QUANTUM HEAT ENGINES AND REFRIGERATORS

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OUTLINE

① Heat engines as driven bi-partite systems : SWAP ENGINE

M. Campisi, JPA 47 24 1 (2014)
M. Campisi, R. Fazio, J. Pekola, NJP 17 035012 (2014)
M. Campisi, R. Fazio, JPA 49 345002 (2016)

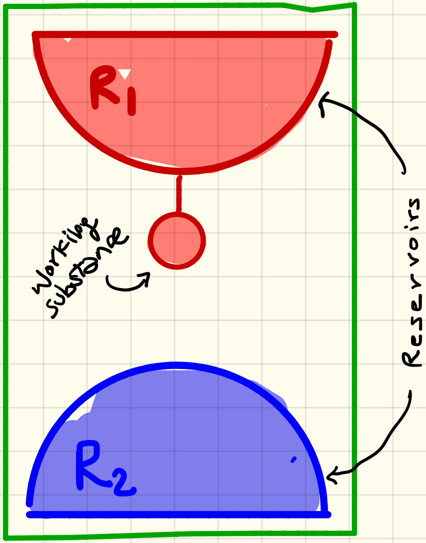
② The Power of a critical heat engine

M. Campisi, R. Fazio, Nature Communications 7 11895 (2016)

③ Josephson quantum heat engine

G. Marchegiani, P. Virtanen, F. Giazotto, M. Campisi, Phys. Rev. Applied 6 054014 (2016)

HEAT ENGINE AS A DRIVEN BIPARTITE SYSTEM



Subsystem 1 = R_1 + WS

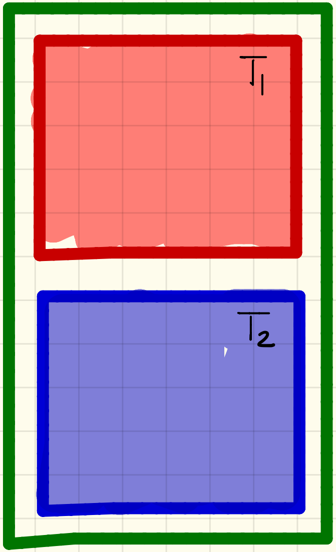
Subsystem 2 = R_2

Driving = Operations on WS and switch ON/OFF couplings

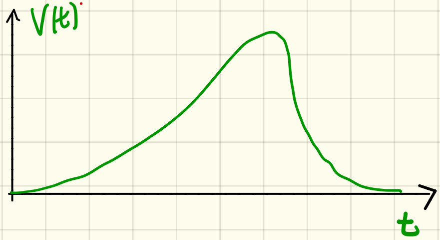
Examples: 4 stroke engines (Carnot, Diesel, etc)

Continuous engines

FLUCTUATION RELATION FOR HEAT ENGINES



$$H(t) = H_1 + H_2 + V(t)$$ ⟵ Work Source



$$\rho_0 = \frac{e^{-\beta_1 H_1}}{Z_1} \otimes \frac{e^{-\beta_2 H_2}}{Z_1}$$

$t = 0$ $\quad E^1_{n_1} \quad E^2_{n_2} \quad P_{n_1 n_2} = \frac{e^{-\beta_1 E^1_{n_1}}}{Z_1} \frac{e^{-\beta_2 E^2_{n_2}}}{Z_2}$

$t = \tau$ $\quad E^1_{m_1} \quad E^2_{m_2} \quad P_{m_1 m_2 | n_1 n_2} = |\langle m_1 m_2 | U | n_1 n_2 \rangle|^2$

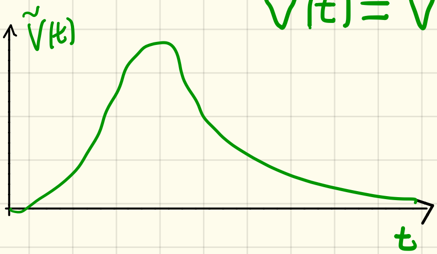
$$\downarrow \qquad \downarrow$$

$$\Delta E_1 + \Delta E_2 = W$$

$$P(\Delta E_1, \Delta E_2) = \sum_{\substack{n_1 n_2 \\ m_1 m_2}} P_{n_1,n_2} \, P_{m_1,m_2|n_1,n_2} \, \delta\left[\Delta E_1 - (E^1_{m_1} - E^1_{n_1})\right] \times$$

$$\times \, \delta\left[\Delta E_2 - (E^2_{m_2} - E^2_{n_2})\right]$$

$$\tilde{V}(t) = V(t - \tau)$$



$$\tilde{P}_{m_1,m_2|n_1,n_2} = P_{n_1,n_2|m_1,m_2}$$

$$\Downarrow$$

$$\frac{P(\Delta E_1, \Delta E_2)}{\tilde{P}(-\Delta E_1, -\Delta E_2)} = e^{+\beta_1 \Delta E_1} \, e^{+\beta_2 \Delta E_2}$$

Change of variables $W = \Delta E_1 + \Delta E_2$

$$\frac{P(\Delta E_1, W)}{\tilde{P}(-\Delta E_1, -W)} = e^{+(\beta_1-\beta_2)\Delta E_1 + \beta_2 W}$$

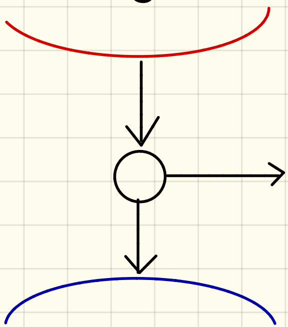
HEAT ENGINE FLUCTUATION RELATION

$$\Rightarrow \left\langle e^{-(\beta_1-\beta_2)\Delta E_1 - \beta_2 W} \right\rangle = 1$$

$$\Rightarrow (\beta_1-\beta_2)\langle \Delta E_1 \rangle - \beta_2 \langle W \rangle \geq 0$$

Sinitsyn JPA 44 405001 (2011)
Campisi JPA 47 245001 (2014)
Campisi et al. NJP 17 035012 (2014)

Heat engine operation



$\langle W \rangle \lesssim 0$

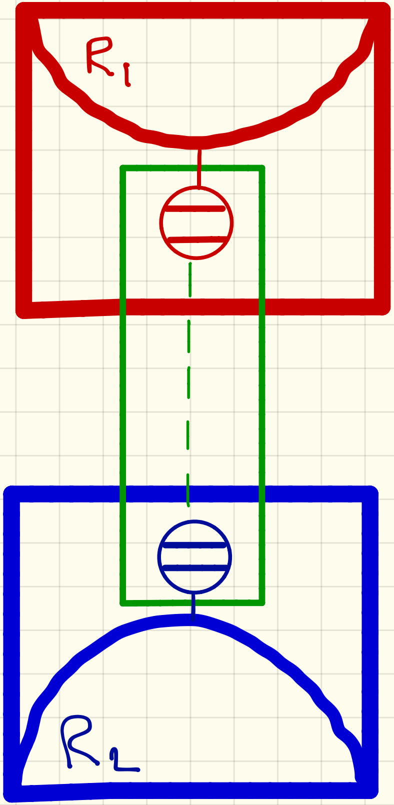
$\langle \Delta E_1 \rangle \lesssim 0$

$$(\beta_1 - \beta_2)\langle \Delta E_1 \rangle - \beta_2 \langle W \rangle \gtrsim 0$$

$$\Rightarrow \quad \eta \doteq \frac{\langle W \rangle}{\langle \Delta E_1 \rangle} \leq 1 - \frac{\beta_2}{\beta_1} \doteq \eta^c$$

