HELLO

DETECTING NON-MARKOVIANITY OF QUANTUM EVOLUTION

SABRINA MANISCALCO
UNIVERSITY OF TURKU
AALTO UNIVERSITY
FINLAND

TURKU QUANTUM TECHNOLOGY GROUP

TCQP

University of Turku

Turku Quantum Technology

S. Maniscalco

TURKU QUANTUM TECHNOLOGY GROUP

Non-Markovian Processes and Complex Systems

J. Piilo

TCQP

Turku Centre for Quantum Physics

www.tcqp.fi

K.-A. Suominen

Quantum Optics and Quantum Dynamics

I. Vilja

Cosmology

DETECTING NON-MARKOVIANITY OF QUANTUM EVOLUTION

with Dariusz Chruscinski and Chiara Macchiavello

Phys. Rev. Lett. 118, 080404 (2017)

“a precise definition of strong coupling and non-Markovianity for open (quantum) systems”

"a precise definition of strong coupling and non-Markovianity for open (quantum) systems"

“a precise definition of strong coupling and non-Markovianity for open (quantum) systems”

“a precise definition of strong coupling and non-Markovianity for open (quantum) systems”

DETECTING

DETECTING

using the properties of the spectrum of the dynamical map

a dynamical analogue to entanglement witnesses

in an experimentally friendly way

Definition(s) of non-Markovianity

The problem of detection

Spectra of dynamical maps

Geometric interpretation

Geometric interpretation

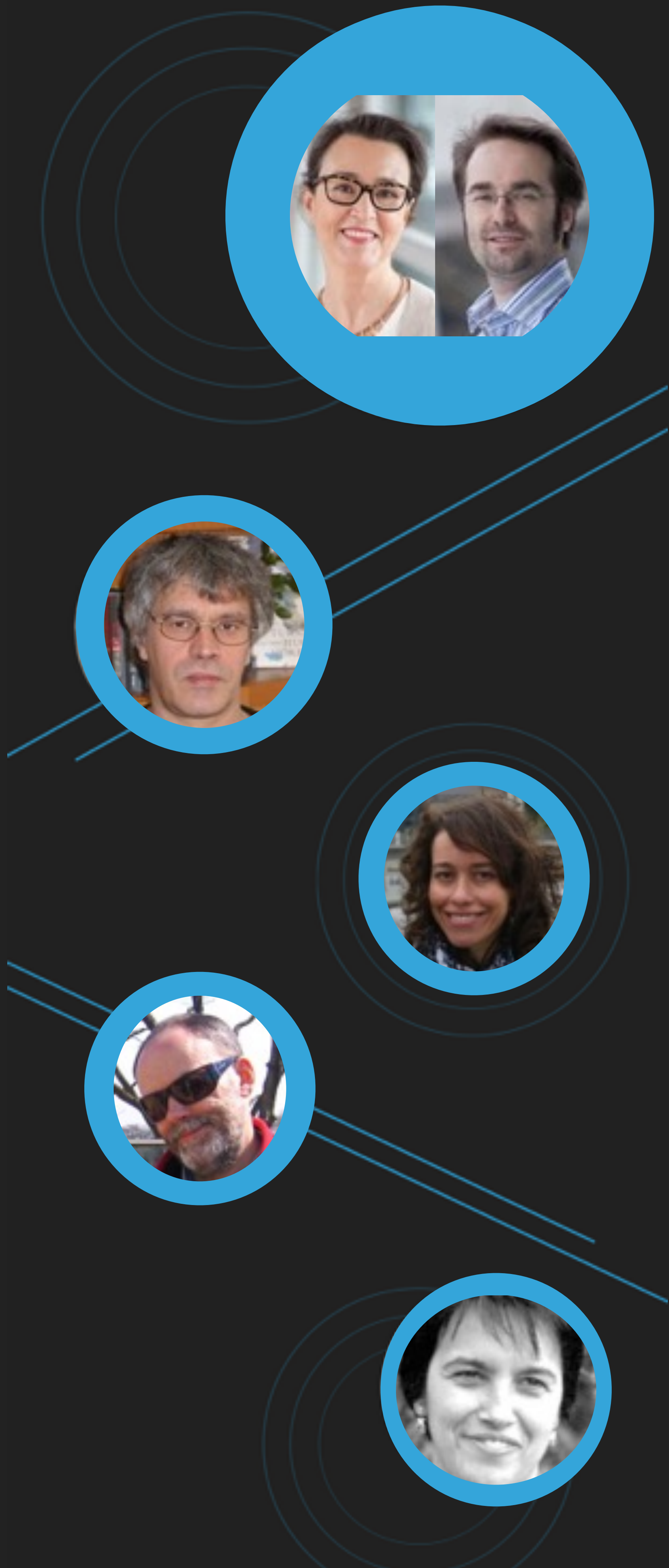
A dynamical analogue of entanglement witnesses

Examples

From zero to infinity

NON-MARKOVIAN QUANTUM DYNAMICS

REVIEWS



A. Rivas, et al., Rep. Prog. Phys. 77, 094001 (2014)

H. -P. Breuer, et al., Rev. Mod. 88, 021002 (2016)

I. de Vega and D. Alonso, Rev. Mod. Phys. 89, 015001 (2017)

PAPERS

D. Chruscinski and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

D. Chruscinski, et al., Phys. Rev. Lett. 118, 080404 (2017)

- **Definition(s) of non-Markovianity**
- The problem of detection
- Spectra of dynamical maps
- Geometric interpretation

