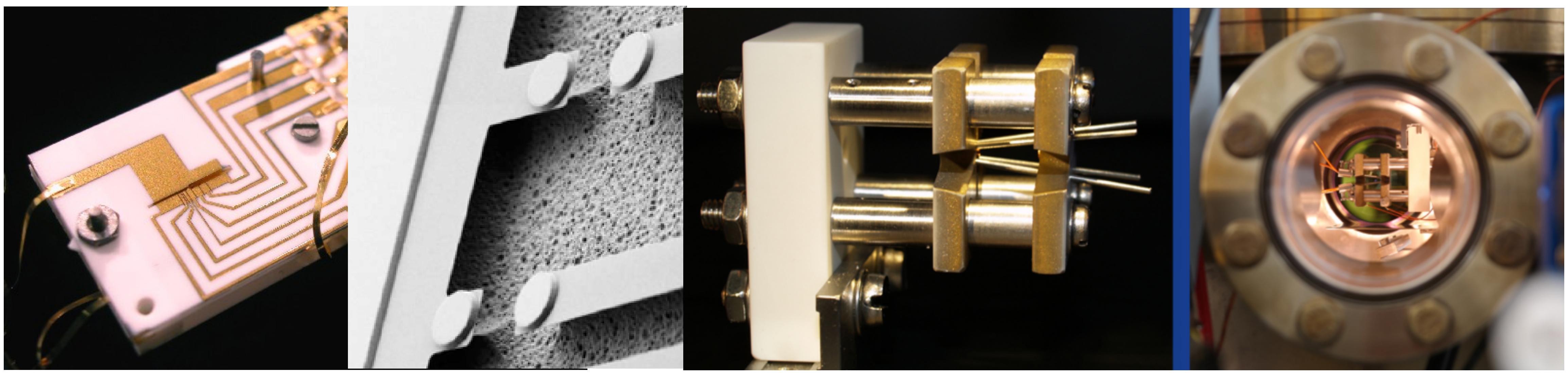
Fundamental corrections to work and power in the strong coupling regime



Jens Eisert, Freie Universität Berlin

Marti Perarnau-Llobet, Henrik Wilming, Arnau Riera, Rodrigo Gallego

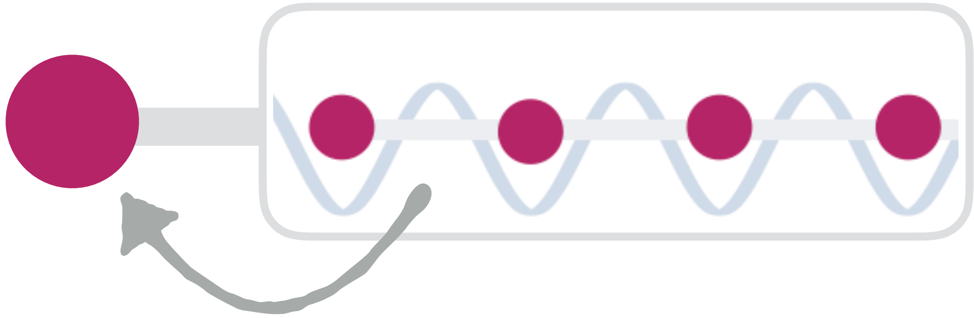
Non-Markovianity and Strong Coupling Effects in Thermodynamics

640. WE-Heraeus Seminar, Bad Honnef, April 2017

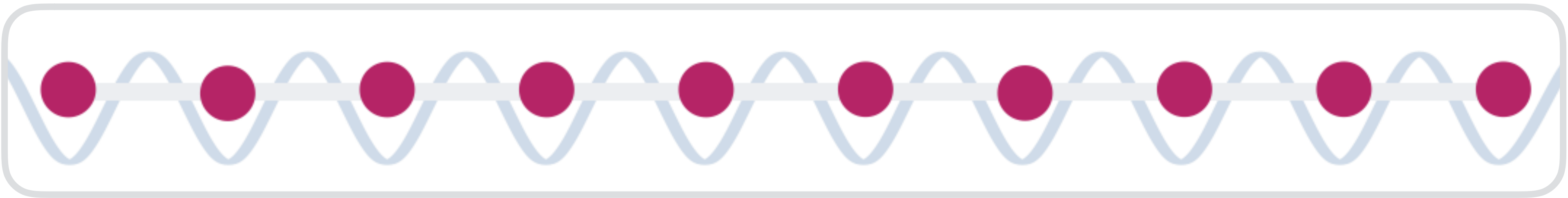
Main questions of this talk

- Strong coupling quantum thermodynamics

- Non-Markovian dynamics



- Equilibration and thermalisation of closed quantum many-body systems



Main questions of this talk

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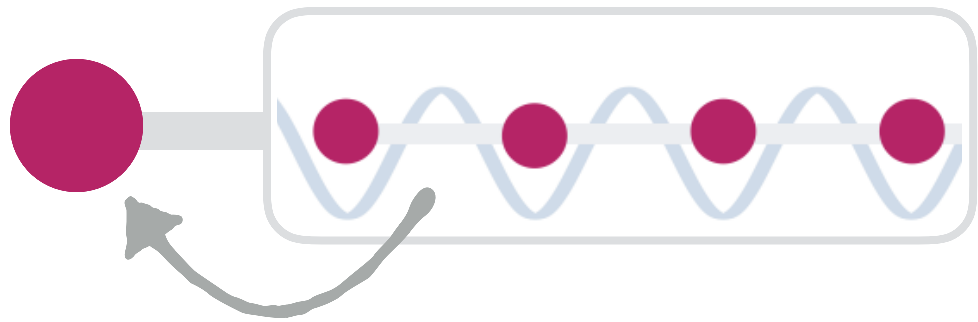


- Equilibration and thermalisation of closed quantum many-body systems

- Compare Massimilano’s, Christopher’s, David’s,etc talks

Main questions of this talk

- Strong coupling quantum thermodynamics

- What implications does strong coupling have to readings of the **second law**?

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)
Gallego, Riera, Eisert, New J Phys 16, 125009 (2014)

Main questions of this talk

- Strong coupling quantum thermodynamics

- How does **work** extraction have to be modified?

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)
Gallego, Riera, Eisert, New J Phys 16, 125009 (2014)

Main questions of this talk

- Strong coupling quantum thermodynamics

- What **power** enhancement due to strong coupling can be achieved?

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)
Gallego, Riera, Eisert, New J Phys 16, 125009 (2014)

Main questions of this talk

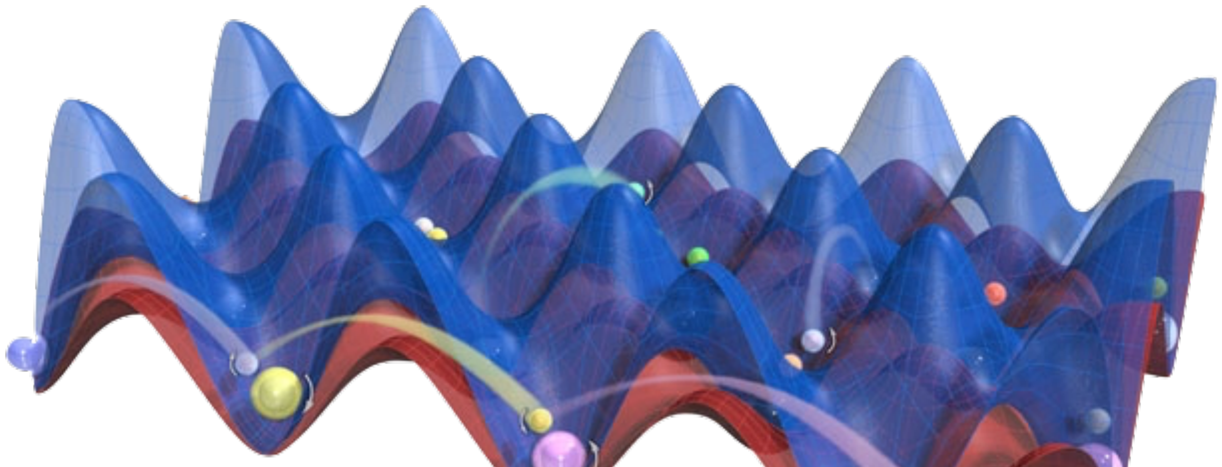
- Strong coupling quantum thermodynamics