Fluctuation Theorem $\Rightarrow$ $\eta \leq \eta^c$

EXAMPLE: TWO QUBIT ENGINE



SIMPLIFYING ASSUMPTION: Driving FAST compared to thermal relaxation

$$H = \underbrace{H_1}_{\frac{\omega_1}{2}\sigma_1^z} + \underbrace{H_2}_{\frac{\omega_2}{2}\sigma_2^z} + V(t) \rightarrow U$$

$$\langle \Delta E_1 \rangle = \mathrm{Tr}_1 \mathrm{Tr}_2\, H_1 (U \rho U^{\dagger} - \rho)$$

$$+$$

$$\langle \Delta E_2 \rangle = \mathrm{Tr}_1 \mathrm{Tr}_2\, H_2 (U \rho U^{\dagger} - \rho)$$

$$=$$

$$\langle W \rangle$$

MAXIMIZATION OF WORK OUTPUT

$$U = \left(\begin{array}{c|cc|c} e^{i\varphi_1} & & & \\ \hline & 0 & e^{i\varphi_3} & \\ & e^{i\varphi_2} & 0 & \\ \hline & & & e^{i\varphi_4} \end{array}\right) \leftarrow \text{SWAP}$$

Basis: $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$

$$U|+-\rangle = e^{i\varphi_3}|-+\rangle$$

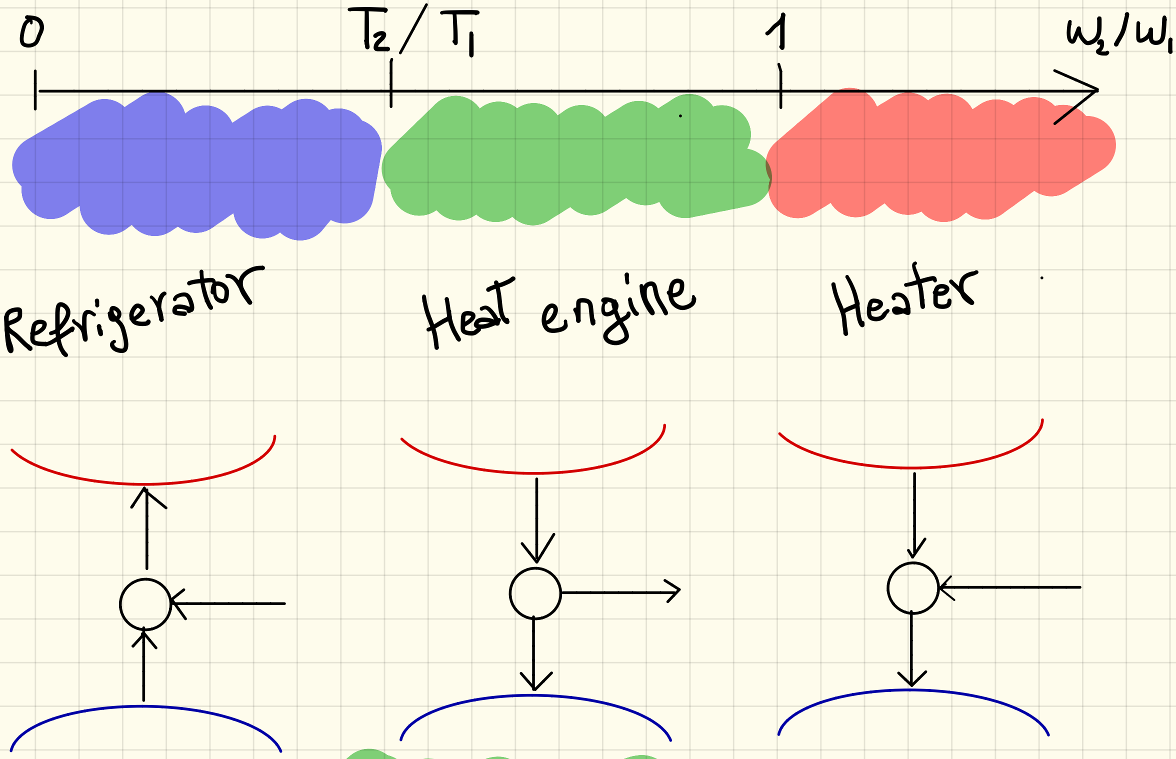
$$U|-+\rangle = e^{i\varphi_2}|+-\rangle$$

$$\langle \Delta E_1 \rangle = -\left(\frac{1}{1+e^{\beta_1 \omega_1}} - \frac{1}{1+e^{\beta_2 \omega_2}}\right) \omega_1$$

$$\langle \Delta E_2 \rangle = \left(\frac{1}{1+e^{\beta_1 \omega_1}} - \frac{1}{1+e^{\beta_2 \omega_2}}\right) \omega_2$$