DYNAMICAL MAP

$$\rho_t = \Lambda_t \rho_0 \qquad \Lambda_t = V_{t,s} \Lambda_s$$

CP-divisibility ⟷ Lindblad-like time local ME

$V_{t,s}$ is CP $\qquad \mathbb{I}_n \otimes \Lambda_t$ positive $\forall n$

k-divisibility

$V_{t,s}$ is k-positive $\qquad \mathbb{I}_n \otimes \Lambda_t$

positive for n =1,2. . . k
not positive for n > k

CP-divisibility

$V_{t,s}$ is CP $\qquad \mathbb{I}_n \otimes \Lambda_t$ positive $\forall n$

k-divisibility

$V_{t,s}$ is k-positive $\qquad \mathbb{I}_n \otimes \Lambda_t$

positive for $n = 1, 2 \ldots k$

not positive for $n > k$

P-divisibility $\qquad$ for $n = 1$

$V_{t,s}$ is positive

NON-MARKOVIANITY

non-Markovianity ⟷ $\Lambda_t = V_{t,s}\Lambda_s$

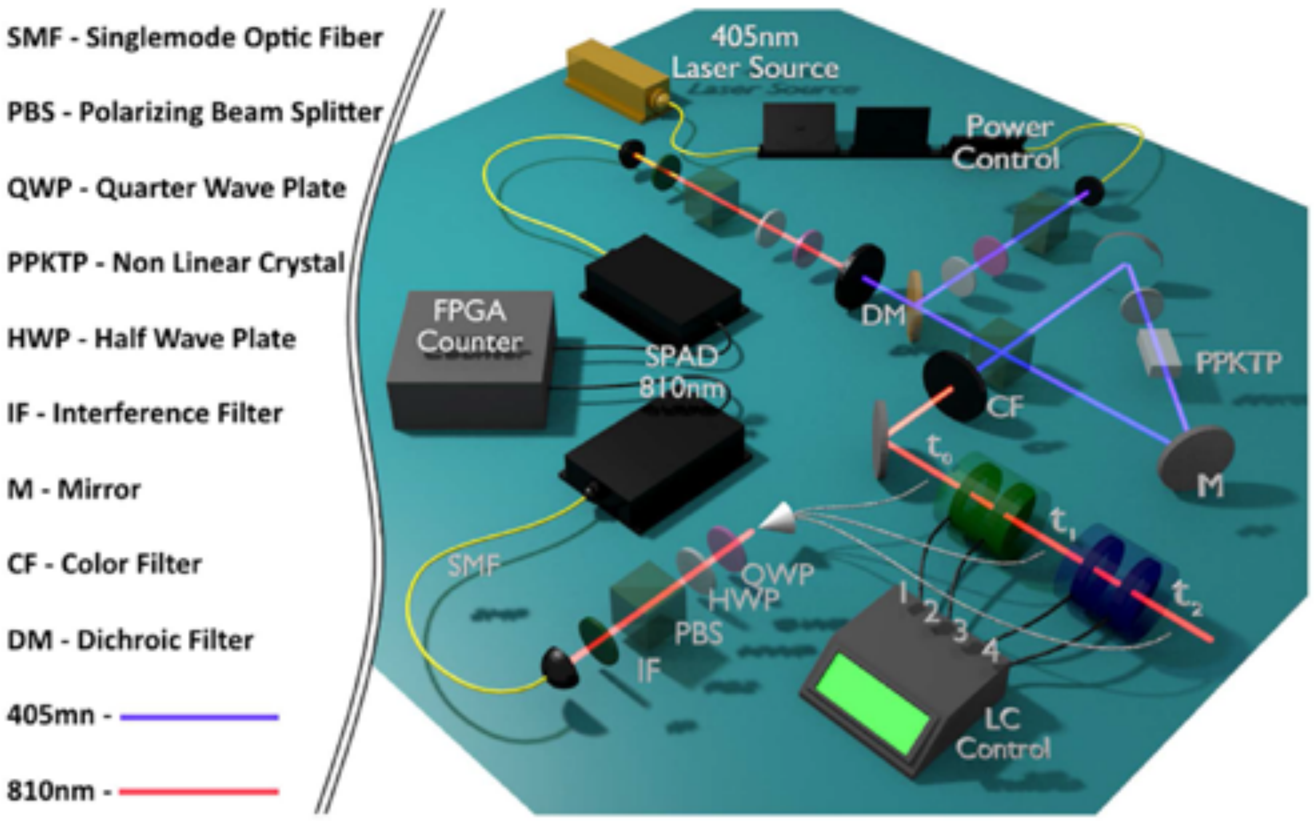
not CP

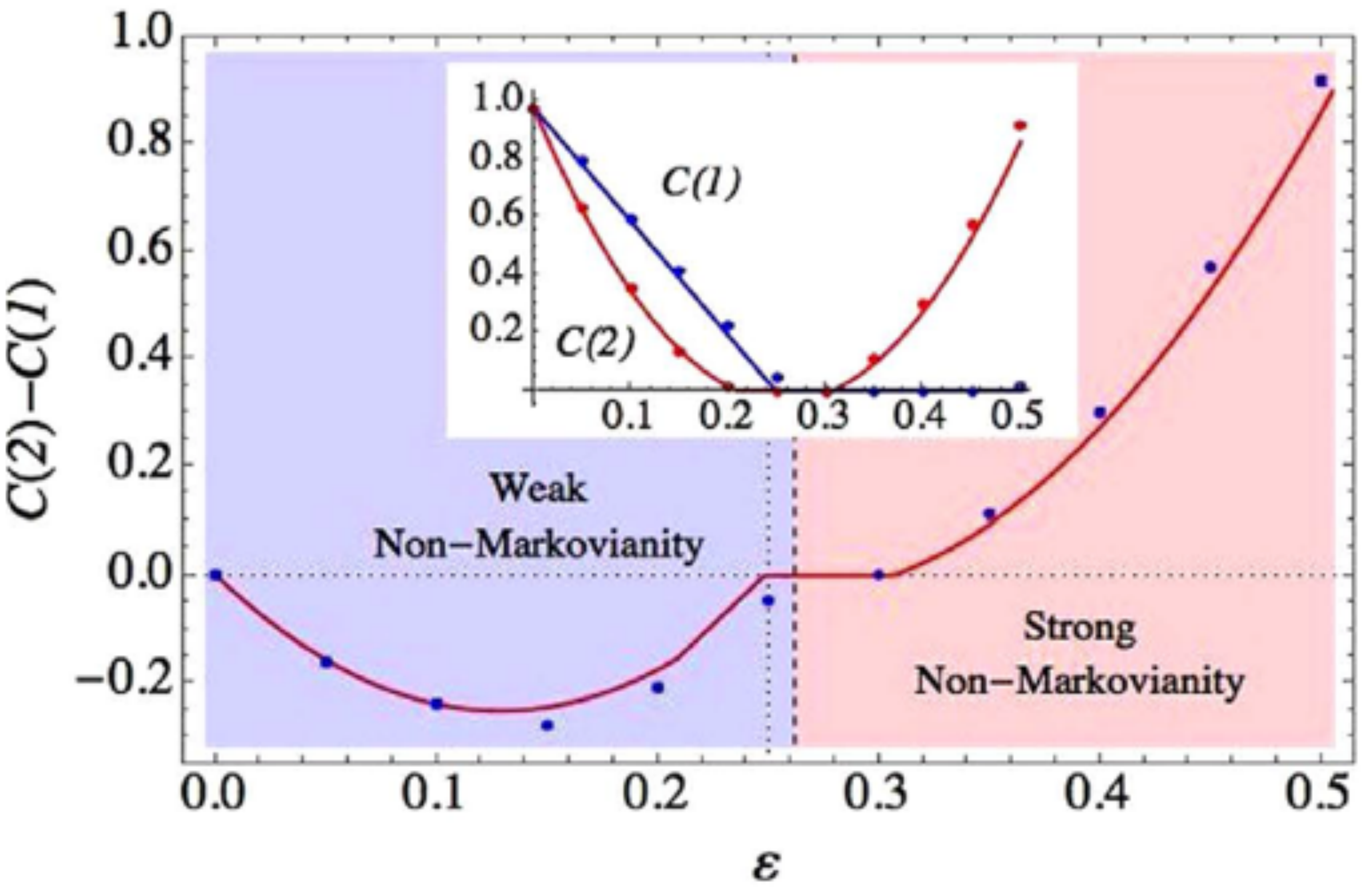
non-Markovinity hierarchy ⟷ not k-positive

essential non-Markovinity ⟷ not even positive

D. Chruscinski and S. Maniscalco, Phys. Rev. Lett. 112, 120404 (2014)

EXPERIMENT





N.K. Bernardes, et al., Scientific Reports 5, 17520 (2015)

REMEMBER!

non-Markovianity is a property of the dynamical map (t-parametrised family of CPTP maps)

