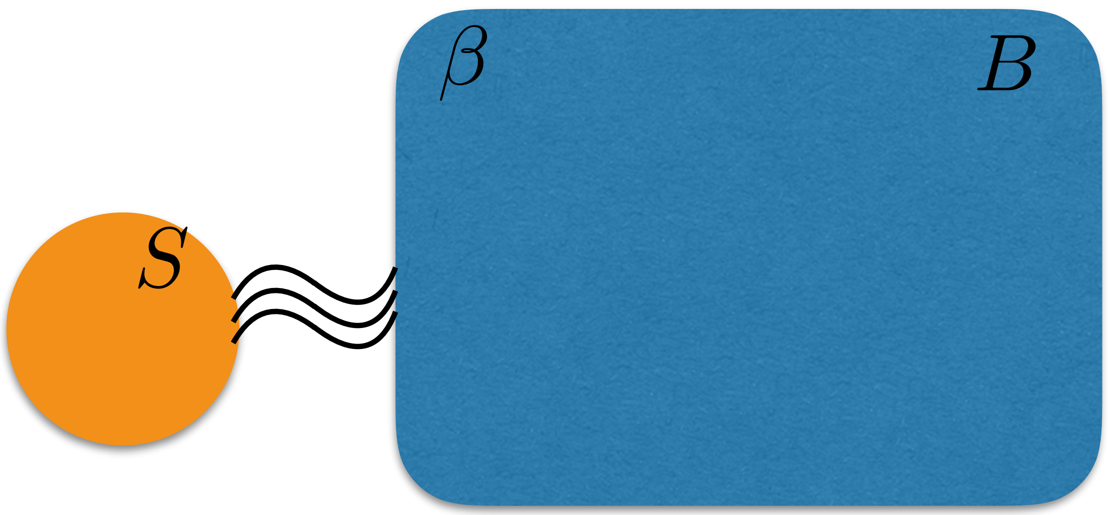
$$\omega_{SB} \propto e^{-\beta H}$$

For generic interacting many-body systems.

Maximum-entropy state for only one conserved quantity (the energy).

Crash course on equilibration

Review: Gogolin, Eisert, Rep. Prog. Phys. 79, 056001 (2016)



$$\dim(H_S) \ll \dim(H_B)$$

$$H = H_S + V + H_B$$

$$U = e^{-iHt} \qquad \rho_S(t) = \mathrm{Tr}_B \left(U \rho_{SB} U^\dagger \right)$$

for most times — time-independent

$$\rho_S(t) \approx \mathrm{Tr}_B(\omega_{SB})$$

General result: If a system equilibrates, then, it equilibrates to the state that maximises the entropy given all consents of motion (diagonal ensemble).

$$\omega_{SB} \propto e^{-\sum_i \mu_i \Pi_i} \qquad H = \sum_i e_i \Pi_i \qquad \mathrm{Tr}(\rho_{SB}\Pi_i) = \mathrm{Tr}(\omega_{SB}\Pi_i)$$

At the level of S, the diagonal ensemble becomes indistinguishable of the…

Gibbs ensemble

$$\omega_{SB} \propto e^{-\beta H}$$

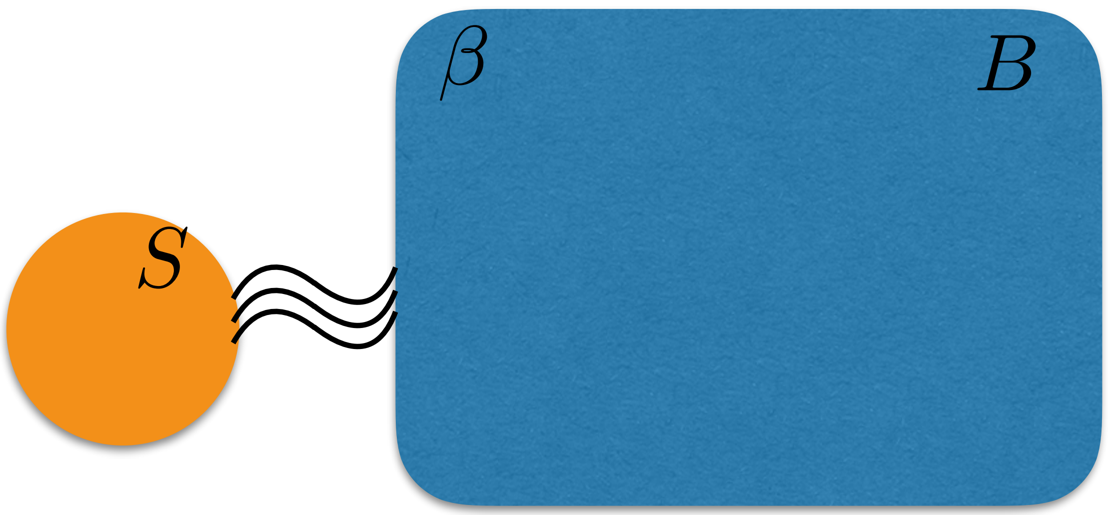
Generalised Gibbs ensemble

$$\omega_{SB} \propto e^{-\sum_i \mu_i Q_i}$$

Further constants of motion become relevant.