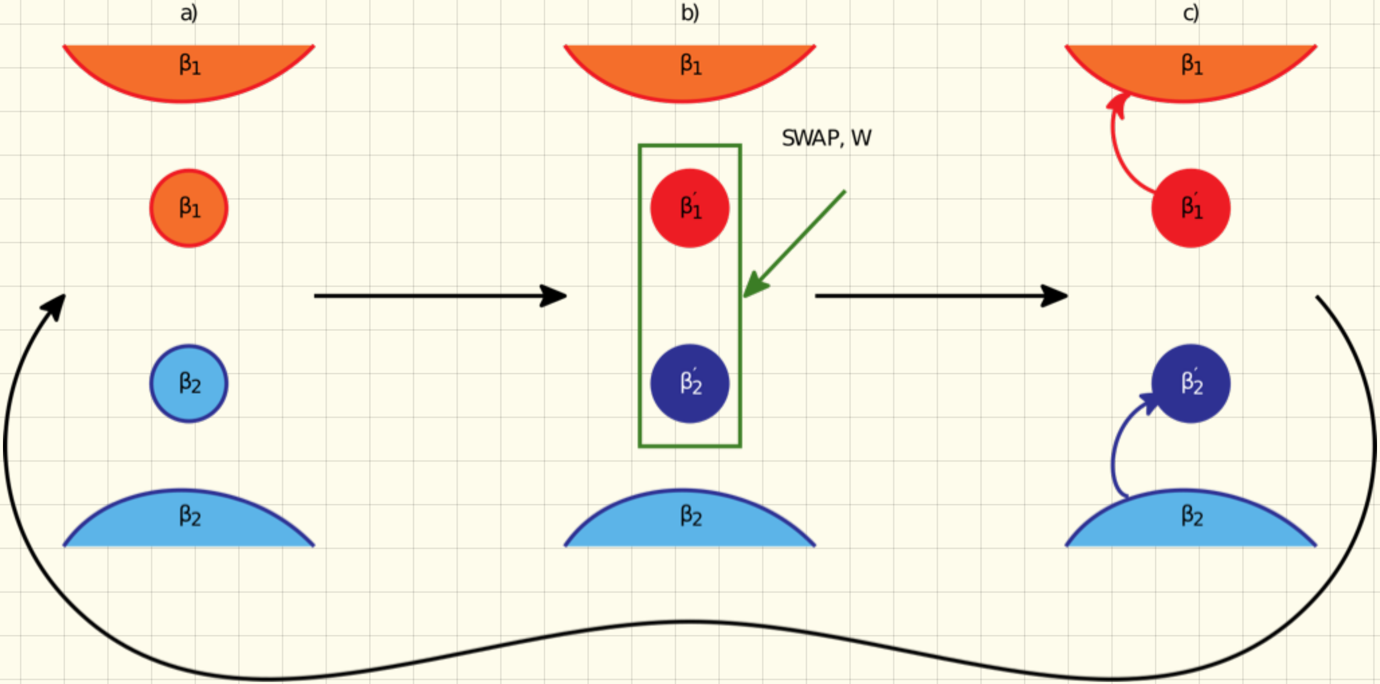
$$\langle W \rangle = \left(\frac{1}{1+e^{\beta_1 \omega_1}} - \frac{1}{1+e^{\beta_2 \omega_2}}\right)(\omega_2 - \omega_1)$$

Allahverdyan et al.
PRE **77** 04118 (2008)



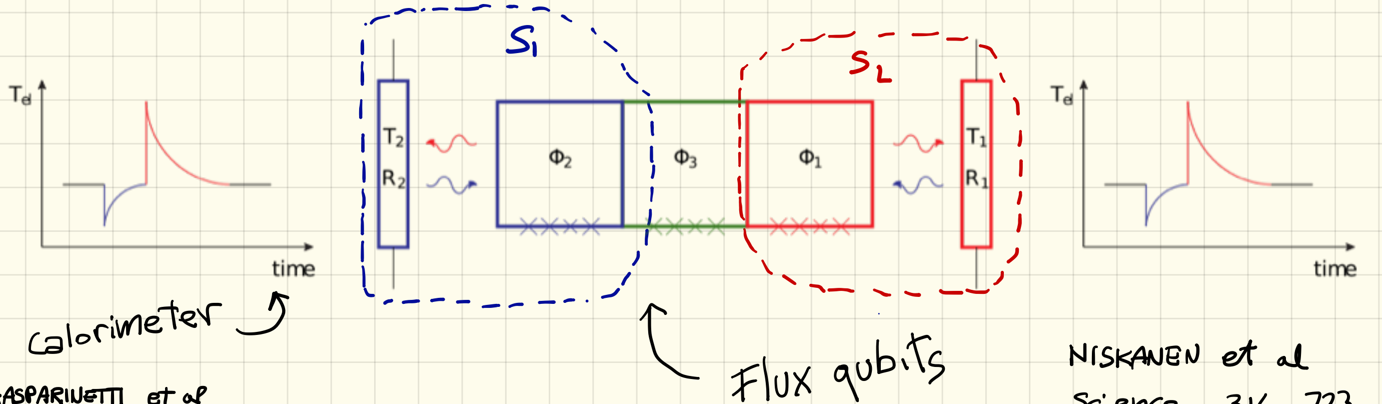
$$\eta = 1 - \frac{W_2}{W_1} \leqslant 1 - \frac{T_2}{T_1}$$

Refrigerator



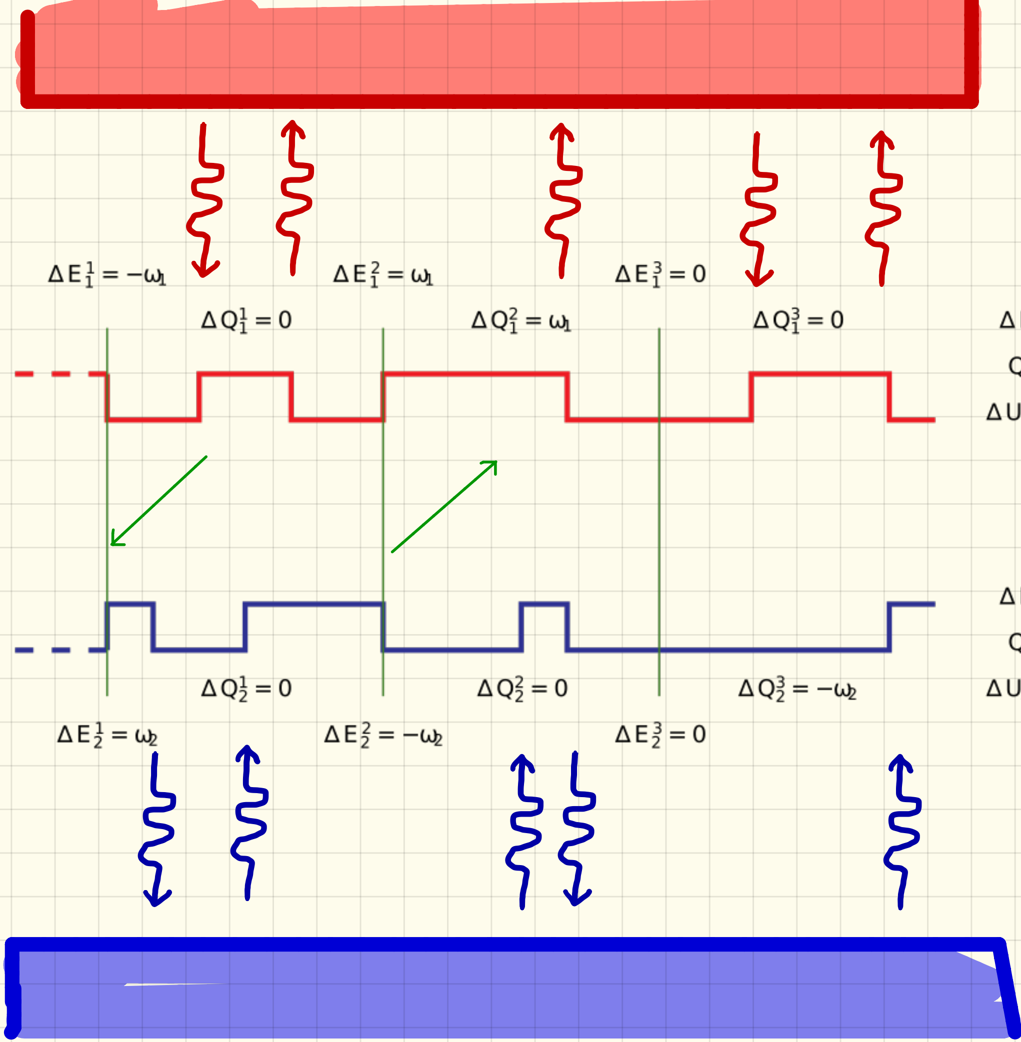
$$\beta_1' = \beta_2 \frac{\omega_2}{\omega_1} \qquad \beta_2' = \beta_1 \frac{\omega_1}{\omega_2}$$