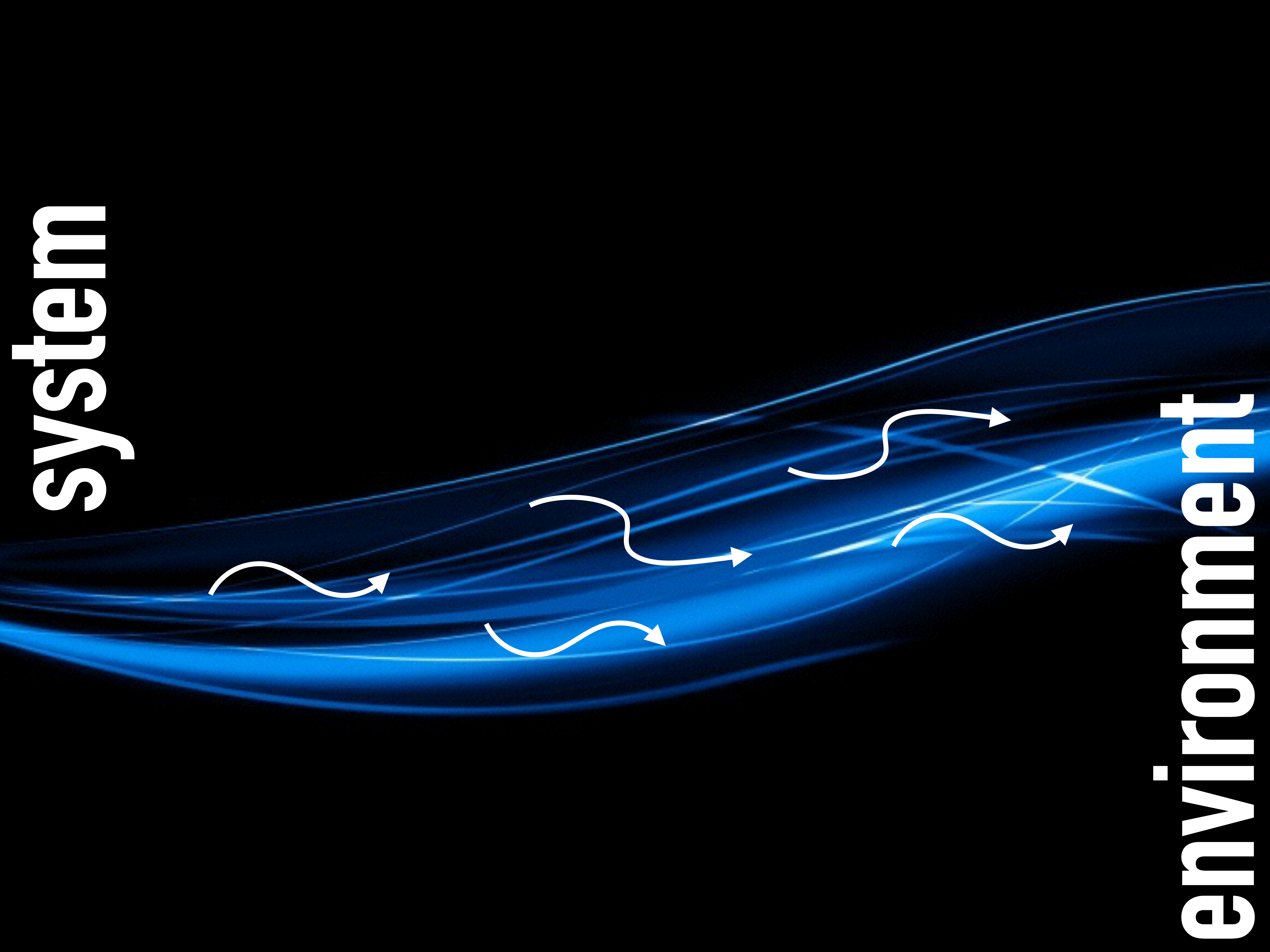
MEMORY EFFECTS & INFORMATION FLOW

BLP MARKOVIANTY

Markovian dynamics

$$\frac{d}{dt}||\Lambda_t[\rho_1 - \rho_2]||_1 \leq 0$$

distinguishability between states decreases monotonically for ANY of initial state


system
environment

BLP NON-MARKOVIANITY

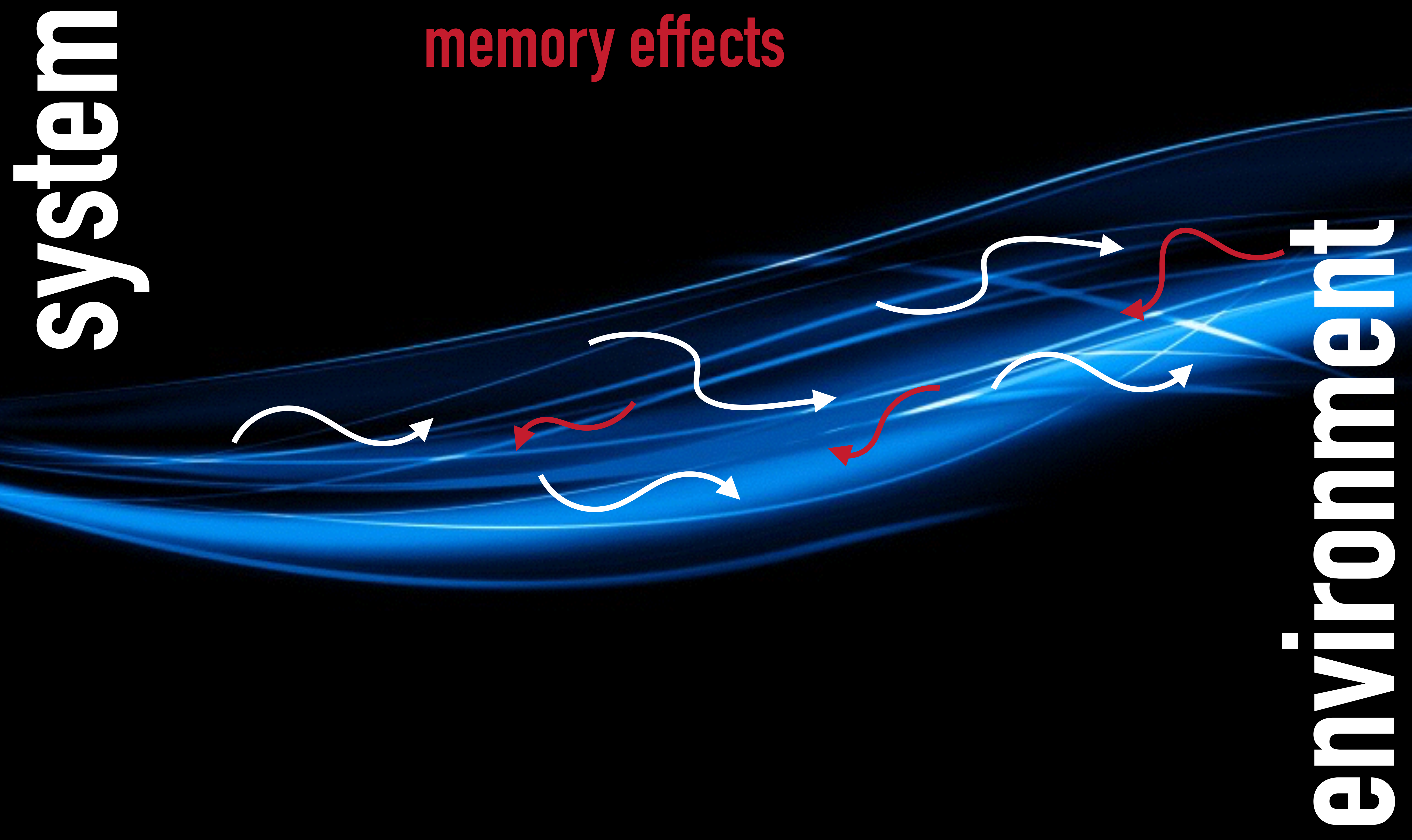
Non-Markovian dynamics

there exist at least one pair of states and time interval for which

$$\frac{d}{dt}||\Lambda_t[\rho_1 - \rho_2]||_1 > 0$$

information backflow

memory effects



THE LINK

$$\Lambda_t = V_{t,s} \Lambda_s$$

is CP

$$\frac{d}{dt} ||\Lambda_t[\rho_1 - \rho_2]||_1 \leq 0$$

BLP Markovian

NOT AN IFF

THE LINK

$$\Lambda_t = V_{t,s} \Lambda_s$$

is CP

$$\frac{d}{dt} ||\Lambda_t[\rho_1 - \rho_2]||_1 \leq 0$$

BLP Markovian

THE LINK

$$\Lambda_t = V_{t,s} \Lambda_s$$

is NOT CP

↑

$$\frac{d}{dt} ||\Lambda_t[\rho_1 - \rho_2]||_1 > 0$$

BLP Non–Markovian

SAME STORY FOR...