Crash course on equilibration

Review: Gogolin, Eisert, Rep. Prog. Phys. 79, 056001 (2016)



$$\dim(H_S) \ll \dim(H_B)$$

$$H = H_S + V + H_B$$

$$U = e^{-iHt} \qquad \rho_S(t) = \mathrm{Tr}_B\left(U\rho_{SB}U^\dagger\right)$$

for most times — time-independent

$$\rho_S(t) \approx \mathrm{Tr}_B(\omega_{SB})$$

General result: If a system equilibrates, then, it equilibrates to the state that maximises the entropy given all consents of motion (diagonal ensemble).

$$\omega_{SB} \propto e^{-\sum_i \mu_i \Pi_i} \qquad H = \sum_i e_i \Pi_i \qquad \mathrm{Tr}(\rho_{SB}\Pi_i) = \mathrm{Tr}(\omega_{SB}\Pi_i)$$

At the level of S, the diagonal ensemble becomes indistinguishable of the…

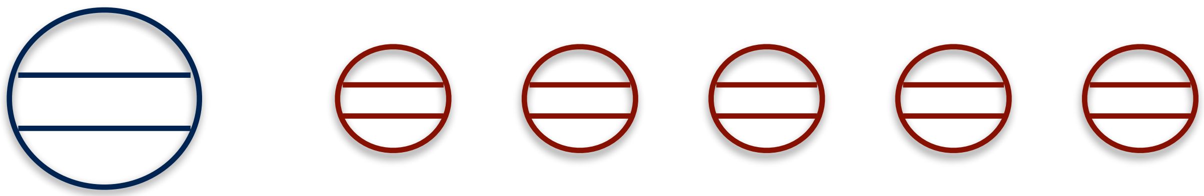
Gibbs ensemble

$$\omega_{SB} \propto e^{-\beta H}$$

Generalised Gibbs ensemble

$$\omega_{SB} \propto e^{-\sum_i \mu_i Q_i}$$

Coming back to the chain of fermions…

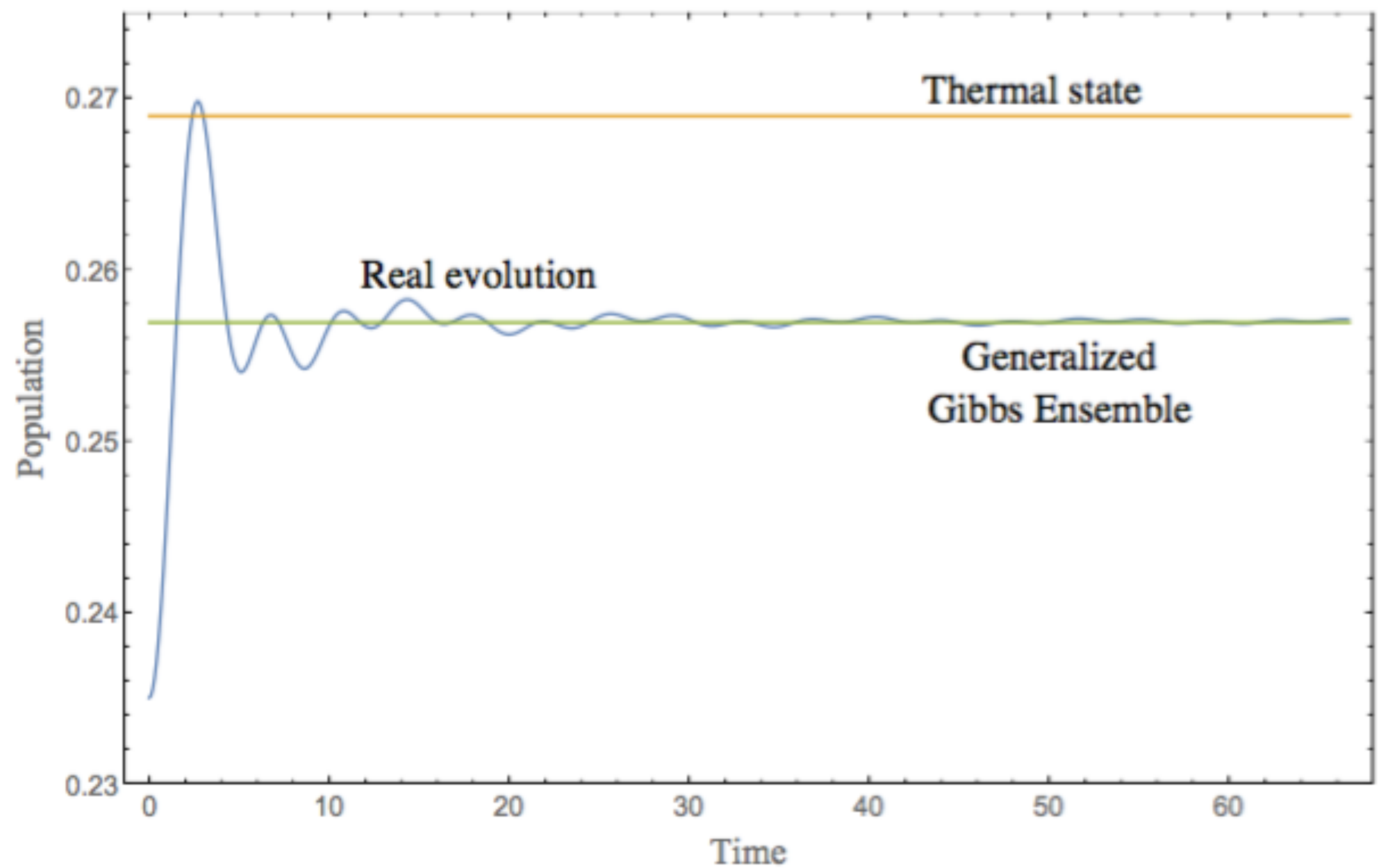


Integrable system

$$H = \epsilon_S a^\dagger a + \sum_i \epsilon_B b_k^\dagger b_k + g(b_k^\dagger b_{k+1} + b_{k+1}^\dagger b_k) = \sum_i E_i c_i^\dagger c_i = \sum_i h_i$$

$$\omega_{SB} = \frac{e^{-\sum_i \mu_i h_i}}{\mathcal{Z}}$$

$$\mathrm{Tr}(h_i \omega_{SB}) = \mathrm{Tr}(h_i \rho_0)$$



Our goal is to incorporate these insights from equilibration theory into the study of thermodynamics (work extraction, heat engines,…)