IMPLEMENTATION



GASPARINETTI et al
Phys. Rev. App. 3 014007 (2015)

NISKANEN et al
Science 316 723 (2007)



$\Delta E_1 = \sum_i \Delta E_1^i = -N\omega_1$

$Q_1 = \sum_i \Delta Q_1^i$

$\Delta U_1 = \Delta E_1 - Q_1$

$\Delta E_2 = \sum_i \Delta E_2^i = N\omega_2$

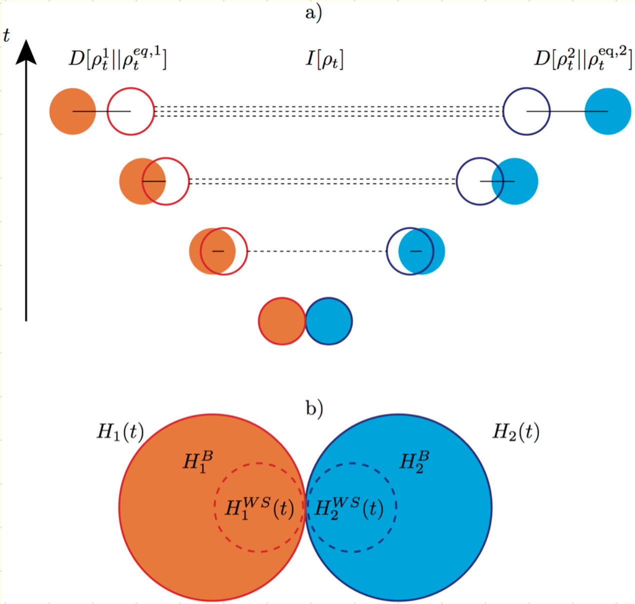
$Q_2 = \sum_i \Delta Q_2^i$

$\Delta U_2 = \Delta E_2 - Q_2$

$\Delta E_2 = -\frac{\omega_2}{\omega_1} \Delta E_1$

$\frac{W}{\Delta E_1} = 1 - \frac{\omega_2}{\omega_1}$

DISSIPATION, CORRELATIONS AND LAGS IN HEAT ENGINES



$$\beta_1 W_{diss}^{(1)} + \beta_2 W_{diss}^{(2)} = D[\rho_t^1||\rho_t^{eq,1}] + D[\rho_t^2||\rho_t^{eq,2}] + I_{1/2}[\rho_t]$$

$$D[\rho||\sigma] = \mathrm{Tr}\,\rho \ln\rho - \mathrm{Tr}\,\rho\ln\sigma$$

$$I_{1/2}[\rho] = \Sigma_i S[\rho^i] - S[\rho] \qquad S[\rho] = -\mathrm{Tr}\,\rho\ln\rho$$

$$W_{diss}^{(i)} = \mathrm{Tr}_i\left[H_i(\tau)\rho_\tau^i - H_i(0)\rho_0^i\right] - \Delta F_i \qquad F_i = -\frac{1}{\beta_i}\ln Z_i$$

$$\rho_0 = \rho_0^1 \otimes \rho_0^2 = \frac{e^{-\beta_1 H_1(0)}}{Z_1} \otimes \frac{e^{-\beta_2 H_2(0)}}{Z_2}$$

For heat engines $\quad \Delta F_i = 0$

$$W_{diss}^{(i)} = Tr_i\left[H_i(\tau)\rho_\tau^i - H_i(0)\rho_0^i\right] = \langle \Delta E_i \rangle$$

$$\beta_{MAX} W_{out}\, \delta\eta \geq D[\rho_t^1 \| \rho_t^{eq,1}] + D[\rho_t^2 \| \rho_t^{eq,2}] + I_{1/2}[\rho_t] \geq 0$$

$W_{out} \geq 0 \qquad \delta\eta = \dfrac{\eta^c}{\eta} - 1 \qquad \eta^c = 1 - \dfrac{\beta_{min}}{\beta_{max}}$