- Fisher information
- Channel capacity
- Mutual information
- Relative entropy

Definition(s) of non-Markovianity

The problem of detection

Spectra of dynamical maps

Geometric interpretation

THE DIFFICULTY

there exist **at least one** pair of states and time interval for which

$$\frac{d}{dt}||\Lambda_t[\rho_1 - \rho_2]||_1 > 0$$

take a pair (or a state)

- evolve
- check for increase of trace distance

THE DIFFICULTY

take a pair (or a state)

→ evolve → check for increase of trace distance

(or any other quantifier of information)

NO

REPEAT

CAN WE DO ANY BETTER?

Can we define a witness that is related to the (spectral) properties of the dynamical map?

WITNESS FOR PROPERTIES OF QUANTUM CHANNELS

$$\rho_t = \Lambda_t \rho_0$$

t-parametrised family of quantum channels

C. Macchiavello and M. Rossi, Phys. Rev. A 88, 042335 (2013)

Choi-Jamolkowski isomorphism

1 to 1 correspondence between CPTP maps Φ in $\mathcal{B}(\mathcal{H})$ and bipartite states

$\dim\mathcal{H} = d$

$$\Phi \longleftrightarrow (\Phi \otimes \mathbb{I})|\alpha\rangle\langle\alpha|$$

channel — **Choi state**

$$|\alpha\rangle = \frac{1}{\sqrt{d}} \sum_{k=1}^{d} |k\rangle|k\rangle$$

C. Macchiavello and M. Rossi, Phys. Rev. A 88, 042335 (2013)

Choi–Jamolkowski isomorphism

1 to 1 correspondence between CPTP maps Φ in $\mathcal{B}(\mathcal{H})$ and bipartite states

$\mathrm{dim}\mathcal{H} = d$

$\Phi \longleftrightarrow (\Phi \otimes \mathbb{I})|\alpha\rangle\langle\alpha|$

channel — Choi state

Consider properties that are based on the convex structure of the channel

Entanglement witness

C. Macchiavello and M. Rossi, Phys. Rev. A 88, 042335 (2013)

PROBLEM(S)!

Can you see why?

Let's see what we can say anyway…

THREE CLASSES

Unital maps

$$\Lambda_t(\mathbb{I}) = \mathbb{I}$$

Normal maps

$$\Lambda_t \Lambda_t^* = \Lambda_t^* \Lambda_t$$

Λ_t^*
dual map

Commutative maps

$$\Lambda_t \Lambda_s = \Lambda_s \Lambda_t$$

THREE CLASSES

Unital maps

$$\Lambda_t(\mathbb{I}) = \mathbb{I}$$

Example: Pauli channels

$$\mathcal{L}_t[\rho] = \frac{1}{2}\sum_{k=1}^{3} \gamma_k(t)(\sigma_k \rho \sigma_k - \rho)$$

$$\Lambda_t[\rho] = \sum_{\alpha} p_\alpha(t)\sigma_\alpha \rho \sigma_\alpha$$

CP-divisible iff

$$\gamma_k(t) \geq 0$$

THREE CLASSES

Normal maps

$$\Lambda_t \Lambda_t^* = \Lambda_t^* \Lambda_t$$

Example: Weyl channels

extension of Pauli channels to d-dimensional systems (d>2)

$$\mathcal{L}_t[\rho] = \sum_{k+l>0}^{d-1} \gamma_{kl}(t)[U_{kl}\rho U_{kl}^\dagger - \rho]$$

$$U_{kl} = \sum_{m=0}^{d-1} \omega^{mk} |m\rangle\langle m+l|$$

$$\Lambda_t[\rho] = \sum_{k,l=0}^{d-1} p_{kl}(t) U_{kl}\rho U_{kl}^\dagger$$

$$\omega = e^{2\pi i/d}$$

THREE CLASSES

Commutative maps

$$\Lambda_t \Lambda_s = \Lambda_s \Lambda_t$$

but not normal!

Example: Amplitude damping

$$\mathcal{L}_t[\rho] = -\frac{is(t)}{2}[\sigma_+\sigma_-, \rho] + \gamma(t)(\sigma_-\rho_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\})$$

CP-divisible iff $\gamma(t) \geq 0$

Definition(s) of non-Markovianity

The problem of detection

Spectra of dynamical maps

Geometric interpretation

Matrix representation

$$\Lambda_t \to F_{\alpha\beta}(t) := \mathrm{Tr}(G_\alpha \Lambda_t[G_\beta])$$

G_α **orthonormal basis in** $\mathcal{B}(\mathcal{H})$

Gell-Mann matrices with $G_0 = \mathbb{I}/\sqrt{d}$ **and** $G_\alpha \quad \alpha = 1, \ldots, d^2 - 1$

$$F(t) = \left(\begin{array}{c|c} 1 & 0 \\ \hline \mathbf{q}_t & \Delta_t \end{array} \right)$$