- Quantum Brownian motion as an example

  Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

- Insights into equilibration and thermalization

  Gluza, Krumnow, Friesdorf, Gogolin, Eisert, Phys Rev Lett 117 (2016)
  Wilming, Krumnow, Goihl, Eisert, this week (2017)
  Gogolin, Eisert, Rep Prog Phys 79, 056001 (2016)
  Eisert, Friesdorf, Gogolin, Nature Physics 11, 124 (2015)
  Trotzky, Chen, Flesch, McCulloch, Schollwöck, Eisert, Bloch, Nature Physics 8, 325 (2012)



Main questions of this talk

- Strong coupling quantum thermodynamics

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  Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

- Insights into equilibration and thermalization

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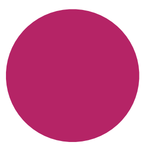
  Eisert, Friesdorf, Gogolin, Nature Physics 11, 124 (2015)
  Trotzky, Chen, Flesch, McCulloch, Schollwöck, Eisert, Bloch, Nature Physics 8, 325 (2012)

- Non-Markovian dynamics

  Groeblacher, Trubarov, Prigge, Cole, Aspelmeyer, Eisert, Nature Comm 6 7606 (2015)
  Wolf, Eisert, Cubitt, Cirac, Phys Rev Lett 101, 150402 (2008)

Setting the scene

- **Thermalisation in thermodynamics:** Abstraction of no interaction

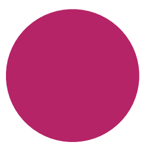


$$\omega_\beta(H_S)$$

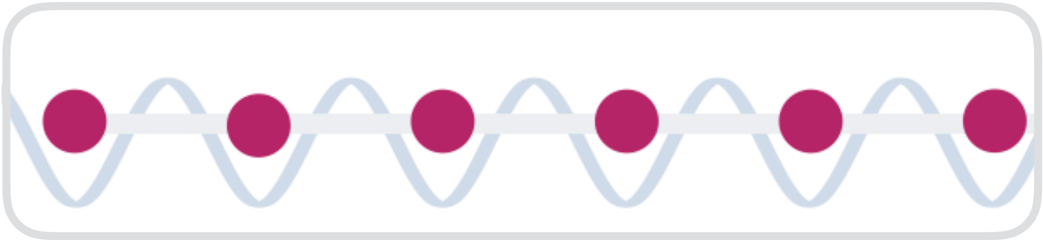
$$\omega_\beta(H_B)$$

$$\omega_\beta(H) = \frac{e^{-\beta H}}{\mathrm{tr}(e^{-\beta H})}$$

- **Thermalisation in thermodynamics:** Abstraction of no interaction



$\omega_\beta(H_S)$



$\omega_\beta(H_B)$

- In ontology of "**thermalisation**"
- **Quasi-static processes** make this assumption implicitly

  Kluge, Neugebauer, Foundations of thermodynamics (1976)
- In **resource-theoretic** formulations, explicitly inbuilt

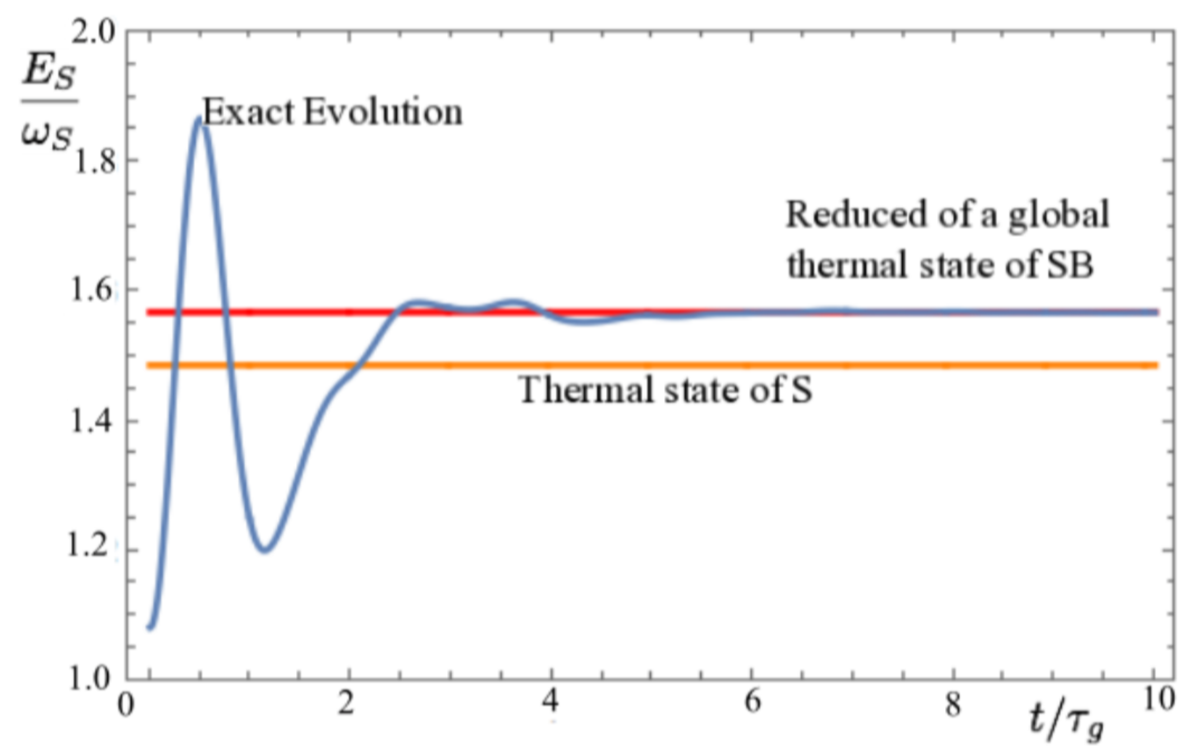
  Janzing, Wocjan, Zeier, Geiss, Beth, Int J Theor Phys 39, 2717 (2000)
  Horodecki, Oppenheim, Nature Comm 4, 2059 (2013)
  Brandao, Horodecki, Ng, Oppenheim, Wehner, PNAS 112, 3275 (2015)
  Ng, Mancinska, Cirstoiu, Eisert, Wehner, New J Phys 17, 08500 (2015)

- Clearly inappropriate in **strong-coupling setting**

$$H = H_S + H_B + V$$

- Expects state to be $\rho_S = \mathrm{tr}_B(\omega_\beta(H))$

- Simple example in quantum Brownian motion



Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)
Gallego, Riera, Eisert, New J Phys 16, 125009 (2014)

Some procrustenation on equilibration

Equilibration

- Systems are expected to equilibrate

$$H = H_S + H_B + V$$

$$\rho(t) = e^{-itH}\rho(0)e^{itH}$$

- Expectation values $\langle S(t)\rangle = \mathrm{tr}(S\rho(t))$ of local observables