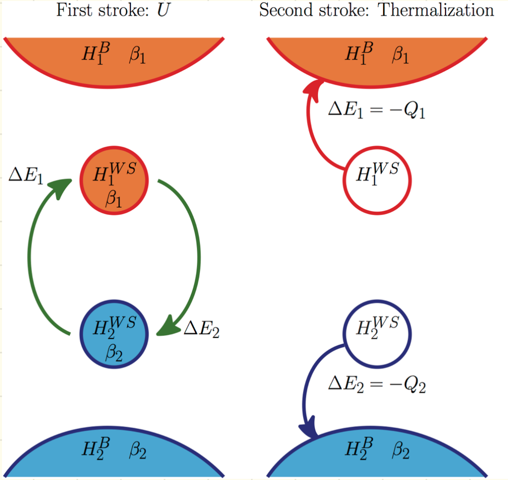
$$\eta \leq \eta^c$$

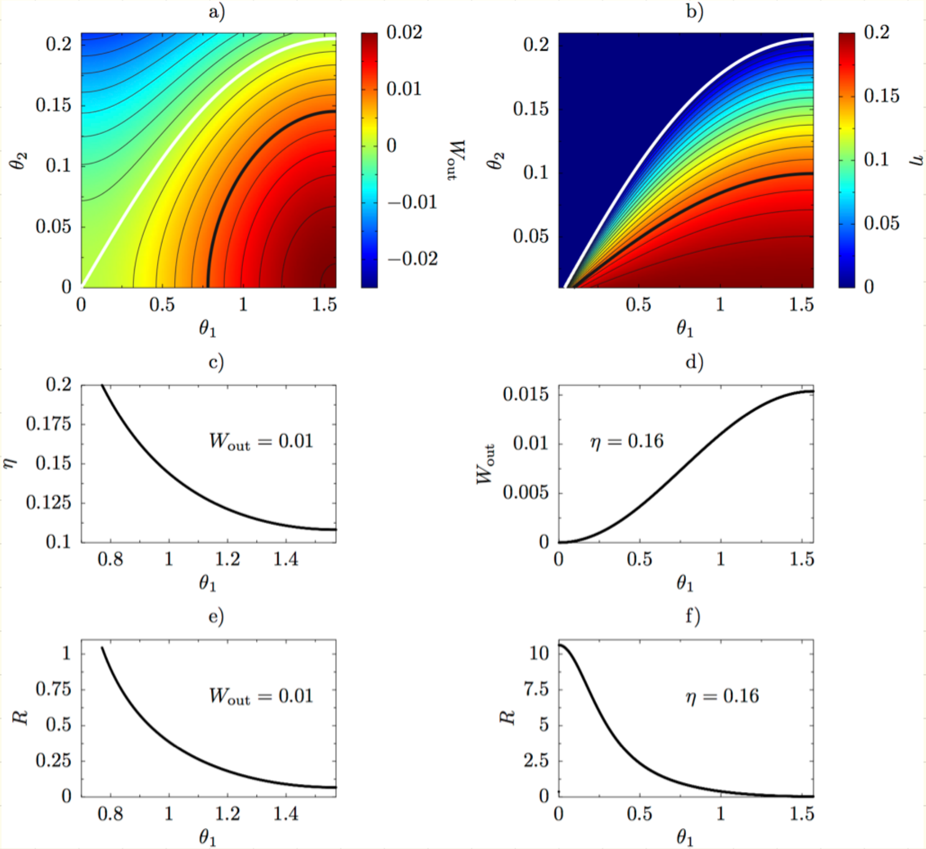
$W_{out} = 0 \quad @ \quad \eta = \eta^c$

$$H(t) = \frac{\omega_1}{2}\sigma_z^1 + \frac{\omega_2}{2}\sigma_z^2 + V(t)$$

$$U = U_{SWAP}(\theta_1) \cdot U_{CNOT}(\theta_2)$$

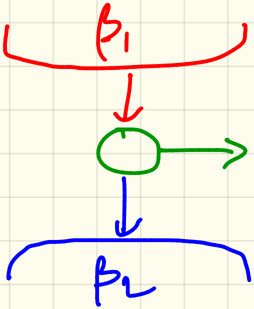
$$R[\rho_\tau] = \frac{I[\rho_\tau]}{D[\rho_\tau^1 \| \rho_\tau^{eq,1}] + D[\rho_\tau^2 \| \rho_\tau^{eq,2}]}$$





The power of a critical heat engine

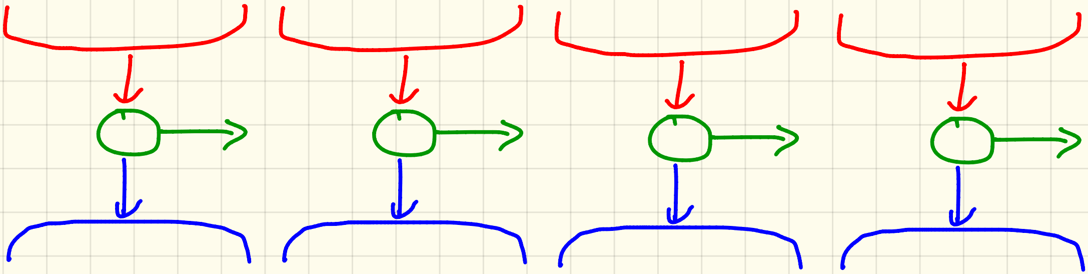
Increase Power without affecting efficiency



η

W_0

P_0

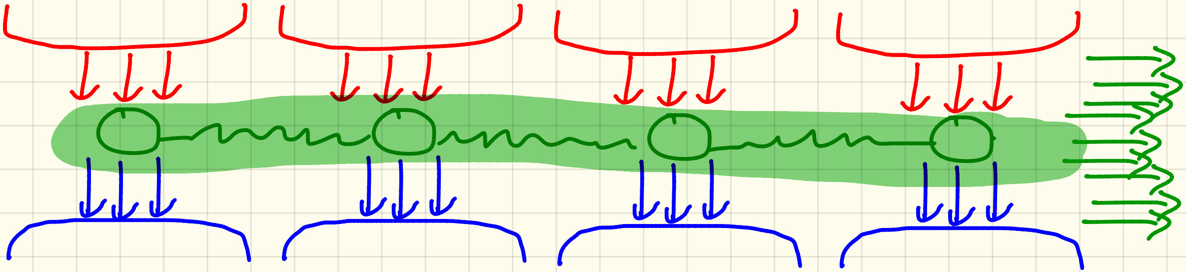


η

$W = N W_0$

$P = N P_0$

Increasing Power AND efficiency via interactions?



Performance $\Pi = \dfrac{w}{\Delta\eta}$

Performance rate $\hat{\Pi} = \dfrac{P}{\Delta\eta}$

$\Delta\eta = \eta^c - \eta$

Parallel Device

$w \sim N$

$\Delta\eta \sim 1 \implies \Pi \sim N$

$P \sim N$

$\Delta\eta \sim 1 \implies \hat{\Pi} \sim N$

- Can interactions boost performance Π ?

- Can interactions boost performance rate $\dot{\Pi}$?

Boosted performance rate $\Rightarrow$ approach to Carnot efficiency

@ fixed power per resource

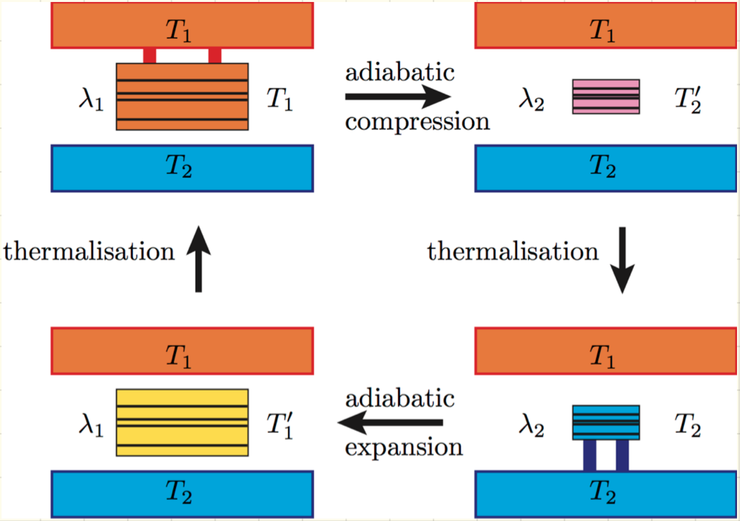
if $\dot{\Pi} = \frac{\mathcal{P}}{\Delta\eta} \sim N^{1+\alpha}$ $(\alpha > 0)$

then $\mathcal{P} \sim N$ with $\Delta\eta \sim \frac{1}{N^{\alpha}} \to 0$ is possible