Our goal is to incorporate these insights from equilibration theory into the study of thermodynamics (work extraction, heat engines,…)

1. **Thermodynamics for systems that thermalise in the strong coupling regime.**

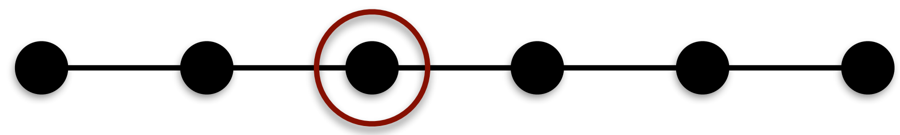
   Thermal machines beyond the weak coupling regime, R. Gallego, A. Riera, J. Eisert, New J. Phys. 16, 125009 (2014).

   Fundamental corrections to work and power in the strong coupling regime, M. P-L., H. Wilming, A. Riera, R. Gallego, J. Eisert —> soon on arxiv.

2. **Thermodynamics for Generalised Gibbs ensembles**

   Work and entropy production in generalised Gibbs ensembles, M. P.-L., A. Riera, R. Gallego, H. Wilming, J. Eisert, New J. Phys. 18, 123035 (2016).

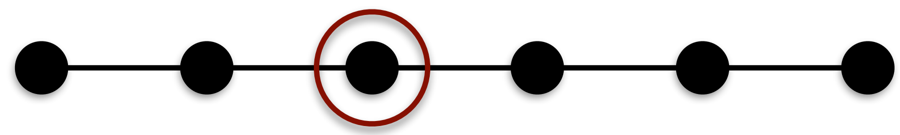
A framework to describe thermodynamic protocols



System: Part that can be controlled

Bath: Rest of the many-body system

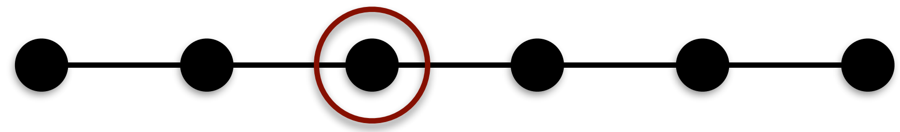
A framework to describe thermodynamic protocols



System: Part that can be controlled

Bath: Rest of the many-body system

A framework to describe thermodynamic protocols



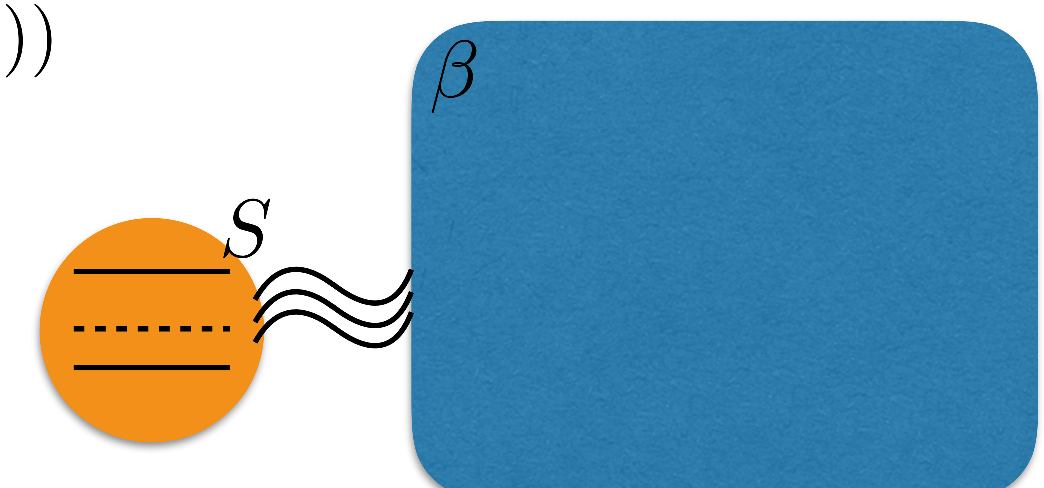
System: Part that can be controlled

Bath: Rest of the many-body system

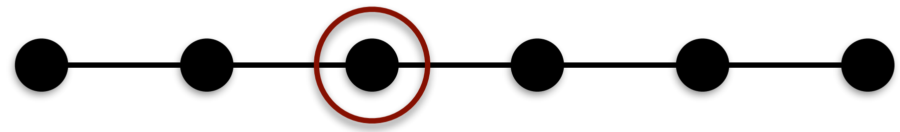
1. Quenches on the system:

$$H_S \to H_S'$$
$$\rho \to \rho$$

$$\langle W \rangle = \mathrm{Tr}(\rho(H_S' - H_S))$$



A framework to describe thermodynamic protocols



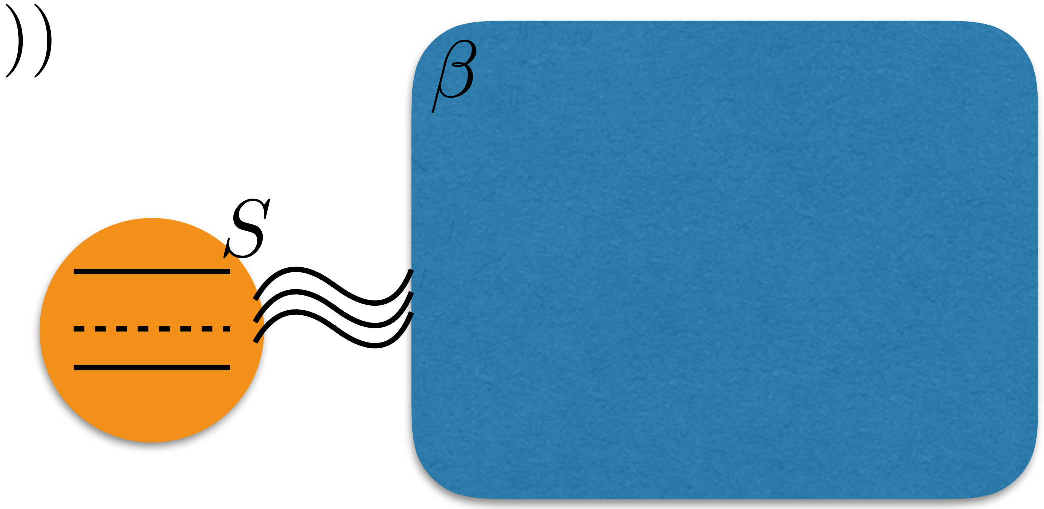
System: Part that can be controlled

Bath: Rest of the many-body system

1. Quenches on the system:

$$H_S \to H'_S \qquad \rho \to \rho$$

$$\langle W \rangle = \mathrm{Tr}(\rho(H'_S - H_S))$$

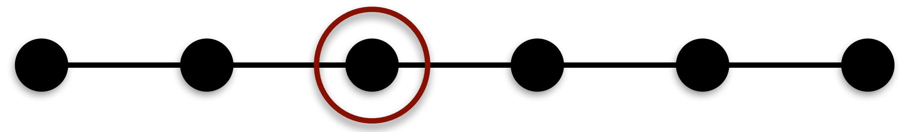


2. Equilibration processes

$$H \to H \qquad \rho_S \to \mathrm{Tr}_B(\omega_{\mathrm{GGE}})$$

$$\langle W \rangle = 0$$

A framework to describe thermodynamic protocols



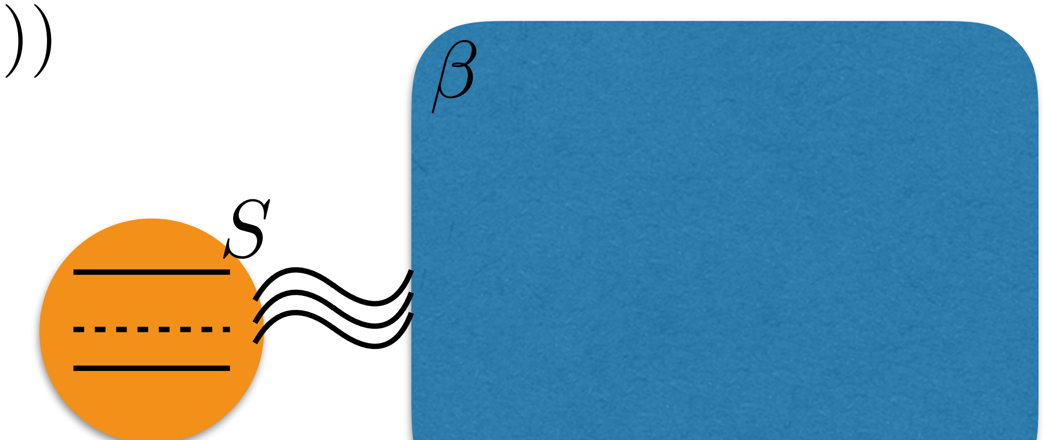
System: Part that can be controlled

Bath: Rest of the many-body system

1. Quenches on the system:

$$H_S \to H_S'$$
$$\rho \to \rho$$

$$\langle W \rangle = \mathrm{Tr}(\rho(H_S' - H_S))$$



2. Equilibration processes

$$H \to H$$
$$\rho_S \to \mathrm{Tr}_B(\omega_{\mathrm{GGE}})$$

$$\langle W \rangle = 0$$

Weak coupling limit: Anders, Govannetti NJP 2013

Strong coupling: Gallego, Wilming, Eisert NJP 2014

A thermodynamic protocol

Quenches $$H^{(0)} \to H^{(1)} \to ... \to H^{(N-1)} \to H^{(N)}$$

Real evolution $$\rho^{(0)} \to \rho^{(1)} \to .... \to \rho^{(N-1)} \to \rho^{(N)}$$

$$\rho^{(i)} = \left(\bigotimes_i U_i \right) \rho \left(\bigotimes_i U_i^\dagger \right)$$

$$U_k = e^{-itH^{(k)}}$$

$$t \gg t_{\mathrm{eq.}}$$

A thermodynamic protocol

Quenches $H^{(0)} \to H^{(1)} \to ... \to H^{(N-1)} \to H^{(N)}$

Real evolution $\rho^{(0)} \to \rho^{(1)} \to .... \to \rho^{(N-1)} \to \rho^{(N)}$

Effective description $\rho^{(0)} \to \omega^{(1)} \to \omega^{(2)} \to ... \to \omega^{(n)}$ Diagonal, GGE or Gibbs states.