$\mathbf{q_t} \in \mathbb{R}^{d^2-1}$

$(d^2-1) \times (d^2-1)$ **real matrix**

Matrix representation

$$F(t) = \left(\begin{array}{c|c} 1 & 0 \\ \hline \mathbf{q}_t & \Delta_t \end{array} \right)$$

singular value decomposition

$$F(t) = \mathcal{O}_1(t)\Sigma(t)\mathcal{O}_2^{-1}(t)$$

rotations

diagonal matrix containing singular values of $F(t)$

Matrix representation

$$F(t) = \left(\begin{array}{c|c} 1 & 0 \\ \hline \mathbf{q}_t & \Delta_t \end{array}\right)$$

$$F(t) = \mathcal{O}_1(t)\Sigma(t)\mathcal{O}_2^{-1}(t)$$

singular values

$$0 < s_k(t) \leq 1$$

$$s_0(t) = 1$$

$$|\mathrm{Det} F(t)| = |\mathrm{Det}\Delta_t| = \mathrm{Det}\Sigma(t) = \prod_{k=1}^{d^2-1} s_k(t)$$

Definition(s) of non-Markovianity

The problem of detection

Spectra of dynamical maps

Geometric interpretation

GENERALISED BLOCH REPRESENTATION

$$\rho = \frac{1}{d}(\mathbb{I} + \sum_{\alpha=1}^{d^2-1} x_\alpha G_\alpha)$$

action of the dynamical map

$$\mathbf{x} \to \mathbf{x}_t = \Delta_t \mathbf{x} + \mathbf{q}_t$$

BLP Markovianity

fully controlled by Δ_t

P-divisibility

controlled by both Δ_t and $\mathbf{q}_t$

CP-divisibility

$$F(t) = F(t,s)F(s)$$

$$\Delta_t = \Delta_{t,s}\Delta_s \qquad \mathbf{q}_t = \mathbf{q}_{t,s} + \Delta_{t,s}\mathbf{q}_s$$

GENERALISED BLOCH REPRESENTATION

$$\rho = \frac{1}{d}(\mathbb{I} + \sum_{\alpha=1}^{d^2-1} x_\alpha G_\alpha)$$

action of the dynamical map

$$\mathbf{x} \to \mathbf{x}_t = \Delta_t \mathbf{x} +$$

UNITAL

BLP Markovianity

fully controlled by Δ_t

P-divisibility

controlled by both Δ_t and $\mathbf{q}_t$

CP-divisibility

$$F(t) = F(t,s)F(s)$$

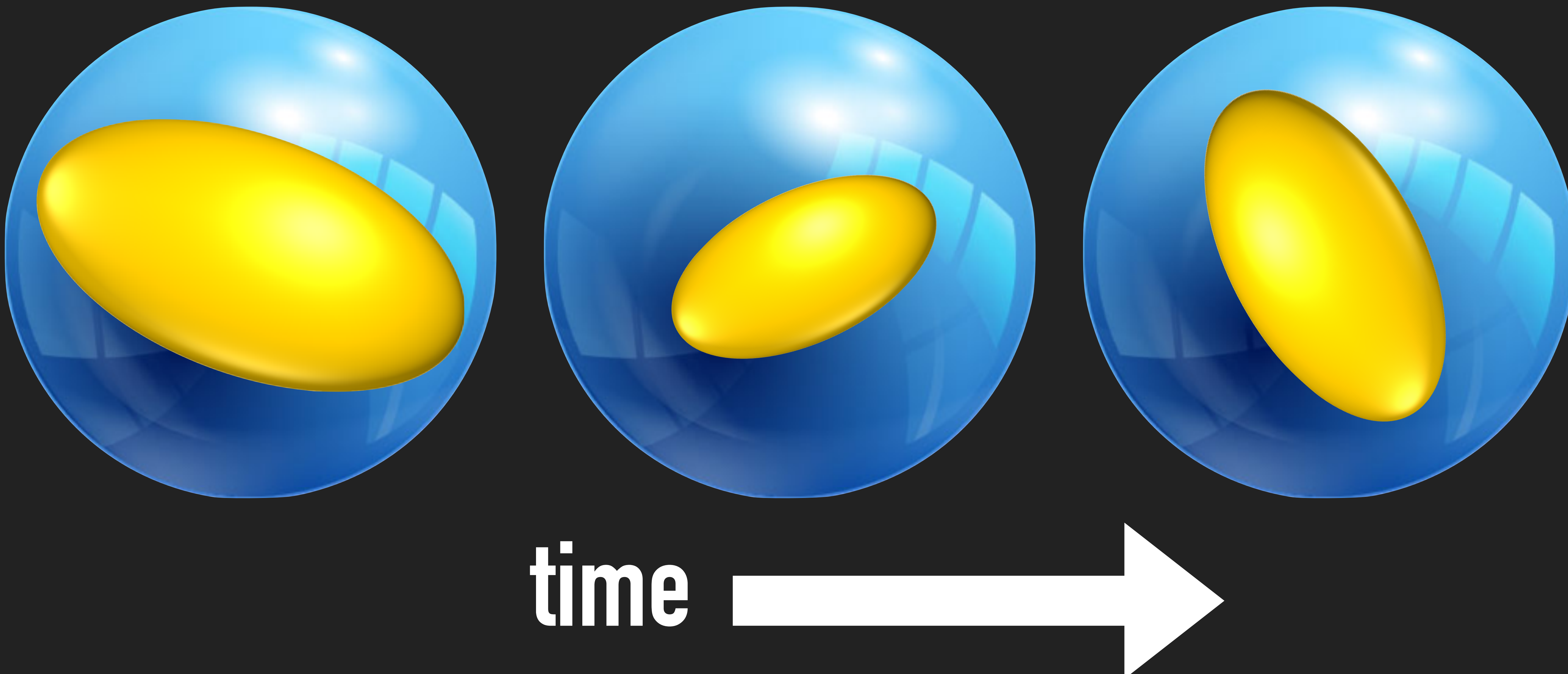
$$\Delta_t = \Delta_{t,s}\Delta_s \qquad \mathbf{q}_t = \quad \Delta_{t,s}\mathbf{q}_s$$