$\to$ one can asymptotically approach $\Delta\eta \to 0$ i.e. $\eta \to \eta^{c}$

"Quantum" Otto engine

Working substance $\quad H^{ws}(t) = \lambda(t)\, K$



$$T_1' = T_2 \frac{\lambda_1}{\lambda_2}$$

$$T_2' = T_1 \frac{\lambda_2}{\lambda_1}$$

$$Q_i = \mp \lambda_i \left[U_K(\beta_2 \lambda_2) - U_K(\beta_1 \lambda_1) \right]$$

$$\left(U_K(\theta) = \frac{\mathrm{Tr}\, K e^{-\theta K}}{\mathrm{Tr}\, e^{-\theta K}} \right)$$

$$W_{out} = Q_1 + Q_2$$

$$= (\lambda_2 - \lambda_1) \left[U_K(\beta_2 \lambda_2) - U_K(\beta_1 \lambda_1) \right]$$

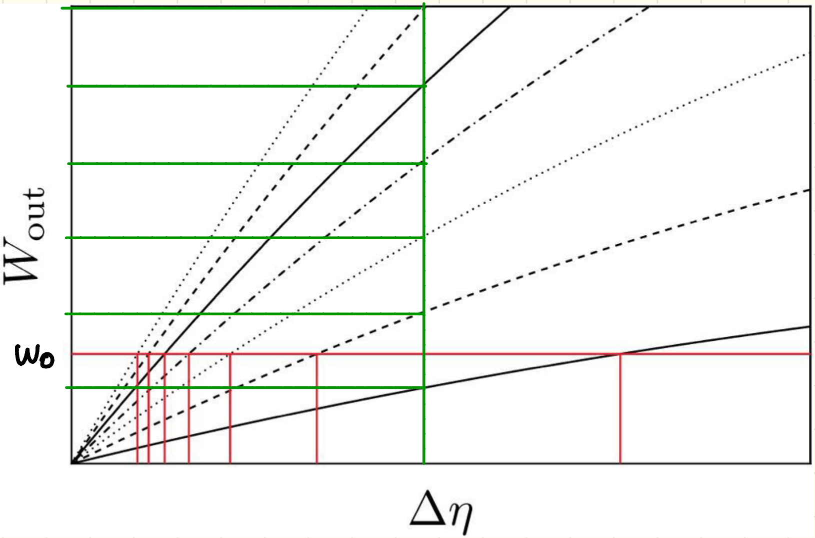
$$\eta = \frac{W_{out}}{Q_1} = 1 - \frac{\lambda_2}{\lambda_1} \leq 1 - \frac{\beta_1}{\beta_2} = \eta^c$$

↳ heat engine regime → $W_{out} \geq 0$, $Q_1 \geq 0$ → $\frac{\lambda_2}{\lambda_1} > \frac{\beta_1}{\beta_2} = \frac{T_2}{T_1}$

N identical quantum Otto engines in parallel.

$H = \Sigma H_i \qquad H_i = \lambda K$

$$W_{out} = N(\lambda_2 - \lambda_1)\left[U_K(\beta_2\lambda_2) - U_K(\beta_1\lambda_1)\right]$$



$W_{out} \sim N$

$\Delta\eta \sim 1$

$\Pi = \frac{W_{out}}{\Delta\eta} \sim N$

$W_{out} = w_0$

$\Delta\eta \sim \frac{1}{N}$

$\Pi = \frac{W_{out}}{\Delta\eta} \sim N$

In general

$$\Pi \sim \left.\frac{\partial W_{out}}{\partial \Delta\eta}\right|_{\Delta\eta=0}$$ (provided linear response applies)

Straightforward calculation

$$\left.\frac{\partial W_{out}}{\partial \Delta\eta}\right|_{\Delta\eta=0} = \underbrace{C_K(\alpha_1\beta_1)}_{\text{Heat capacity}} = N \underbrace{c_K(\alpha_1\beta_1)}_{\text{Specific heat}}$$

$$C_K = -\theta^2 \frac{\partial U_K}{\partial \theta}$$

Boost of performance Π
if Heat capacity C_K scales
more than linearly

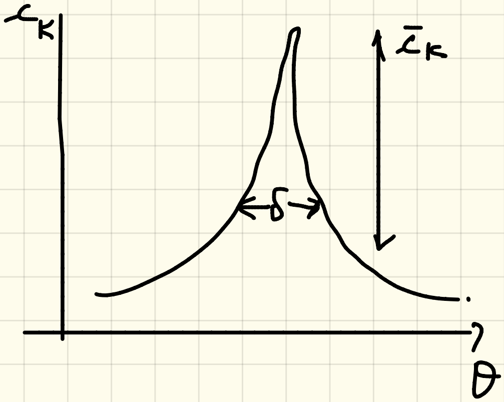
$\downarrow$

Thermodynamic limit violated

e.g. ① The system is at a phase transition
of 1^{st} or 2^{nd} order

② The system is "small"

2^{ND} order phase transition



$\bar{c}_K \sim N^{\alpha/d\nu}$

$\delta \sim N^{-1/d\nu}$

$\alpha \rightarrow$ Specific heat critical exponent

$\nu \rightarrow$ Correlation length critical exponent

$d \rightarrow$ # of dimensions

$$\Rightarrow \Pi \sim N^{1+\alpha/d\nu}$$

$$P = \frac{W_{out}}{T} \Rightarrow \dot{\Pi} = \frac{\Pi}{T}$$

$T \sim$ relaxation to equilibrium $\sim N^{z/d}$

$$\Rightarrow \dot{\Pi} \sim N^{1+(\alpha-z\nu)/d\nu}$$

$z \rightarrow$ Dynamical critical exponent

2^{ND} order phase transition

$$\dot{\pi} \sim N^{1+(\alpha - z\nu)/d\nu}$$

Boost if $\alpha - z\nu > 0$

$\Rightarrow \quad \theta \sim N$

$$\Delta\eta \sim \frac{1}{N^{(\alpha - z\nu)/d\nu}}$$

It asymptotically occurs if

$\alpha - z\nu \geq 1$ $\quad \left(\delta \sim \frac{1}{N^{d\nu}} \right)$

$\alpha - z\nu > 0$ ← transient

$\alpha - z\nu \geq 1$ ← Asymptotic

Possible ?