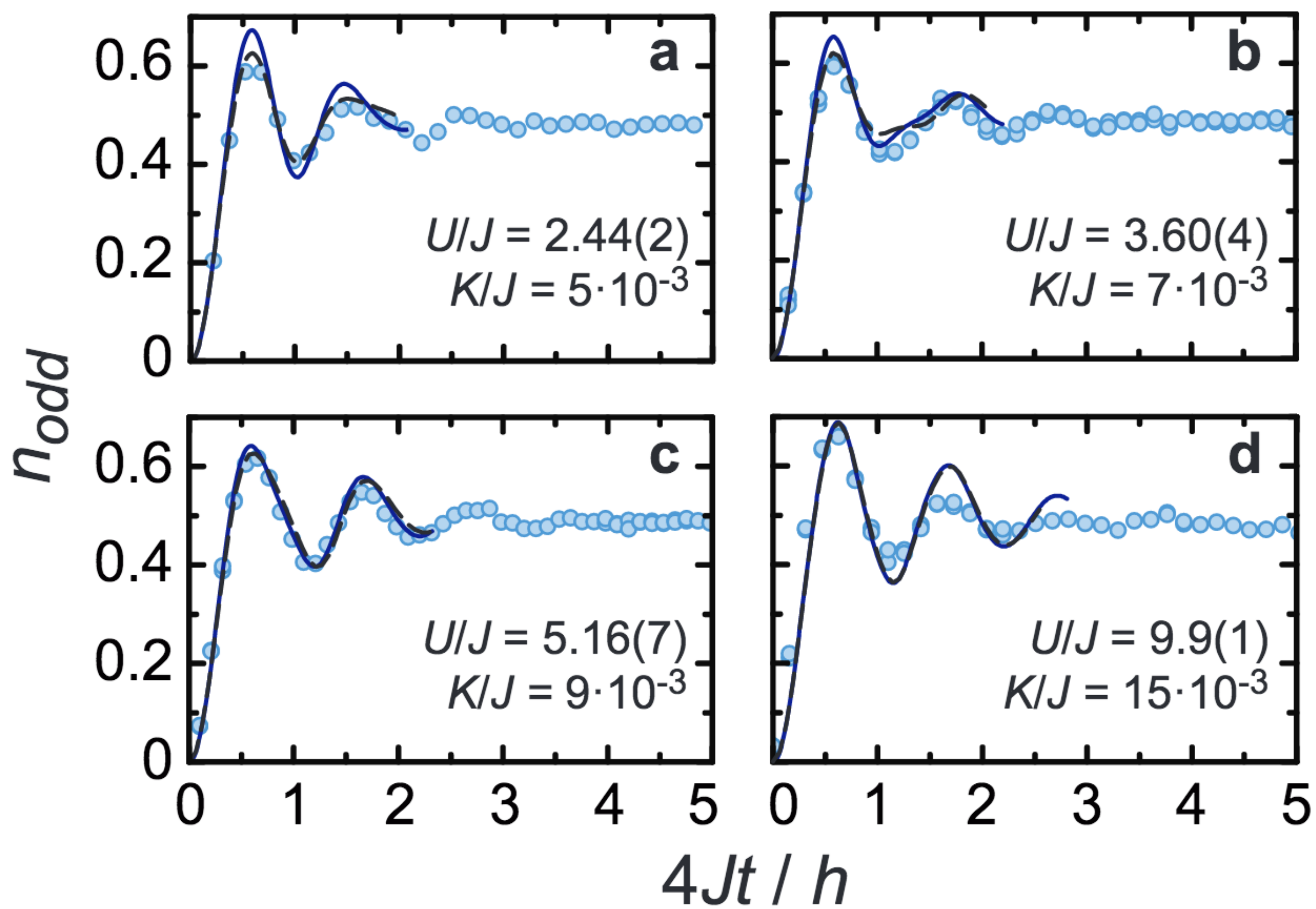
$$|\langle S(t)\rangle - \mathbb{E}_t(\langle S(t)\rangle)| < \varepsilon$$

take values of **time average** for overwhelming times

Linden, Popescu, Short, Winter, Phys Rev E 79, 061103 (2009)
Short, Farrelly, New J Phys 14, 013063 (2012)
Gogolin, Eisert, Rep Prog Phys 79, 056001 (2016)
Reimann, Kastner, New J Phys 14, 043020 (2012)

- Systems are expected to equilibrate

- Compelling experimental evidence (i.e., cold atoms)



Trotzky, Chen, Flesch, McCulloch, Schollwöck, Eisert, Bloch, Nature Physics 8, 325 (2012))

Equilibration

- Systems are expected to equilibrate

$$H = H_S + H_B + V$$

$$\rho(t) = e^{-itH} \rho(0) e^{itH}$$

- In expectation

$$\mathbb{E}_t |\langle S(t)\rangle - \mathbb{E}_t \langle S(t)\rangle| \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d_{\text{eff}}}}, \quad d_{\text{eff}}^{-1} = \sum_k |\langle E_k|\psi(0)\rangle|^4$$

Linden, Popescu, Short, Winter, Phys Rev E 79, 061103 (2009)
Short, Farrelly, New J Phys 14, 013063 (2012)
Gogolin, Eisert, Rep Prog Phys 79, 056001 (2016)

- Can be lower bounded for local Hamiltonians

Farrelly, Brandao, Cramer, arXiv:1610.01337

Equilibration

- Systems are expected to equilibrate

$$H = H_S + H_B + V$$

$$\rho(t) = e^{-itH}\rho(0)e^{itH}$$

- With equilibration times: Based on reasonable assumptions

$$|\langle S(t)\rangle - \mathbb{E}_t\langle S(t)\rangle| \leq$$

Wilming, Krumnow, Goihl, Eisert, this week (2017)
Oliveira, Jonathan, Charalambous, Lewenstein, Riera, same (2017)

Equilibration

- Systems are expected to equilibrate

$$H = H_S + H_B + V$$

$$\rho(t) = e^{-itH} \rho(0) e^{itH}$$

- With equilibration times: Based on reasonable assumptions

$$|\langle S(t) \rangle - \mathbb{E}_t \langle S(t) \rangle| \leq \min \left\{ 2\|A\|, \frac{C_k(\delta, \tau)}{t^k} \right\} + \delta$$

$$= \sum_{E_i \neq E_j} \langle E_i | A | E_j \rangle \langle E_j | \rho | E_i \rangle e^{i(E_i - E_j)t} = \sum_{\Delta \neq 0} z_\Delta e^{i\Delta t}$$

- Use rigorous results on local Hamiltonians
- Convolute, to make harmonic analysis applicable

Wilming, Krumnow, Goihl, Eisert, this week (2017)
Oliveira, Jonathan, Charalambous, Lewenstein, Riera, same (2017)

Equilibration

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- Non-interacting models equilibrate

Gluza, Krumnow, Friesdorf, Gogolin, Eisert, Phys Rev Lett 117 (2016)
Cramer, Dawson, Eisert, Osborne, Phys Rev Lett 100, 030602 (2008)
Calabrese, Essler, Fagotti, Phys Rev Lett 106, 227203 (2011)

- So closed systems equilibrate. Do they thermalise?

Wilming, Krumnow, Goihl, Eisert, this week (2017)
Oliveira, Jonathan, Charalambous, Lewenstein, Riera, same (2017)

Thermalisation

- Systems are expected to equilibrate

$$H = H_S + H_B + V$$

$$\rho(t) = e^{-itH}\rho(0)e^{itH}$$

- Conventionally, yes, eigenstate thermalisation hypothesis

Deutsch, Phys Rev A 43, 2046 (1991)
Srednicki, Phys Rev E 50, 888 (1994)

$$\mathrm{tr}_{S^c}|E_k\rangle\langle E_k|) \sim \mathrm{tr}_{S^c}(e^{-\beta H})$$

Thermalisation

- **Theorem:** For all lattices and all local models, there ex a **threshold temp** (model-independent) above which all correlations cluster exponentially

  Kliesch, Gogolin, Kastoryano, Riera, Eisert, Phys Rev X 4, 031019 (2014)

- **Theorem:** Any local perturbation of a high temperature Gibbs state will locally relax to the reduced of a Gibbs state

  Farrelly, Brandao, Cramer, arXiv:1610.01337

- Conventionally, yes, eigenstate thermalisation hypothesis

  Deutsch, Phys Rev A 43, 2046 (1991)
  Srednicki, Phys Rev E 50, 888 (1994)

$$\mathrm{tr}_{S^c}|E_k\rangle\langle E_k|) \sim \mathrm{tr}_{S^c}(e^{-\beta H})$$

- For initially high temperature baths, this is settled

Thermalisation

$$\rho(t) = e^{-itH}\rho(0)e^{itH}$$

- Non-integrable systems-bath systems are expected to **thermalise** in strong coupling

Recent reviews →

Eisert, Friesdorf, Gogolin, Nature Physics 11, 124 (2015)
Gogolin, Eisert, Rep Prog Phys 79, 056001 (2016)