A thermodynamic protocol

Quenches $$H^{(0)} \to H^{(1)} \to ... \to H^{(N-1)} \to H^{(N)}$$

Real evolution $$\rho^{(0)} \to \rho^{(1)} \to .... \to \rho^{(N-1)} \to \rho^{(N)}$$

Effective description $$\rho^{(0)} \to \omega^{(1)} \to \omega^{(2)} \to ... \to \omega^{(n)}$$ Diagonal, GGE or Gibbs states.

Exact equilibrium state,

$$\omega^{(i+1)} = \omega_{GGE}\left(\rho^{(i)}, H, \{Q_i\}\right)$$

We take instead the efficient approximate description,

$$\omega^{(i+1)} = \omega_{\mathrm{GGE}}\left(\omega^{(i)}, H, \{Q_i\}\right)$$

A thermodynamic protocol

Quenches $H^{(0)} \to H^{(1)} \to ... \to H^{(N-1)} \to H^{(N)}$

Real evolution $\rho^{(0)} \to \rho^{(1)} \to .... \to \rho^{(N-1)} \to \rho^{(N)}$

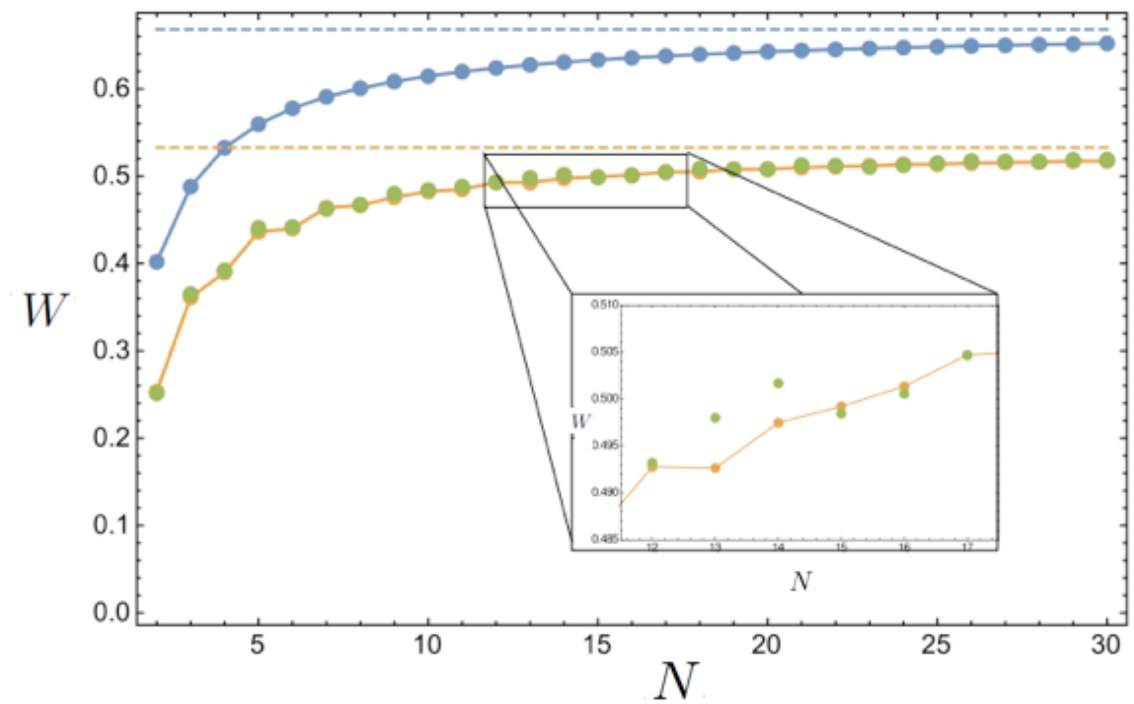
Effective description $\rho^{(0)} \to \omega^{(1)} \to \omega^{(2)} \to ... \to \omega^{(n)}$ Diagonal, GGE or Gibbs states.

When does the effective description works?