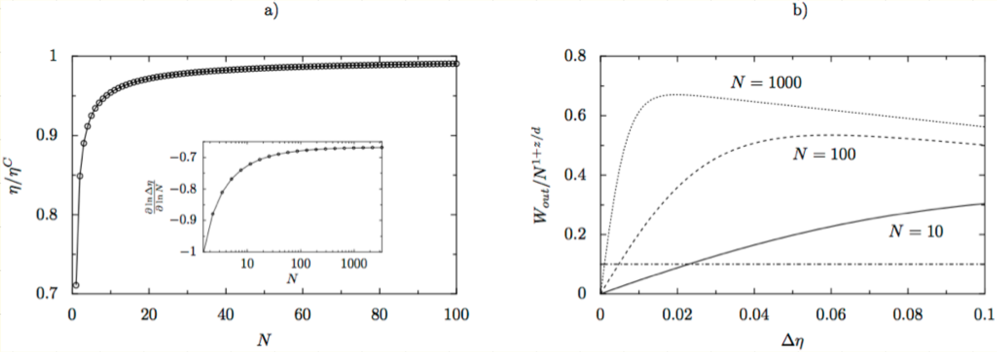
Ising 3D

$\alpha \simeq 0.12$
$\nu \simeq 0.63$ $\Rightarrow$ $\alpha - z\nu \simeq -0.28$
$z \simeq 2.35$

$Dy_2Ti_2O_7$ Spin Ice

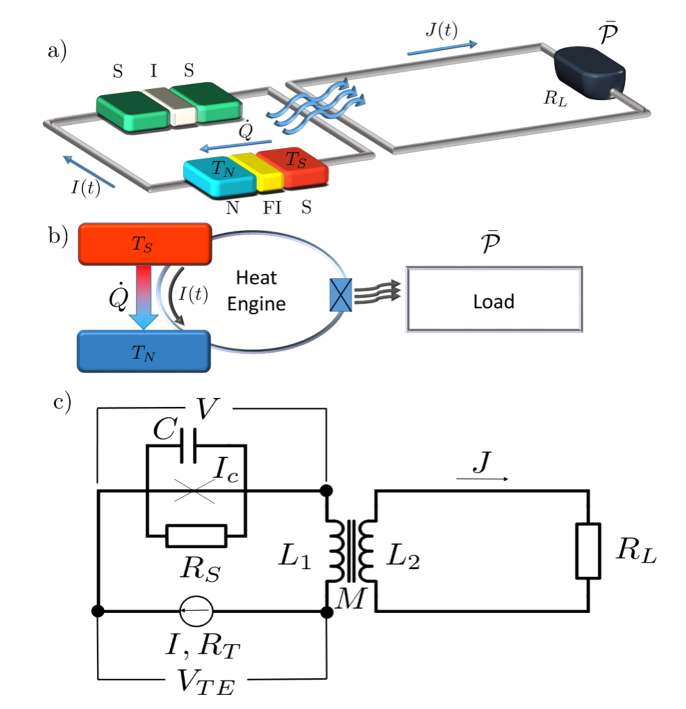
$z\nu \simeq -0.7$ $\leftarrow$ Grams et al, Nat. Comm. 5 4853 (2014)
$\alpha \gtrsim 0.38$ $\leftarrow$ Higashinaka et al, J Phys Soc Jap 73 2845 (2004)

$\Rightarrow$ $\alpha - z\nu \gtrsim 1.08$

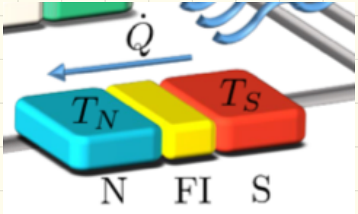


Josephson quantum heat engine

G. Marchegiani, P. Virtanen, F. Giazotto, M. Campisi, Phys. Rev. Applied 6 054014 (2016)



The N-FI-S element



Giazotto et al
Phys. Rev. Applied 4 044016 (2015)

$$N_{\uparrow,\downarrow}(E) = \frac{1}{2}\left|\mathrm{Re}\left[\frac{E + i\Gamma \pm h_{exc}}{\sqrt{(E + i\Gamma \pm h_{exc})^2 - \Delta^2}}\right]\right|$$

$$I_{TE}(V_{TE}) = \frac{1}{eR_T}\int_{-\infty}^{+\infty} dE\mathcal{N}(E)[f_S(T_S) - f_N(V_{TE}, T_N)]$$

$$\mathcal{N} = N_+ + PN_-$$

$$P = (G_\uparrow - G_\downarrow)/(G_\uparrow + G_\downarrow) \qquad N_\pm = N_\uparrow \pm N_\downarrow$$

$$f_N(V_{TE}, T_N) = [1 + \exp(E + eV_{TE}/k_BT_N)]^{-1}$$

$$f_S(T_S) = [1 + \exp(E/k_BT_S)]^{-1}$$

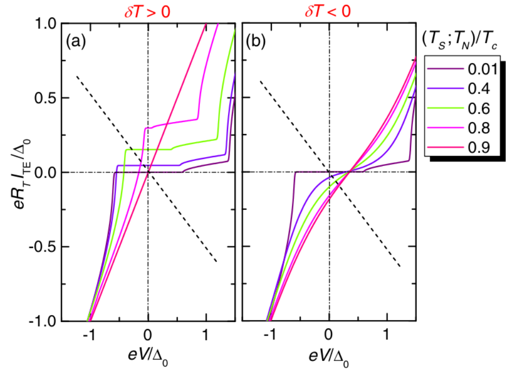
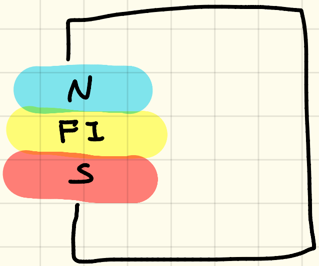


FIG. 3. Characterization of the TE for $T_S \neq T_N$. (a) I_{TE}-V characteristic of TE calculated for several values of T_S (see top legend) at $T_N = 0.01T_c$, $P = 0.9$, and $h_{exc} = 0.4\Delta_0$. (b) The same as in panel (a) calculated for several T_N values at $T_S = 0.01T_c$. Dashed lines in panels (a) and (b) represent the current (I_{JJ}) flowing through the Josephson element when it is operated in the resistive regime, $I_{JJ} = -V/R_{JJ}$.