Protocols

Step A: Switching interaction

- Thermodynamic protocols consist of the three steps

$$H_{SB}^{(i)} = H_S^{(i)} + H_B + V$$

- **A: Turning on/off the interaction**
  - Expected work cost

$$W_{\text{on}}^{(i)} = \operatorname{tr}(\rho_{SB}^{(i)} V) = -W_{\text{off}}^{(i)}$$

as state does not change

- Thermodynamic protocols consist of the three steps

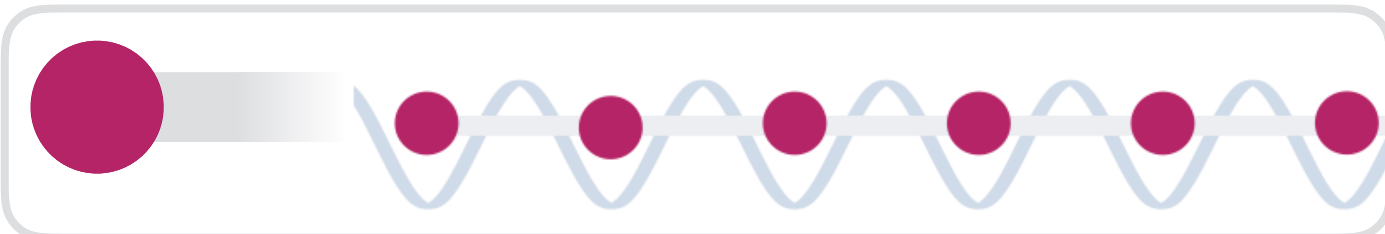
$$H_{SB}^{(i)} = H_S^{(i)} + H_B + V \mapsto H_{SB}^{(i+1)} = H_S^{(i+1)} + H_B + V$$

- **A: Turning on/off the interaction**
- **B: Fast quenches**
  - Expected work cost

$$W^{(i)} = \mathrm{tr}(\rho_S^{(i)}(H_S^{(i)} - H_S^{(i+1)}))$$

Step C: Equilibration

- Thermodynamic protocols consist of the three steps



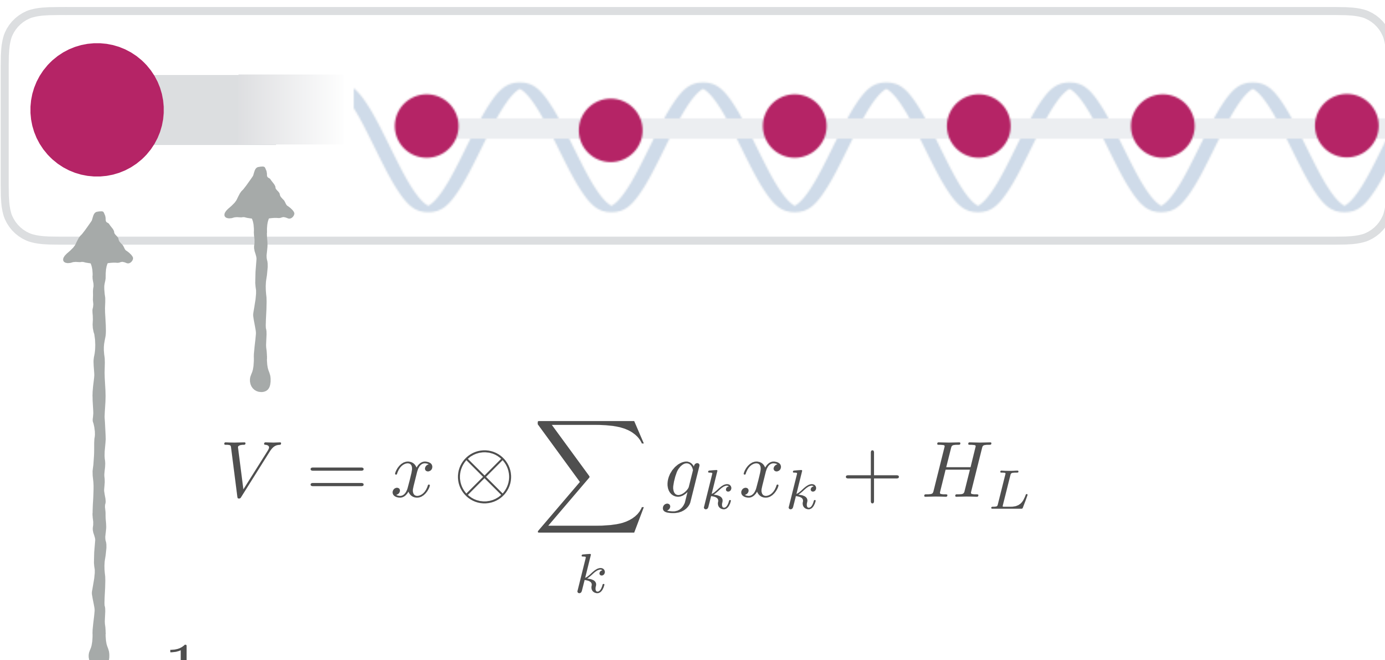
$$\rho_S^{(i+1)} = \mathrm{tr}_B(\omega_\beta(H_{SB}^{(i+1)}))$$

- A: Turning on/off the interaction
- B: Fast quenches
- C: Equilibration

Example

- **Quantum Brownian motion** as ubiquitous example

Caldeira, Leggett, Physica A 121, 587 (1983), Ullersma, Physica 23, 56 (1966)



Harmonic bath

$$V = x \otimes \sum_k g_k x_k + H_L$$

$$H_S = \frac{1}{2}(m\omega^2 x^2 + p^2/m)$$

- Highly relevant in **quantum thermodynamics, opto-mechanics**, etc

Allahverdyan, Nieuwenhuizen, Phys Rev Lett 85, 1799 (2000)
Groeblacher, Trubarov, Prigge, Cole, Aspelmeyer, Eisert, Nature Comm 6, 7606 (2015)
Rivas, Plato, Huelga, Plenio, New J Phys 12, 113032 (2010)
Strasberg, Schaller, Lambert, Brandes, New J Phys 18, 073007 (2016)

Example

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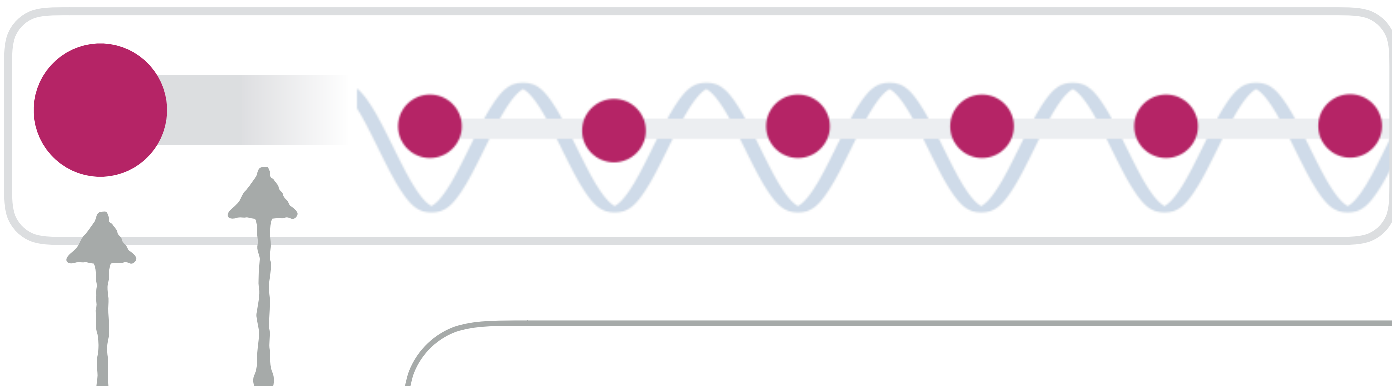