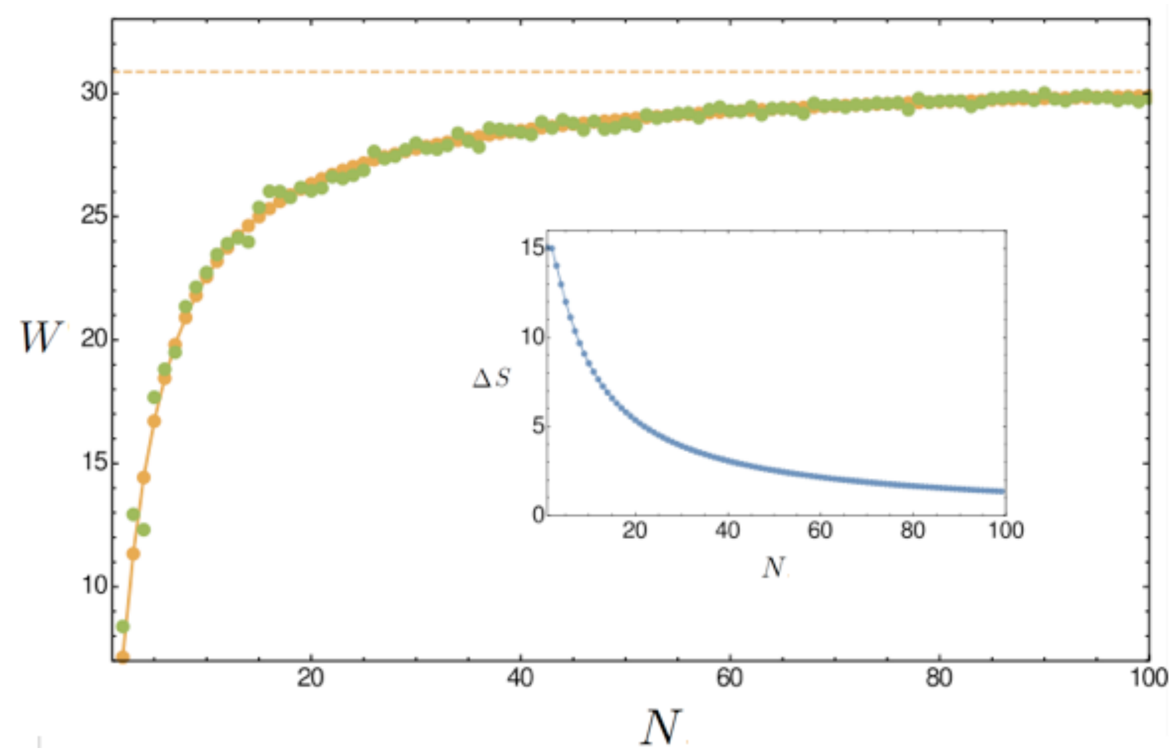
$$\mathrm{Tr}(Q_i^{(j+1)}\rho^{(j)}) \approx \mathrm{Tr}(Q_i^{(j+1)}\omega_{\mathrm{GGE}}^{(j)})$$

A comparison between the effective description and the exact unitary dynamics: Thermodynamic protocols on the fermonic chain

Local quenches
+
Initial Gibbs state



Global quenches
+
Initial GGE state



Result: A framework to describe in an effective way concatenation of equilibration processes.

* Good agreement with exact dynamics for quadratic fermionic systems.

* The framework is flexible enough to incorporate other many-body systems.

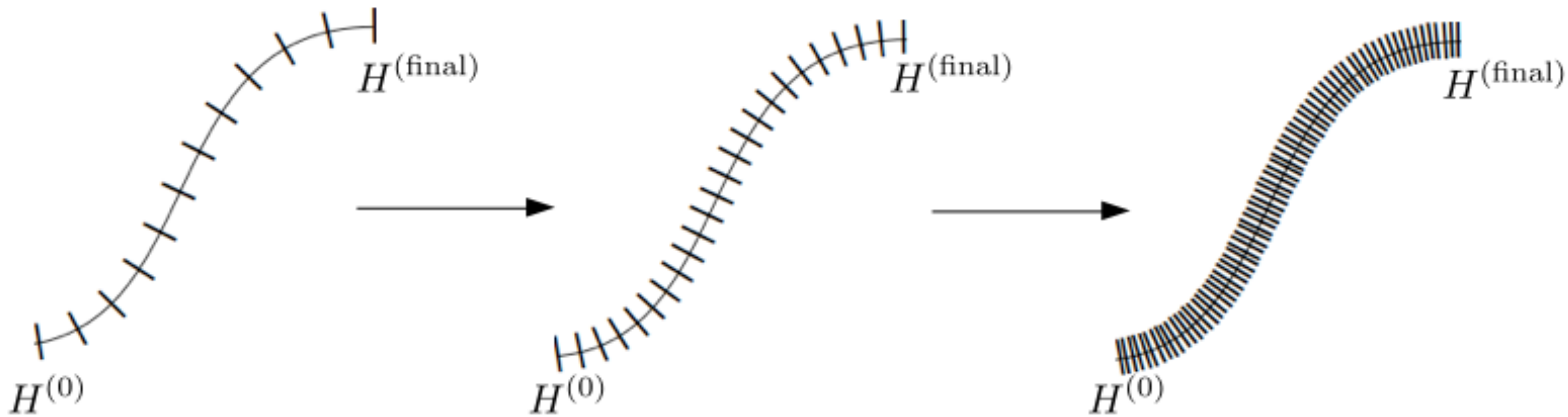
Entropy production as lost of information

In the real description: $$S(\rho^{(j+1)}) = S(\rho^{j})$$

In the effective description: $$S(\omega^{(j+1)}) \geq S(\omega^{j})$$

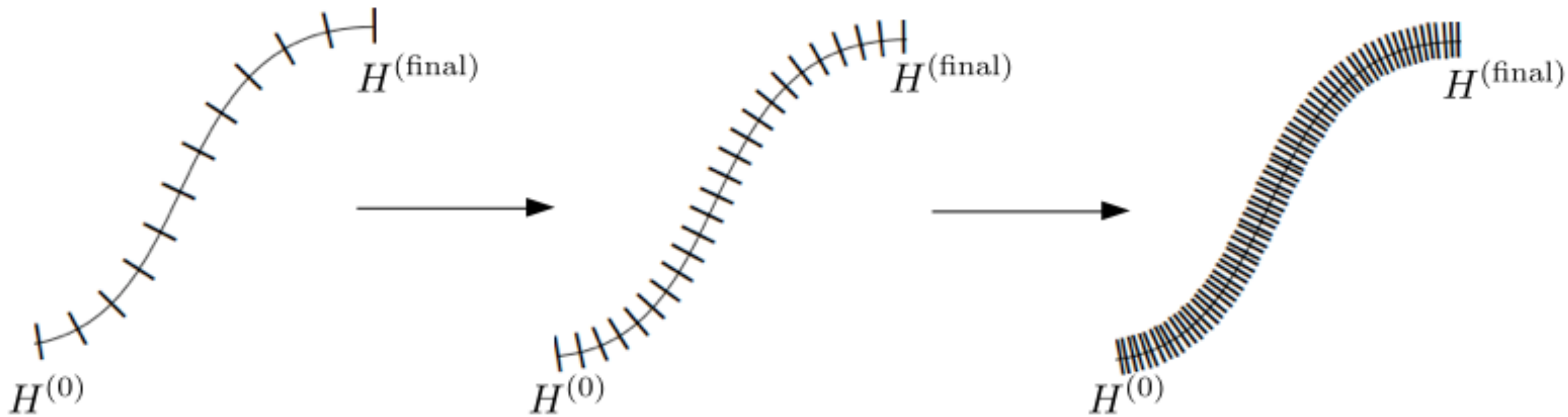
$$S(\rho) = -\mathrm{Tr}(\rho \ln \rho)$$