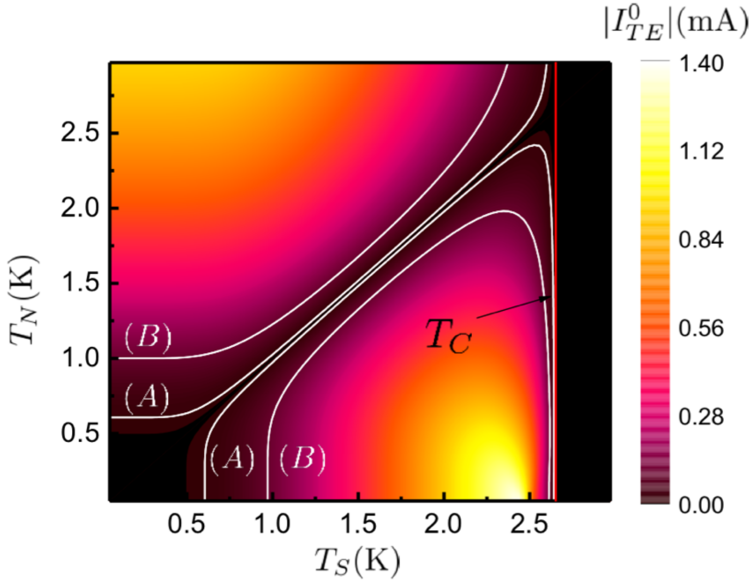
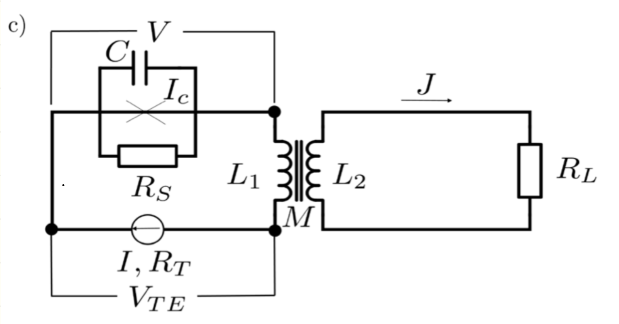


FIG. 2. (color online). Density plot of zero voltage thermoelectric current provided by the N-FI-S element. Parameters are $P = 0.9$, $T_{C0} = 3.0$ K, $R_T = 0.1\ \Omega$, $\Gamma = 10^{-4}\ \Delta_0$ and $h_{exc} = 0.4\ \Delta_0$, where $\Delta_0 = 1.764\ k_B T_{C0}$ and T_{C0} is the zero-field critical temperature. In the black region $|I^0_{TE}| < 3\ \mu$A. The contour lines for $|I^0_{TE}| = 10\ \mu$A (A) and $|I^0_{TE}| = 100\ \mu$A (B) are plotted in white.

Circuit equations



1) $I = I_c \sin\varphi + \frac{V}{R_s} + C\dot{V}$

2) $\dot{\varphi} = \frac{2\pi}{\Phi_0} V$

3) $V_{TE} = L_1 \dot{I} - M\dot{J} + V$

4) $M\dot{I} = L_2 \dot{J} + R_L J$

Ansatz: $V_{TE}(t) = \overline{V_{TE}} + \delta V_{TE}(t)$ $\quad \langle \delta V_{TE} \rangle = 0$

Approx: $Z(\omega) \simeq Z(0) = R_T$

5) $I(t) = -I_{TE}(\overline{V_{TE}}) - \frac{\delta V_{TE}(t)}{R_T}$

Unknowns:

$I, J, \varphi, \delta V_{TE}, V$

$\overline{V_{TE}}$ → tuned to the point where $\langle \delta V_{TE} \rangle = 0$

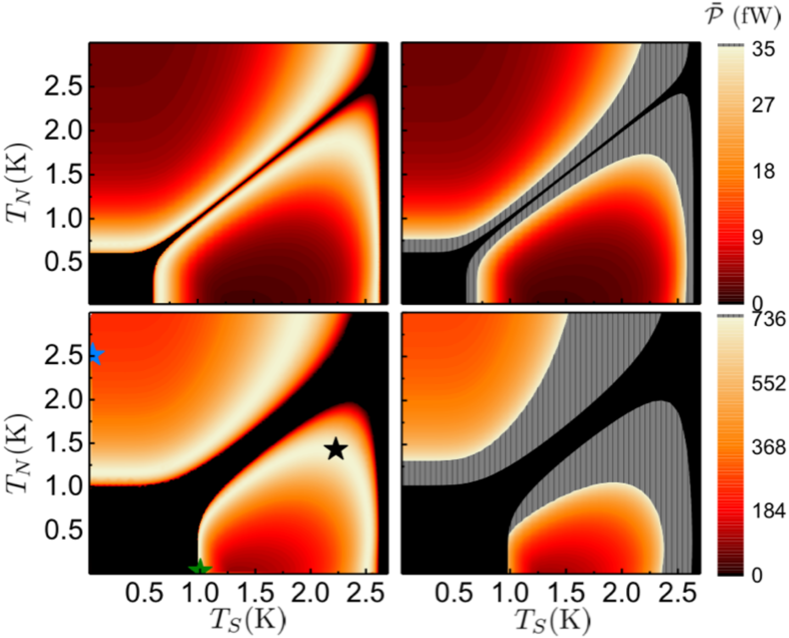


FIG. 3. (color online). Transmitted power vs thermal gradient for $I_c = 10\ \mu$A (top) and $I_c = 100\ \mu$A (bottom). Numerical data (left panels) are compared to the expression Eq. (9) (right panels). The regions where Eq. (9) gives a higher estimate than the largest value of $\bar{\mathcal{P}}$ in the corresponding numerical graph are drawn in grey. The stars denote the points investigated in Fig. 4. Parameters are $C = 100$ fF, $L_1 = L_2 = 100$ pH, $M = 10$ pH, $R_S = 1\ \Omega$ and $R_L = 10\ \Omega$.

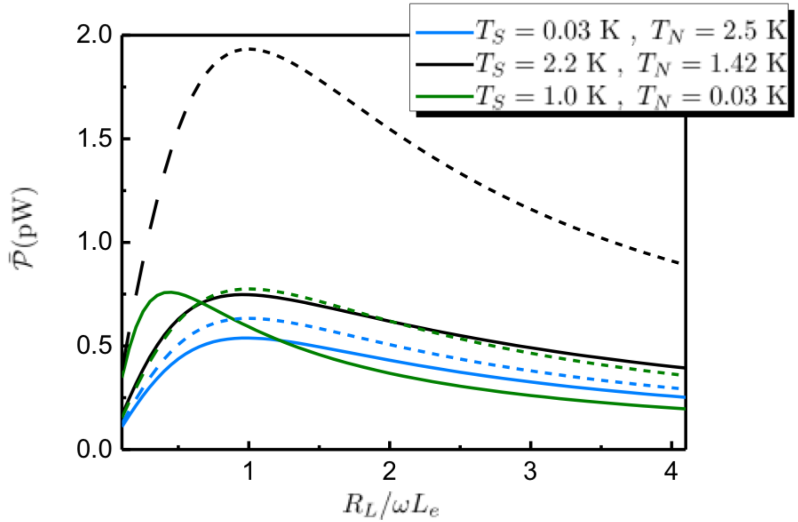


FIG. 4. (color online). Transmitted power vs load resistance for different working point in the (T_S, T_N) space, marked by the stars in Fig. 3. Numerical data (solid lines) are compared with the expression Eq.(9) (dash lines). Parameters are $C = 100$ fF, $L_1 = L_2 = 100$ pH, $M = 10$ pH, $R_S = 1\ \Omega$, and $I_c = 100\ \mu$A.