$V = $

$H_S = \frac{1}{2}(m\omega^2$

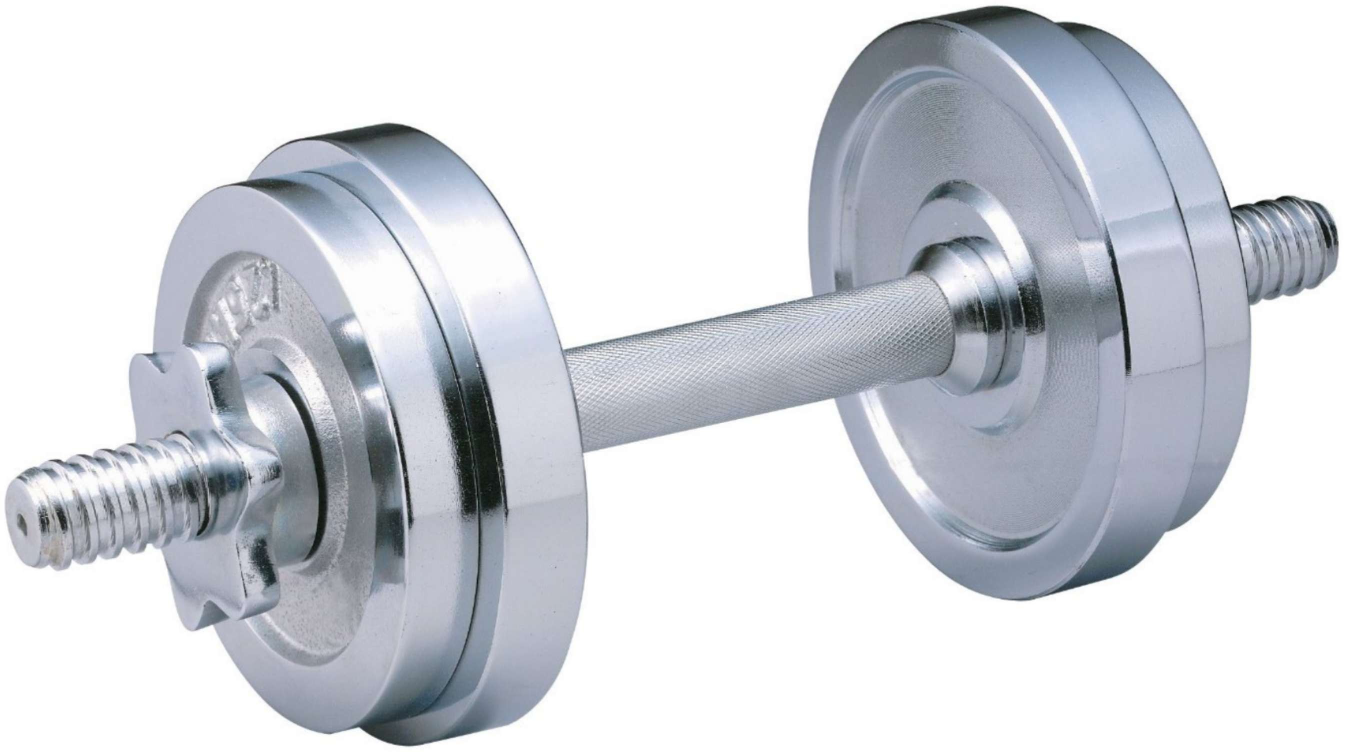
- Highly relevant

Allahverdyan, Nieuwenhuizen,
Groeblacher, Trubarov, Prigge
Rivas, Plato, Huelga, Plenio, N
Strasberg, Schaller, Lambert, B

- Spectral density $J(w) = \frac{\pi}{2}\sum_k \frac{g_k^2}{\omega_k^2}\delta(\omega - \omega_k)$
  - "**Ohmic**": $J(\omega) \sim \omega$ for small ω
  - "Sub-Ohmic": $J(\omega) \sim \omega^k,\ k < 1$
  - "Supra-Ohmic": $J(\omega) \sim \omega^k,\ k > 1$

as approximation in continuum limit

Work extraction



Isothermal processes

- **Isothermal processes:** $N \to \infty$ quenches (B) and equilibrations (C)

Campisi, Talkner, Haenggi, Phys Rev Lett 102, 210401 (2009)
Gallego, Riera, Eisert, New J Phys 16, 125009 (2014)

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Campisi, Talkner, Haenggi, Phys Rev Lett 102, 210401 (2009)
Gallego, Riera, Eisert, New J Phys 16, 125009 (2014)

Weak coupling work extraction

- **Isothermal processes:** $N \to \infty$ quenches (B) and equilibrations (C)

- For an initial state in local equilibrium $\rho_S^{(1)} = \mathrm{tr}_B(\omega_{SB}^{(1)})$ the expected work for $H_{SB}^{(1)}$ and $H_{SB}^{(N)}$ is

$$W^{(\mathrm{isoth})} = F(\omega_{SB}^{(1)}, H_{SB}^{(1)}) - F(\omega_{SB}^{(N)}, H_{SB}^{(N)})$$

- (Non-equilibrium) free energy

$$F(\rho, H) = \mathrm{tr}(H\rho) - S(\rho)/\beta$$

Campisi, Talkner, Haenggi, Phys Rev Lett 102, 210401 (2009)
Gallego, Riera, Eisert, New J Phys 16, 125009 (2014)

Weak coupling work extraction

- **Isothermal processes:** $N \to \infty$ quenches (B) and equilibrations (C)

- For an initial state in local equilibrium $\rho_S^{(1)} = \mathrm{tr}_B(\omega_{SB}^{(1)})$, in **weak coupling**, the expected work for $H_{SB}^{(1)}$ and $H_{SB}^{(N)}$ is

$$W^{(\mathrm{weak})} = F(\rho_S, H_S) - F(\omega_\beta(H_S), H_S)$$

- Simply change in **free energy**

Campisi, Talkner, Haenggi, Phys Rev Lett 102, 210401 (2009)
Gallego, Riera, Eisert, New J Phys 16, 125009 (2014)

Strong coupling work extraction

- To be fair, start with $H_0 := H_{SB}^{(0)} = H_S + H_B$ and $\rho_0 := \rho_{SB}^{(0)} = \rho_S \otimes \omega_\beta(H_B)$

- Start by quenching to $H_S^{(1)}$ and end by quenching back from $H_S^{(N)}$ to H_S

- How is work extraction changed in the strong coupling regime?

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)
Compare Esposito, Lindenberg, Van den Broeck, New J Phys 12, 013013 (2010)

Strong coupling work extraction

- To be fair, start with $H_0 := H_{SB}^{(0)} = H_S + H_B$ and $\rho_0 := \rho_{SB}^{(0)} = \rho_S \otimes \omega_\beta(H_B)$

- Start by quenching to $H_S^{(1)}$ and end by quenching back from $H_S^{(N)}$ to H_S

- **Observation** (Maximum extractable work): The maximum extractable work is

$$W_{\max} = W^{(\mathrm{weak})} - \Delta F_{\min}^{(\mathrm{res})} - \Delta F_{\min}^{(\mathrm{irrev})}$$

with $\Delta F_{\min}^{(\mathrm{irrev})} = \min_{H_S^{(1)}} \Delta F^{(\mathrm{irrev})}$ and $\Delta F_{\min}^{(\mathrm{res})} = \min_{H_S^{(N)}} \Delta F^{(\mathrm{res})}$

- Here, $W^{(\mathrm{weak})}$ is the extractable work in weak coupling and

$$\Delta F^{(\mathrm{irrev})} := F(\rho_0, H_{SB}^{(1)}) - F(\omega_{SB}^{(1)}, H_{SB}^{(1)})$$

$$\Delta F^{(\mathrm{res})} := F(\omega_{SB}^{(N)}, H_0) - F(\omega_{SB}^{(0)}, H_0)$$

with $H_{SB}^{(1)/(N)} = H_S^{(1)/(N)} + H_B + V$

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

- To be fair, start with $H_0 := H_{SB}^{(0)} = H_S + H_B$ and $\rho_0 := \rho_{SB}^{(0)} = \rho_S \otimes \omega_\beta(H_B)$

- **Observation** (Maximum extractable work): The maximum extractable work is

- Up to choice of $H_S^{(1)}$ and $H_S^{(N)}$ cyclic processes of three steps are optimal
  - (i) A quench to $H_S^{(1)}$ followed by turning on V
  - (ii) An isothermal process from $H_S^{(1)}$ to $H_S^{(N)}$
  - (iii) A quench back to the original Hamiltonian H_S after turning off V

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

Strong coupling work extraction

- To be fair, start with $H_0 := H_{SB}^{(0)} = H_S + H_B$ and $\rho_0 := \rho_{SB}^{(0)} = \rho_S \otimes \omega_\beta(H_B)$