Quasi-static processes



*smoothness of the path and the Lagrange multipliers being finite.

Quasi-static processes

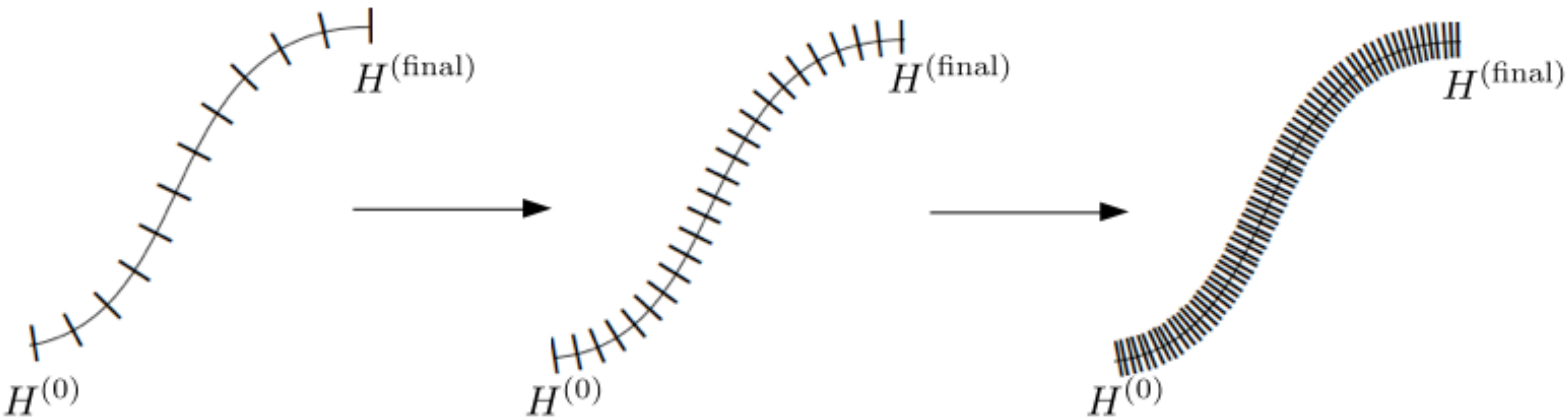


Result: Under mild conditions,* in a quasi-static process there is no entropy production in the effective description,

$$S(\omega^{(N)}) = S(\omega^1) + \mathcal{O}(1/N)$$

*smoothness of the path and the Lagrange multipliers being finite.

Quasi-static processes



Result: Under mild conditions,* in a quasi-static process there is no entropy production in the effective description,