A WEAK WITNESS

based on spectral properties of the map

with a clear geometric interpretation

VOLUME OF ACCESSIBLE STATES



S. Lorenzo, et al., Phys. Rev. A 88, 020102(R) (2013)

VOLUME OF ACCESSIBLE STATES

B **space of the density operators**

$B(t) = \Lambda_t[B]$ **body of accessible states**

P-divisibility $\Rightarrow$ $$\frac{d}{dt}\mathrm{Vol}(t) \leq 0,$$

$$\mathrm{Vol}(t)/\mathrm{Vol}(0) = |\mathrm{Det}F(t)| = |\mathrm{Det}\Delta_t| = \mathrm{Det}\Sigma(t) = \prod_{k=1}^{d^2-1} s_k(t)$$

S. Lorenzo, et al., Phys. Rev. A 88, 020102(R) (2013)

WHAT MORE?

THREE CLASSES

Unital maps

$$\Lambda_t(\mathbb{I}) = \mathbb{I}$$

Normal maps

$$\Lambda_t \Lambda_t^* = \Lambda_t^* \Lambda_t$$

Λ_t^* dual map

Commutative maps

$$\Lambda_t \Lambda_s = \Lambda_s \Lambda_t$$

Unital maps

P-divisibility $\Rightarrow$ $$\frac{d}{dt}s_k(t) \leq 0$$

The body of accessible states shrinks monotonically $B(t) \subset B(s)$ $t > s$

stronger conditions than $$\frac{d}{dt}\mathrm{Vol}(t) = \frac{d}{dt}\prod_{k=1}^{d^2-1} s_k(t)\mathrm{Vol}(0)$$

Example: Pauli channel $B(t)$ $$\frac{x_1^2}{s_1^2(t)} + \frac{x_2^2}{s_2^2(t)} + \frac{x_3^2}{s_3^2(t)} \leq 1$$

Normal maps

$$\Lambda_t \Lambda_t^* = \Lambda_t^* \Lambda_t$$ ➡ $F(t)$ normal matrix

$\mathbf{q}_t = 0$ and Δ_t normal

normal maps are always unital

$$s_k(t) = |\lambda_k(t)|$$

eigenvalues of the dynamical map

P-divisibility ➡ $$\frac{d}{dt}|\lambda_k(t)| \leq 0$$

Normal maps

$$\Lambda_t \Lambda_t^* = \Lambda_t^* \Lambda_t$$

spectral representation of normal maps

$$\Lambda_t[\rho] = \sum_{k=0}^{d^2-1} \lambda_k(t) F_k(t) \mathrm{Tr}(F_k^\dagger(t)\rho)$$

same for the dual map

with $\lambda_k(t) \to \lambda_k^*(t)$

Hermitian maps $\lambda_k^*(t) = \lambda_k(t)$

P-divisibility → $\frac{d}{dt}\lambda_k(t) \leq 0$

Geometric interpretation

A dynamical analogue of entanglement witnesses

Examples

From zero to infinity

Entanglement witnesses

W in $\mathcal{H} \otimes \mathcal{H}$ such that $\mathrm{Tr}(W \rho_{sep}) \geq 0$

$\mathrm{Tr}(W \rho) < 0$

for some entangled states

any such operator may be constructed as

$$W := (\mathbb{I} \otimes \Phi)|\alpha\rangle\langle\alpha| \quad \text{with} \quad |\alpha\rangle = \frac{1}{\sqrt{d}} \sum_{k=1}^{d} |k\rangle|k\rangle$$

positive but not CP map

Non-Markovianity witness

for Hermitian maps

P-divisibility → $$\frac{d}{dt}\langle\alpha|(\mathbb{I}\otimes\Lambda_t)[P^+]|\alpha\rangle \leq 0$$

with $P^+ = |\alpha\rangle\langle\alpha|$

$$|\alpha\rangle = \frac{1}{\sqrt{d}}\sum_{k=1}^{d}|k\rangle|k\rangle$$

Experimentally friendly!

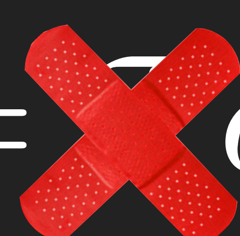
For 2-dimensional systems it requires only a single projection onto a Bell state (useful for linear optical systems)

Commutative maps $\Lambda_t \Lambda_s = \Lambda_s \Lambda_t$

more general than unital and normal

Commutativity implies time-independent eigenvectors of the dynamical map and its dual

$$\Lambda_t[F_\alpha] = \lambda_\alpha(t) F_\alpha \ , \quad \Lambda_t^*[G_\alpha] = \lambda_\alpha^*(t) G_\alpha$$

$\Lambda_t =$  $e^{\int_0^t \mathcal{L}_\tau d\tau}$

eigenvalues of $\mathcal{L}_t$

eigenvalues of Λ_t

$$\lambda_\alpha(t) = e^{\int_0^t \mu_\alpha(\tau) d\tau}$$

Commutative maps $\Lambda_t \Lambda_s = \Lambda_s \Lambda_t$

P-divisibility $\Rightarrow$ $\frac{d}{dt}|\lambda_\alpha(t)| \leq 0$ $\quad \mathrm{Re}\, \mu_\alpha(t) \leq 0$

$$\langle \alpha | (\mathbb{I} \otimes \mathcal{L}_t)[P^+] | \alpha \rangle \leq 0$$

compare with the witness for Hermitian maps

$$\frac{d}{dt} \langle \alpha | (\mathbb{I} \otimes \Lambda_t)[P^+] | \alpha \rangle \leq 0$$