- **Observation** (Maximum extractable work): The maximum extractable work is

$$W_{\max} = W^{(\mathrm{weak})} - \Delta F_{\min}^{(\mathrm{res})} - \Delta F_{\min}^{(\mathrm{irrev})}$$

with $\Delta F_{\min}^{(\mathrm{irrev})} = \min_{H_S^{(1)}} \Delta F^{(\mathrm{irrev})}$ and $\Delta F_{\min}^{(\mathrm{res})} = \min_{H_S^{(1)}} \Delta F^{(\mathrm{res})}$

$W \leq W^{(\mathrm{weak})}$ $\quad$ $\Delta F^{(\mathrm{irrev})/(\mathrm{res})} \geq 0$

- Since $F(\rho, H) - F(\omega_\beta(H), H) = S(\rho || \omega_\beta(H)) \geq 0$

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

- To be fair, start with $H_0 := H_{SB}^{(0)} = H_S + H_B$ and $\rho_0 := \rho_{SB}^{(0)} = \rho_S \otimes \omega_\beta(H_B)$

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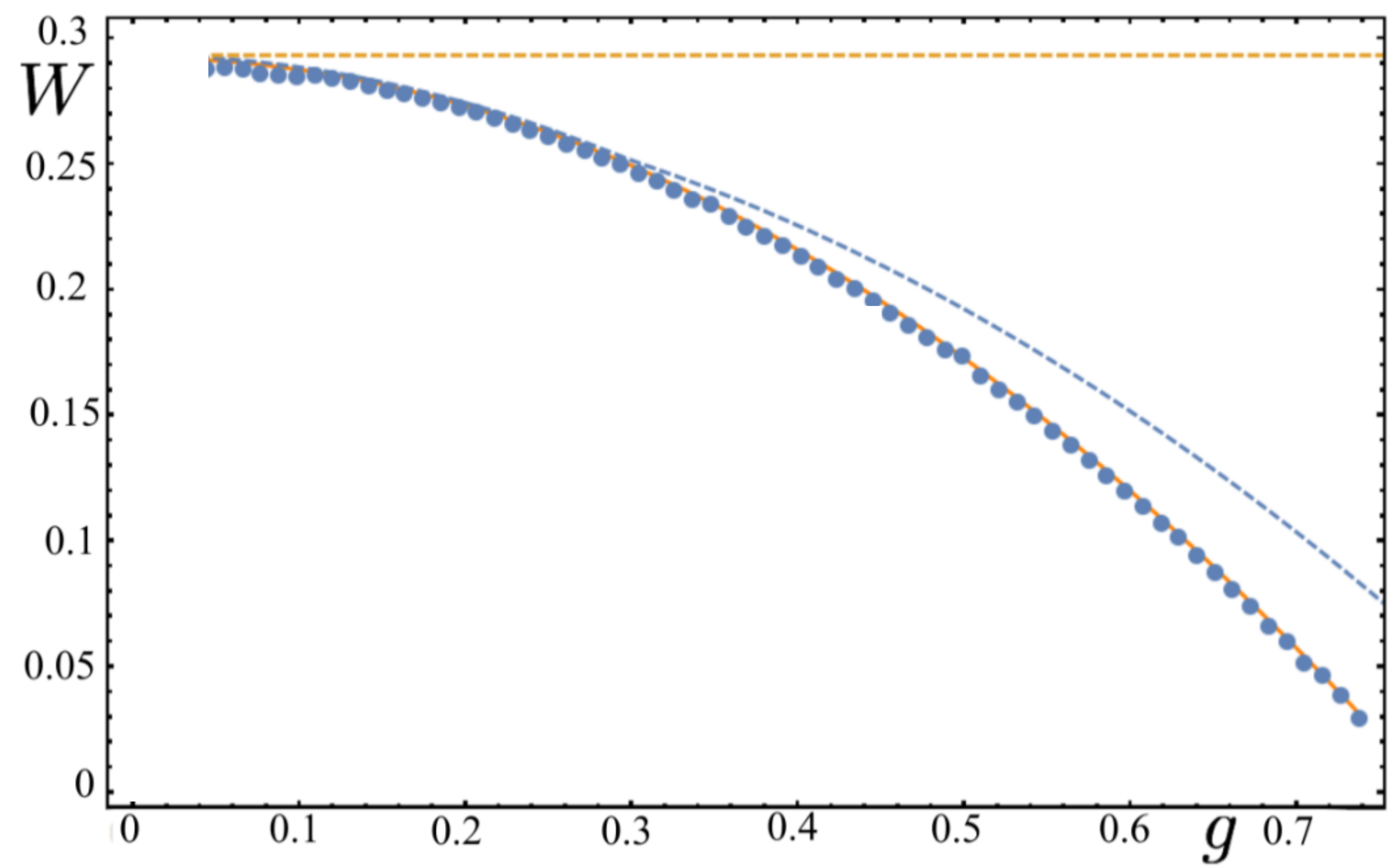
with $\Delta F_{\min}^{(\text{irrev})} = \min_{H_S^{(1)}} \Delta F^{(\text{irrev})}$ and $\Delta F_{\min}^{(\text{res})} = \min_{H_S^{(1)}} \Delta F^{(\text{res})}$

- In weak coupling limit, $H_S^{(1)} = \beta^{-1} \log \rho_S$ and $H_S^{(N)} = H_S$ let penalty vanish

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

- Strong coupling has significant influence on maximum work extraction
- In general $W_{\mathrm{max}} < W^{(\mathrm{weak})}$

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)



Work vs. interaction. Blue dots: Exact results by computing the unitary evolution of SB for a work extraction protocol which becomes optimal in the weak coupling regime. Orange line: same protocol but using our framework. Dashed blue: $W_{\max}$. Dashed orange: $W^{(\text{weak})}$

- Strong…
- In gen…

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Dissipated heat

- Dissipated heat is $Q = T\Delta S - (\Delta F_B^{(\mathrm{res})} + TI(S:B) + \Delta F^{(\mathrm{irrev})})$

- $\Delta S = S(\mathrm{tr}_B \omega_{SB}^{(N)}) - S(\rho_S)$ gain of entropy of S
- $\Delta F_B^{(\mathrm{res})} = S(\mathrm{tr}_S \omega_{SB}^{(N)} || \omega_\beta(H_B))$ is gain of free energy of bath
- $I(S:B)$ is mutual information between system and bath

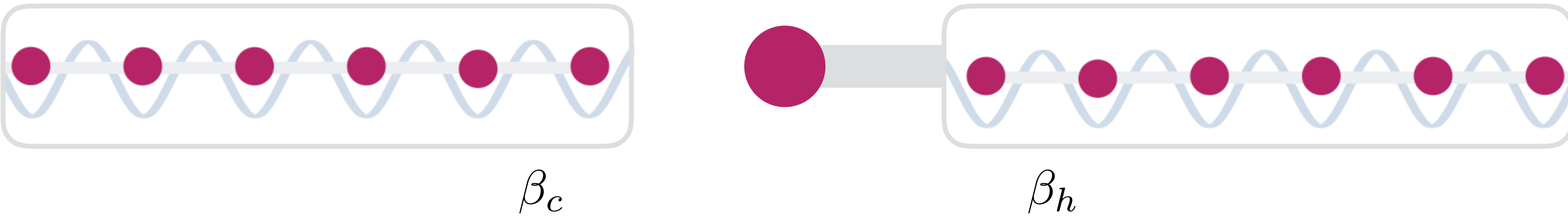
- Writing this as $-T\Delta S = -Q + T\left(S(\omega_B^{(N)}||\omega_B^{(0)}) + I(\omega_{SB}^{(N)}) + S(\rho_0||\omega_{SB}^{(1)})\right)$ gives **Landauer bound**

$$-Q \geq -T\Delta S$$

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

Heat engines

- Efficiency of heat engines in strong coupling

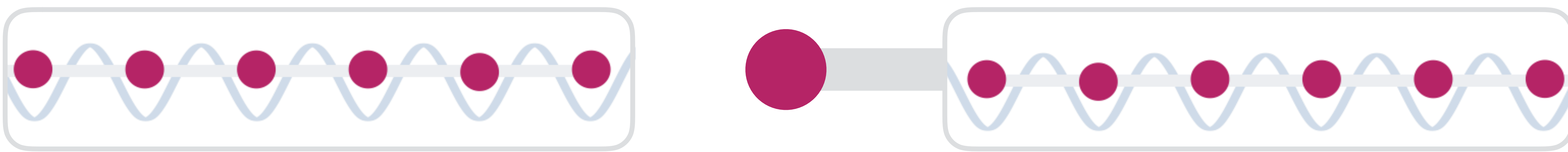


- **Observation** (Corrections to Carnot): The efficiency of a Carnot cycle

$$\eta = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c(1+x_c)}{T_h(1-x_h)} < \eta^C$$

where T_h and T_c are the temperatures of the hot and cold baths, and $\eta^C = 1 - T_c/T_h$

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)



Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

Power in strong coupling

Optimal power?