$$S(\omega^{(N)}) = S(\omega^1) + \mathcal{O}(1/N)$$

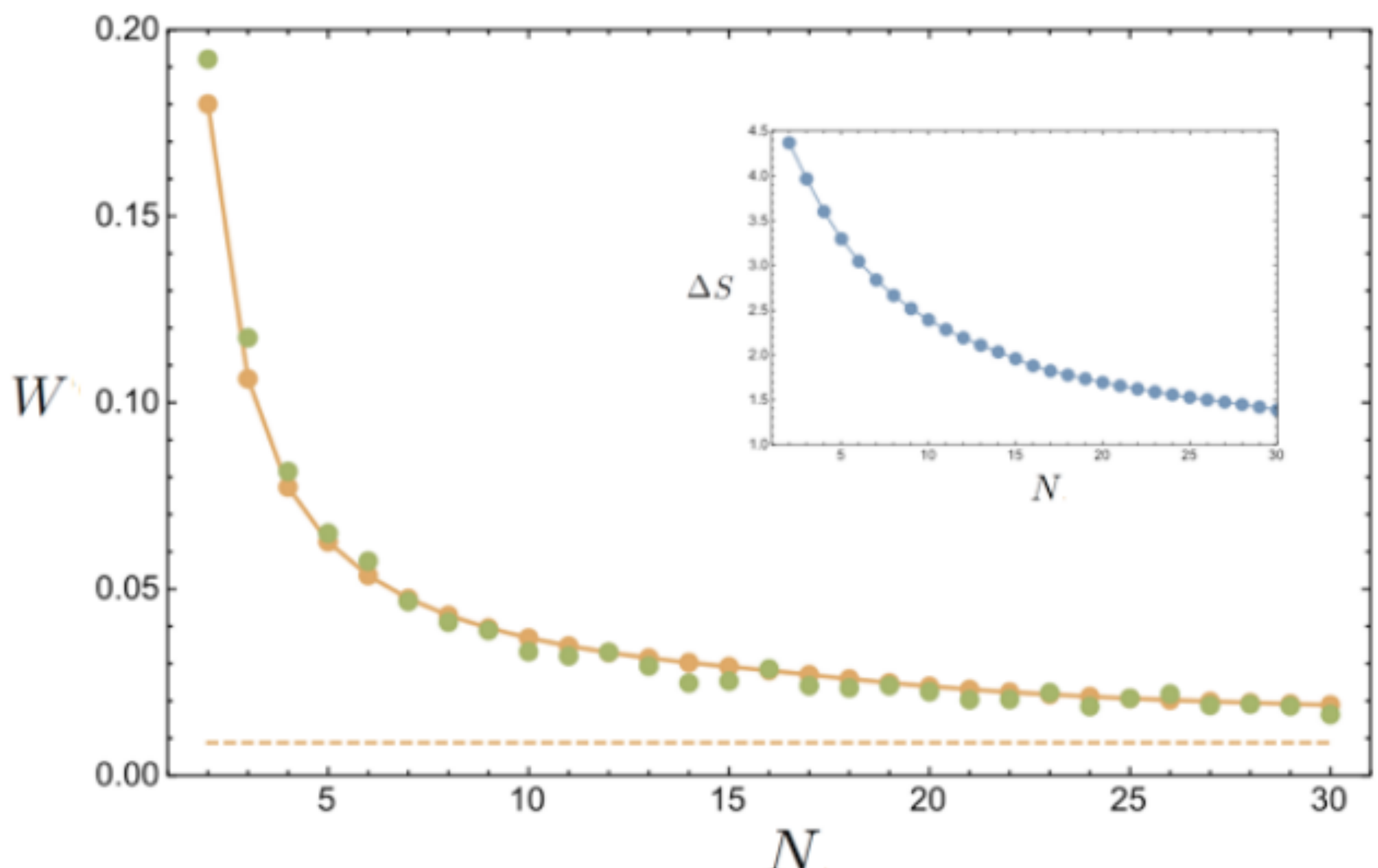
Slow processes ⟶ No info. is lost ⟶ Reversible

*smoothness of the path and the Lagrange multipliers being finite.

Result: Validity of the minimal work principle for the different models of equilibration.

Gibbs states: It holds in general.

GGE states: It can break down (case study: free fermions).



Summary

1. We built an effective description of thermodynamic protocols based on equilibrium states (Time-averaged, Gibbs, GGE).

Summary

1. We built an effective description of thermodynamic protocols based on equilibrium states (Time-averaged, Gibbs, GGE).

2. Good agreement between the effective description and the unitary dynamics for a case study.

Summary

1. We built an effective description of thermodynamic protocols based on equilibrium states (Time-averaged, Gibbs, GGE).

2. Good agreement between the effective description and the unitary dynamics for a case study.

3. In general, the entropy of the effective description always increases, and becomes constant in slow reversible processes.

Summary

1. We built an effective description of thermodynamic protocols based on equilibrium states (Time-averaged, Gibbs, GGE).

2. Good agreement between the effective description and the unitary dynamics for a case study.

3. In general, the entropy of the effective description always increases, and becomes constant in slow reversible processes.

4. Break down of the minimal work principle for Generalised Gibbs ensembles.

Summary

1. We built an effective description of thermodynamic protocols based on equilibrium states (Time-averaged, Gibbs, GGE).

2. Good agreement between the effective description and the unitary dynamics for a case study.

3. In general, the entropy of the effective description always increases, and becomes constant in slow reversible processes.

4. Break down of the minimal work principle for Generalised Gibbs ensembles.

Work and entropy production in generalised Gibbs ensembles, M. P.-L., A. Riera, R. Gallego, H. Wilming, J. Eisert, New J. Phys. 18, 123035 (2016).

Thank you for your attention!

$$\rho_S(t) = \mathrm{Tr}_B\left(U \rho_{SB} U^\dagger\right)$$

$$\rho_S(t) = \mathrm{Tr}_B\left(U\rho_{SB}U^\dagger\right)$$

Equilibrates to the
diagonal ensemble

$$\rho_S(t) \approx \mathrm{Tr}_B\left(\omega_{\mathrm{SB}}\right)$$

$$\omega_{SB} \propto e^{-\sum_i \mu_i \Pi_i}$$

Exact but
impractical result

$$\rho_S(t) = \mathrm{Tr}_B \left(U \rho_{SB} U^\dagger \right)$$

Equilibrates to the
diagonal ensemble

$$\rho_S(t) \approx \mathrm{Tr}_B \left(\omega_{\mathrm{SB}} \right)$$

$$\omega_{SB} \propto e^{-\sum_i \mu_i \Pi_i}$$

Exact but
impractical result

At the level of S, this states
becomes indistinguishable from….

$$\omega_{SB} \propto e^{-\beta H}$$

For “generic” systems.

$$\rho_S(t) = \mathrm{Tr}_B\left(U\rho_{SB}U^\dagger\right)$$

Equilibrates to the diagonal ensemble

$$\rho_S(t) \approx \mathrm{Tr}_B\left(\omega_{\mathrm{SB}}\right)$$

$$\omega_{SB} \propto e^{-\sum_i \mu_i \Pi_i}$$

Exact but impractical result

At the level of S, this states becomes indistinguishable from….

$$\omega_{SB} \propto e^{-\beta H}$$

For “generic” systems.

$$\omega_{SB} \propto e^{-\sum_i \mu_i Q_i}$$

When further integrals of motion/ symmetries become relevant.