Geometric interpretation

A dynamical analogue of entanglement witnesses

Examples

From zero to infinity

EXAMPLE

Commutative but generally not normal

$$\mathcal{L}_t[\rho] = -\frac{is(t)}{2}[\sigma_+\sigma_-, \rho] + \gamma(t)(\sigma_-\rho_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\})$$

$$\Lambda_t[\rho] = \begin{pmatrix} \rho_{11} + (1 - |G(t)|^2)\rho_{22} & G(t)\rho_{12} \\ G^*(t)\rho_{21} & |G(t)|^2\rho_{22} \end{pmatrix}$$

$$s(t) = -2\mathrm{Im}\frac{\dot{G}(t)}{G(t)}$$

$$\gamma(t) = -2\mathrm{Re}\frac{\dot{G}(t)}{G(t)}$$

Eigenvalues

$$\lambda_0(t) = 1 \quad \lambda_1(t) = G(t) \quad \lambda_2(t) = G^*(t) \quad \lambda_3(t) = |G(t)|^2$$

depends on $J(\omega)$

$$\frac{d}{dt}|\lambda_k(t)| \leq 0 \Rightarrow \gamma(t) \geq 0 \Rightarrow$$ **Markovianity**

EXAMPLE

Commutative but generally not normal

$$\mathcal{L}_t[\rho] = -\frac{is(t)}{2}[\sigma_+\sigma_-, \rho] + \gamma(t)(\sigma_-\rho_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\})$$

$$\Lambda_t[\rho] = \begin{pmatrix} \rho_{11} + (1 - |G(t)|^2)\rho_{22} & G(t)\rho_{12} \\ G^*(t)\rho_{21} & |G(t)|^2\rho_{22} \end{pmatrix}$$

$$s(t) = -2\mathrm{Im}\frac{\dot{G}(t)}{G(t)}$$

$$\gamma(t) = -2\mathrm{Re}\frac{\dot{G}(t)}{G(t)}$$

Eigenvalues

$$\lambda_0(t) = 1 \quad \lambda_1(t) = G(t) \quad \lambda_2(t) = G^*(t) \quad \lambda_3(t) = |G(t)|^2$$

depends on $J(\omega)$

$$\frac{d}{dt}|\lambda_k(t)| \leq 0 \iff \gamma(t) \geq 0 \iff$$ **Markovianity**

EXAMPLE

Commutative but generally not normal

$$\Lambda_t[\rho] = \begin{pmatrix} \rho_{11} + (1 - |G(t)|^2)\rho_{22} & G(t)\rho_{12} \\ G^*(t)\rho_{21} & |G(t)|^2\rho_{22} \end{pmatrix}$$

Eigenvalues

$$\lambda_0(t) = 1 \qquad \lambda_1(t) = G(t) \qquad \lambda_2(t) = G^*(t) \qquad \lambda_3(t) = |G(t)|^2$$

depends on $J(\omega)$

Lorentzian spectral density (on resonance)

$$J(\omega) = \frac{\gamma_M \lambda^2}{2\pi[(\omega - \omega_c)^2 + \lambda^2]}$$

$G(t)$ **REAL**

$$\frac{d}{dt}\langle\alpha|(\mathbb{I} \otimes \Lambda_t)[P^+]|\alpha\rangle \leq 0 \iff \gamma(t) \geq 0 \iff \text{Markovianity}$$

SUMMARIZING....

P-divisibility →

Unital maps

$$\frac{d}{dt}s_k(t) \leq 0$$

Normal&Commutative maps

$$\frac{d}{dt}|\lambda_k(t)| \leq 0$$

Hermitian maps

$$\frac{d}{dt}\lambda_k(t) \leq 0$$

↓

Geometric interpretation

SUMMARIZING....

P-divisibility

Hermitian maps

$$\frac{d}{dt}\langle\alpha|(\mathbb{I}\otimes\Lambda_t)[P^+]|\alpha\rangle \leq 0$$

Commutative maps

$$\frac{d}{dt}\langle\alpha|(\mathbb{I}\otimes\Lambda_t)[P^+]|\alpha\rangle \leq 0$$

Geometric interpretation

A dynamical analogue of entanglement witnesses

Examples

From zero to infinity

2ND ARTIC SCHOOL ON OPEN QUANTUM SYSTEMS

Kevo Subartic Research Station – Kevo, 10-15 September 2017

http://arcticschool.aalto.fi
arcticschool@aalto.fi

Lecturers

Michel Brune (Laboratoire Kastler Brossel – Paris)
Michele Campisi (Scuola Normale Superiore – Pisa)
Dariusz Chruscinski (Nicolaus Copernicus University – Torun)
Elisabetta Paladino (University of Catania)
Sorin Paraoanu (Aalto University)
Matteo Paris (University of Milano)
Jyrki Piilo (University of Turku)
Kilian Singer (University of Kassel)

Local Organisers:
Rebecca Schmidt (University of Turku and Aalto University)
Sabrina Maniscalco (University of Turku and Aalto University)
Antti Kupiainen (University of Helsinki)
Paolo Muratore Gianneschi (University of Helsinki)
Jukka Pekola (Aalto University)

THANKS

QuProCS

European Commission

SCIENTIARUM SOCIETAS FENNICA 1838

Magnus Ehrnrooth foundation

ACADEMY OF FINLAND