- Maybe unsurprisingly, strong coupling is a burden for **work extraction** and **efficiency**

- How about the power?

Power from equilibration

- Guess that equilibration time $\tau \sim \|V\|^{-1}$?
- Careful analysis shows $\tau \geq \delta Q / r$
    - δQ energy change of the bath during equilibration
    - $r := \|[H_B, V]\|$ maximum rate in which system and bath can exchange energy

- Implies upper bound to power

Power from equilibration

- **Observation** (Upper bound to power): The power of a Carnot engine arbitrarily strongly coupled to its baths is upper bounded by

$$P := \frac{W}{\delta t} \leq \frac{r_c \eta}{1 - \eta + r_c/r_h} < t_h \eta$$

where W is the work produced in a cycle, Δt is its length in time, η is the efficiency of the machine, $r_{c/h} := \|[H_{B_{c/h}}, V_{c,h}]\|$ is the maximum rate at which the cold/hot bath with Hamiltonian $H_{B_{c/h}}$ can lose/gain energy and $V_{c/h}$ is the interaction that couples S to the cold/hot bath

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)
Compare Shiraishi, Saito, Tasaki, Phys Rev Lett 117, 190601 (2016)
Shiraishi, Tajima, arXiv:1701.01914

Macroscopic limit and non-negligibly weak coupling

Area laws

- In macroscopic limit

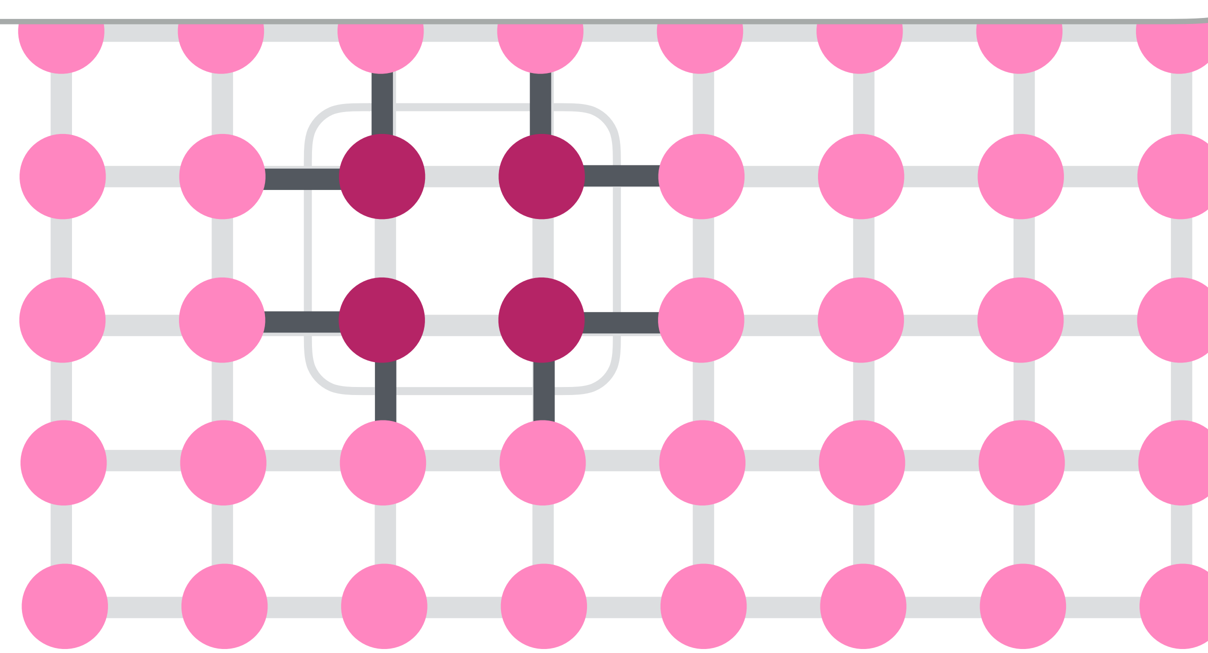
$$\Delta F_{\min}^{(\mathrm{res})/(\mathrm{irrev})} \leq 2\|V\|$$

Perarnau-Llobet, Wilming, Riera, Gallego, Eisert, this week (2017)

$$TI(S:B) \leq 2\|V\|$$ (from mutual info area law)

Wolf, Verstraete, Hastings, Cirac, Phys Rev Lett 100, 070502 (2008)

- As interactions are **"area law"-like** and non-extensive, they are negligible in the macroscopic limit



Non-negligible weak coupling

- Replace V by gV with $0 < g \ll 1$
- Obviously, $\omega_\beta(H)$ is unique minimiser for $\rho \mapsto S(\rho\|\omega_\beta(H))$
- $\Delta F^{(\mathrm{res})}$ and $\Delta F^{(\mathrm{irrev})}$ take unique minimum for $g = 0$

$$W(g) = W^{(\mathrm{weak})} - K_w T g^2 + O(g^3)\,,\ K_w > 0$$

$$Q(g) = T\,\Delta S - K_q T g^2 + O(g^3),\ K_q > 0$$

- Carnot efficiency

$$\eta = \eta^{\mathrm{C}} - g^2 \frac{T_c}{T_h} \left(\frac{T_h K_q^{(h)} + T_h K_q^{(c)}}{Q_h^{(\mathrm{weak})}} \right) + O(g^3)$$

- The second law is “robust” in that lowest order contributions vanish

- Replace V by gV with $0 < g \ll 1$

- Power gives

$$P(g) \leq g \frac{r_c \, \eta^{\mathrm{C}}}{1 - \eta^{\mathrm{C}} + r_c / r_h} - O(g^3)$$

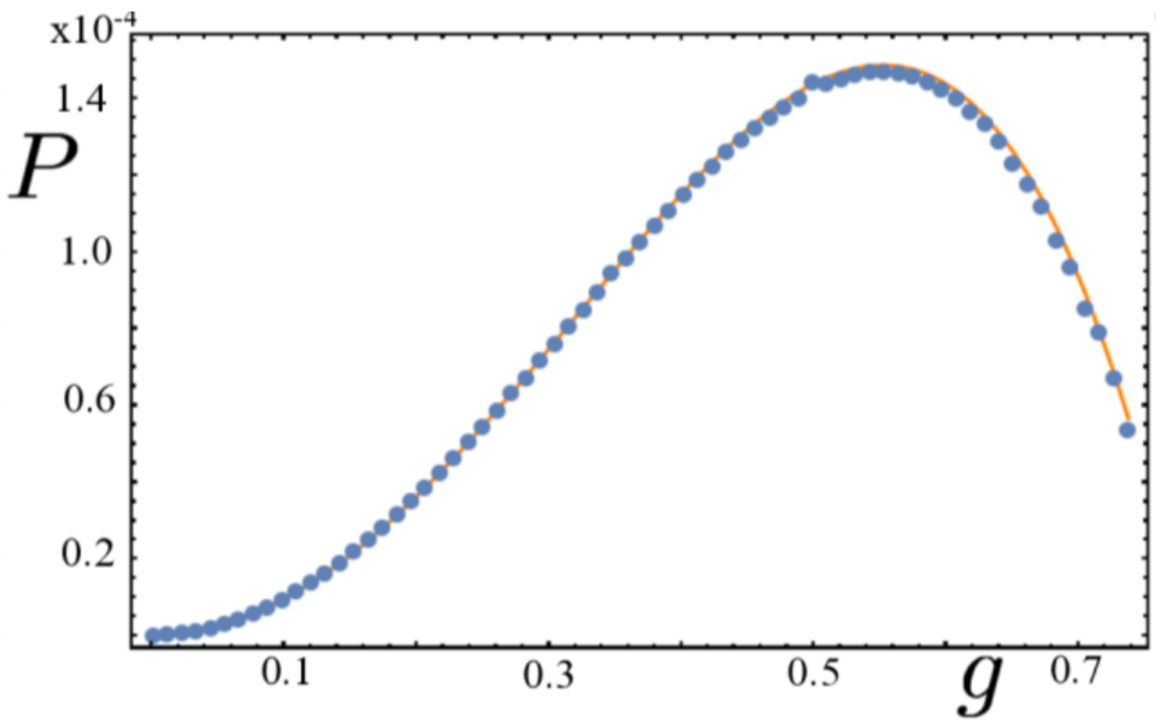
linear for small g and monotone decreasing for large g

- But **optimal power** is obtained for finite coupling strength

- Replace V by gV with $0 < g \ll 1$

- Pow

line



Power vs. interaction. Blue dots: exact unitary evolution. Orange line: Effective description using our framework. Parameters for both figures: B is made up of n = 165 oscillators, $\omega_{\max} = 1.2$and $\beta = 3.5$. For S $\beta_S = 1,\ \omega_S = 1,\ m = 1$. The protocol consistsof 200 quenches, with a waiting time $10/g^2$ when computing the exact unitary dynamics

- But **optimal power** is obtained for finite coupling strength

Non-negligible weak coupling

- But **optimal power** is obtained for finite coupling strength

Role of non-Markovian behavior

- Dynamics is obviously **non-Markovian** and not forgetful in strong coupling

- A process is Markovian if dynamical map $\rho(0) \mapsto \rho(t) = T_t(\rho(0))$ satisfies

$$T_s \circ T_t = T_{s+t},\ \ s,t \geq 0\,,\, T_0 = 1$$

- Deviation from **divisibility** can be made measure…

Wolf, Eisert, Cubitt, Cirac, Phys Rev Lett 101, 150402 (2008)
Rivas, Huelga, Plenio, Phys Rev Lett 105, 050403 (2010)

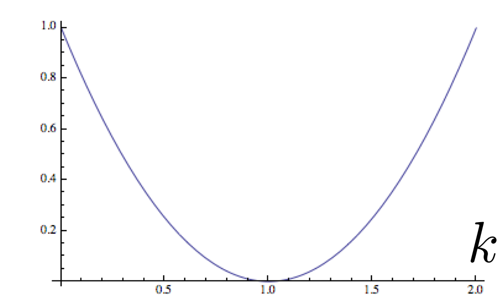
… or one based on **information backflow**

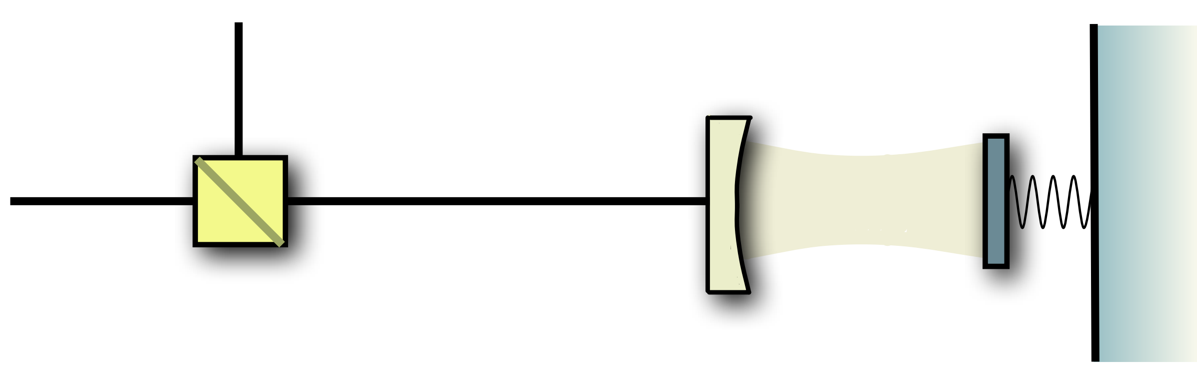
Breuer, Laine, Piilo, Phys Rev Lett 103, 210401 (2009)

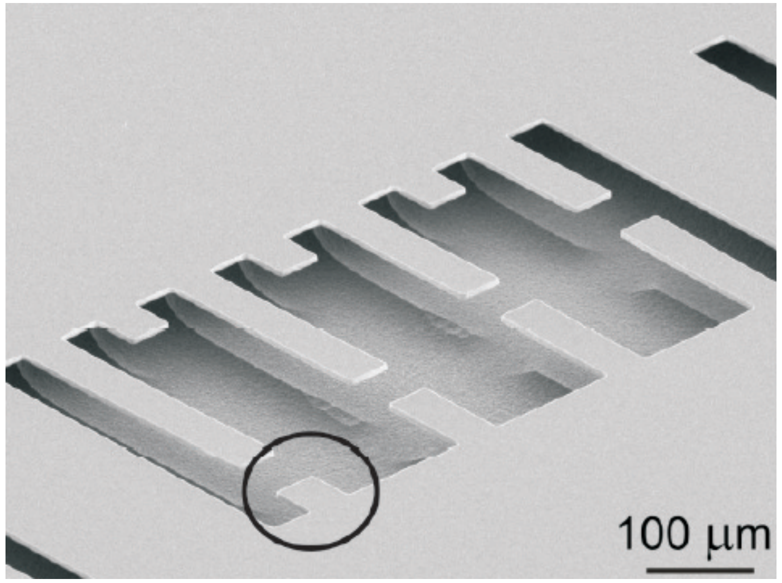
Non-Markovianity

- Dynamics is obviously **non-Markovian** and not forgetful in strong coupling

- Quantitative from infinite divisibility for **quantum Brownian motion**
- Interestingly, $I(\omega) = \omega^k,\ \omega < \Lambda$ approximates Markovian dynamics for weak coupling arbitrarily well precisely for $k = 1$







Groeblacher, Trubarov, Prigge, Cole, Aspelmeyer, Eisert, Nature Comm 6 7606 (2015)

- Dynamics is obviously **non-Markovian** and not forgetful in strong coupling

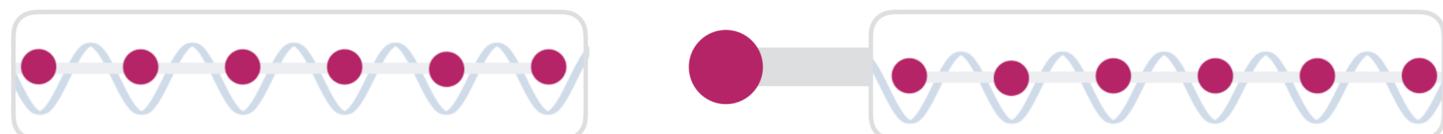
- Strong coupling probes **deviation from Markovian dynamics** in quantum Brownian motion

Outlook

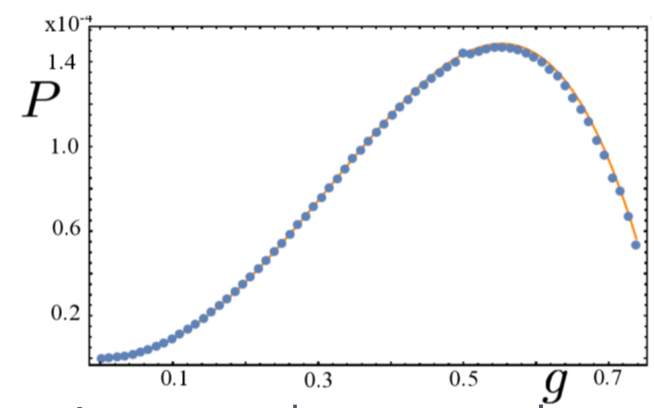
Summary

Strong coupling effects in quantum thermodynamics

Insights from equilibration and thermalisation



Strong coupling detrimental for work extraction



Intermediate coupling optimises power

- **Open questions:**
- Simultaneous couplings?
- Substantiated effect of non-Markovianity
- Strong coupling resource theories?

Thanks for your attention

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http://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-eisert